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Sharpness of Rickman's Picard theorem

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One of the classical theorems in complex analysis is Picard's theorem stating that a non-constant entire holomorphic map from the complex plane to the Riemann sphere omits at most two points.

In the late 1960's and early 1970's, results of Reshetnyak and Martio-Rickman-Väisälä showed that mappings of bounded distortion, also called as quasiregular mappings, can be viewed as a counterpart for holomorphic mappings in quasiconformal geometry. One of the natural goals from the very beginning in this theory was obtain Picard-type results. In 1980, Rickman showed that a non-constant quasiregular mapping from the Euclidean n-space to the n-sphere omits only finitely many points, where the number depends only on the dimension and distortion. The sharpness of Rickman's theorem was not as simple issue as in the classical Picard theorem. In 1984, Rickman showed by a surprising and elaborate construction that given any finite set in the 3-sphere there exists a quasiregular from the Euclidean 3-space into the 3-sphere omitting exactly that set.

In this talk, I will discuss joint work with Pekka Pankka on the sharpness of Rickman's Picard theorem in all dimensions. Especially, I will discuss the role of bilipschitz geometry in the proof which leads to a stronger statement on the metric properties of the map and is a crucial ingredient in dimensions n > 3.