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Well posedness of continuity equations and flows in metric measure spaces

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I will illustrate some recent work written in collaboration with a PhD student at SNS, Dario Trevisan. In this work we extend the DiPerna-Lions theory of well posedness for continuity and transport equations (at the PDE level) and of flows (at the ODE level) associated to Sobolev vector fields to a much more general setup, namely for metric measure structures. This requires a new point of view, not only because the notions of 'Sobolev vector field' and 'solution to the ODE' have to be properly understood, in this abstract context, but also because some estimates of the classical theory heavily rely on the Euclidean structure. Using tools and notions from Gamma-calculus we are able to develop a more intrinsic approach, based on the heat flow, which brings also a new light on the Euclidean theory.

The above mentioned topics are the object of research work, partly in progress, carried out within the ERC Advanced Grant project "Gemethnes".