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Dimension distortion by Sobolev and quasiconformal mappings on the Heisenberg group

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I will describe recent joint work with Zoltan Balogh and Kevin Wildrick. We study the distortion of Hausdorff dimension of generic affine subspaces in the domain of a Sobolev mapping. Such conclusions generalize and extend classical results such as the ACL property of Sobolev mappings and the universal dimension distortion bounds enjoyed by quasiconformal mappings. In Euclidean domains we consider all affine subspaces parallel to a fixed linear subspace. When the domain is a Heisenberg group equipped with a sub-Riemannian metric, we naturally focus on cosets of horizontal or vertical subgroups. Some of our conclusions are consequences of a general theory of dimension distortion for generic fibers of well-behaved 'projection-type' mappings between metric spaces. However, in the most difficult (but most relevant) case of left coset horizontal foliations, our sharpest results come from a careful analysis of the geometric structure of the foliation, with proofs based on old techniques pioneered by Gehring and Mostow coupled with new geometric measure theory ingredients such as Mattila's projection and slicing machinery.