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Editorial

Evolving dynamical networks

1. Introduction

Networks of dynamical systems are common models for many problems in physics, engineering, chemistry, biology, and social sciences [1–3]. Recently, considerable attention has been devoted to algebraic, statistical, and graph theoretical properties of networks, and their relationship to network dynamics (see [1–4] and references therein). In particular, the interplay between network structure and synchronization has been extensively studied, as synchronization has been shown to play an important role in the function or dysfunction of a wide spectrum of technological and biological networks (see [5–14] for a sampling of this large field).

Notwithstanding the vast literature on networks of dynamical systems, the majority of the existing studies focus on static networks whose connectivity and coupling strengths are constant in time. However, in many realistic networks the coupling strength or the connection topology may vary in time, according to a dynamical rule, whether deterministic or stochastic. The recent efforts [15–43] are among the few to consider time-dependent couplings. Researchers are only now starting to investigate the connection between the evolving structure and the overall network dynamics. This is currently a hot research topic due to its potential in a variety of emerging applications.

The idea of organizing this special issue was inspired by a two-part mini-symposium of the same title that was held at the 2011 SIAM Conference on Applications of Dynamical Systems in Snowbird, Utah. This special issue contains a collection of research papers from a broad spectrum of topics related to modeling, analysis, and control of evolving dynamical networks. We hope that this collection will generate a significant interest among the mathematics, physics, and engineering audiences of the journal. Junior researchers might also find this collection useful as an inspiration to start graduate research in this exciting field of research.

This highly interdisciplinary special issue integrates new research contributions from different areas in applied mathematics, physics, neuroscience, and engineering, including stability and bifurcation theory, information and ergodic theory, averaging methods, and mathematical control theory. It can be roughly divided into three themes.

The first theme is that of *evolving neuronal networks and phase models*, including epileptic brain networks [44], networks of theta neurons with time-varying excitability [45], and Kuramoto models with slow adaptive coupling [46] and coupling plasticity [47].

The second theme is represented by the *general theory and applications of adaptive and switching networks*, including the use of: (i) causation entropy for identifying indirect influences in switching/blinking networks [48]; (ii) composite centrality to evaluate node and edge centrality in an evolving network [49]; (iii) moment-closure approximations for discrete adaptive networks, such as an adaptive voter model [50]; and (iv) a multiscale adaptive network model for the evolution of leadership in collective migration [51].

The third theme is related to *monitoring, control, and optimization in evolving engineering networks*. This includes: (i) the use of adaptive synchronization for detecting changes in the topology of a mobile robotic network [52]; (ii) graph optimization for a multi-agent leader–follower problem [53]; (iii) the consensus problem in a controlled network with a delay-dependent coupling that tunes its strength against the delay, to prevent instability [54]; and (iv) a class of time-varying port-Hamiltonian systems for studying problems at the intersection of statistical mechanics and control of physical systems [55].

In this brief survey, we introduce the subject and discuss current directions of research in dynamical networks, with emphasis on rigorous analysis. We propose a definition of an evolving network and present several important classes of dynamical networks with time-varying structures. We leverage our prior work in this area to offer salient examples of each theme addressed in this special issue and, most importantly, we draw the reader's attention to the papers contributing to this special issue by discussing their main results and challenges.

2. What is an evolving network?

Static network models lack two fundamental characteristics that are displayed by many complex systems: (i) the dynamic nature of the components and their interactions that embody a particular function and (ii) the possible evolution of the underlying network structure. Attempts have been made to complement static networks with these features [4,32,56]. Although in most cases, the dynamics of the network nodes and the evolution of the network structure have been considered separately. Unfortunately, for most systems this is not the case, with network topology, dynamics, and evolution all affecting one another. To overcome these limitations, *adaptive networks* and *evolving dynamical networks* have been proposed as more realistic models of complex systems [43,57].

In these classes of networks, the local dynamical process that takes place over the network structure is coupled to the evolutionary rules of the network itself. Therefore, the dynamics influence the evolution and vice versa. Thus, a wide range of dynamical processes have been considered, including those of ordinary differential equations, probabilistic mappings, and games on networks (see [92] and references therein). Here, we focus on evolving networks as defined in [43], which can be regarded as an attempt to merge the concept of dynamic graphs, first introduced in the pioneering work by Siljak [58], with that of complex adaptive systems, presented by Holland in the late seventies [59,60].

Following [43], we begin by considering a directed graph $\mathcal{G}_t = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ is a set of nodes and $\mathcal{E} = \{e_1,$

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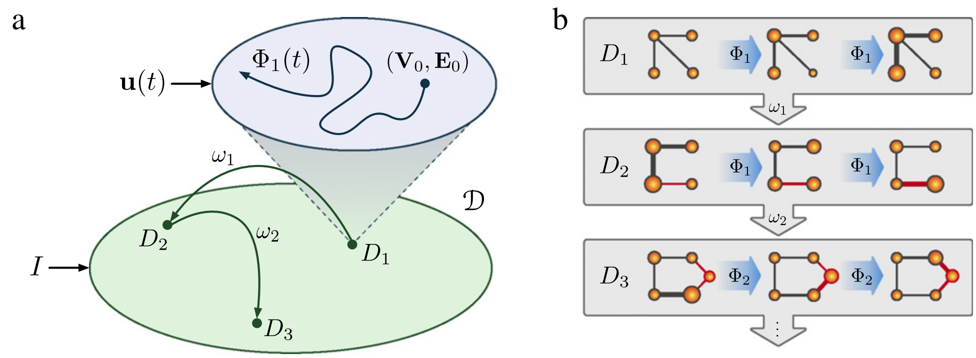

Fig. 1. (a) Schematic of the two levels of an evolving dynamical network. (b) Example of a specific EDN exhibiting dynamics and evolution. The set $\mathcal{D}$ represents all possible generalized dynamical graphs. Of those reached by the evolutionary process $D_1, D_2, D_3 \in \mathcal{D}$, each consists of a state space over which the network dynamics can take place. This is in accordance with the associated dynamical mappings, here denoted by Φ_1 and Φ_2. The evolution of the underlying network topology, dynamics mapping and state space occur through the application of the evolutionary operator τ with $\omega_1 = \tau(I, D_1)$ and $\omega_2 = \tau(I, D_2)$.
Source: The figure is reproduced from [43] with permission from *Complexity* [John Wiley & Sons].

Table 1
EDN models as multi-level complex systems: key elements.

	Neural networks	Communication networks	Ecological systems
$\mathcal{V}$	Neurons	Routers	Species/patches
$\mathcal{E}$	Synapses	Communication lines	Prey–predator relationships/dispersal
$\mathbf{V}$	Neural firing	Internal router state	Population size
$\mathbf{E}$	Synaptic strength	Capacity and usage	Predation/dispersal rates
$\mathbf{U}$	External stimuli	Central congestion control	External invasions
Ω	Addition/removal of neurons and synapses, synaptic plasticity	Addition/removal of routers and communication links, alteration of router dynamics and link capacities	Creation/extinction of species/patches and prey–predator relationships, changes to predation and dispersal rates
I	Developmental signals	User demands, network dynamics	Environmental conditions, population dynamics

$e_2, \ldots, e_m\}$ where $e_i \in \mathcal{V} \times \mathcal{V}$ is a set of directed edges. To enable dynamics to take place on this fixed network structure, a general state is associated with each node and edge. To simplify the notation, the current state of a node v_i or edge e_i is denoted by $\mathbf{v}_i \in \mathbf{V}_i$ or $\mathbf{e}_i \in \mathbf{E}_i$ respectively, with $\mathbf{V}_i$ and $\mathbf{E}_i$ being the sets of admissible state values. These states are typically functions of time, whereby $\mathbf{v}_i(t)$ and $\mathbf{e}_i(t)$ define the network dynamics. In general, we also consider a set of external control inputs $\mathbf{U} = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p\}$, where $\mathbf{u}_i \in \mathbf{U}_i$ can affect the node and edge state dynamics. Thus, the overall evolution is described by an appropriate evolution operator, say $\Phi : (\mathbf{V} \times \mathbf{E} \times \mathbf{U} \times \mathbf{T}) \to \mathbf{V} \times \mathbf{E}$, where $\mathbf{T}$ is the time interval. We define the collection $(\mathcal{G}, \mathcal{V}, \mathbf{E}, \mathbf{U}, \mathbf{T}, \Phi)$ as a *generalized dynamic graph* (GDG).

While GDGs allow for the description of many features of a complex system, they are still constrained by a fixed underlying topology $(\mathcal{V}, \mathcal{E})$ and form of dynamics Φ. To explore feasible alternatives, we consider the set of all possible generalized dynamic graphs $\mathcal{D}$, and we define a complex adaptive system over this set. We view actions taken by the complex adaptive system to embody evolution of the system occurring concurrently with the network dynamics (even if at different time scales).

Specifically, as illustrated in Fig. 1, we define an *evolving dynamical network* (EDN) as a collection $(\mathcal{D}, \Omega, I, \tau)$, where $\mathcal{D} = \{D_1, D_2, \ldots\}$ is the set of generalized dynamic graphs under consideration (each characterized by its structure, and dynamics on the nodes and edges); $\Omega = \{\omega_1, \omega_2, \ldots\}$ is the set of structural operators ω_i mapping a GDG in $\mathcal{D}$ to a different GDG in $\mathcal{D}$; I is the set of inputs from the environment or control inputs; and, following the concept of a complex adaptive system proposed in [59,60], $\tau : I \times \mathcal{D} \to \Omega$ is the evolutionary plan determining the operator $\omega_i \in \Omega$ to be applied when transitioning from one structure to the next.

According to this definition, an EDN can be seen as a multi-level complex system as shown in Fig. 1 where the intrinsic dynamics of the nodes and the evolution of the network structure are strongly intertwined, giving rise to complex, emerging behavior.

3. Evolving dynamical networks of neurons and phase oscillators

3.1. Example: neural networks

Numerous examples of complex systems can be modeled as an EDN (see Table 1) [43]. A notable example is that of neural networks in the brain which are known to adapt and evolve their structure as well as the strength of their synaptic connections to perform different functions [61]. An EDN model can describe all of these features in an integrated manner. In particular, referring to the framework above, such a neural network can be described as an EDN where $\mathcal{V}$ is the set of neurons; $\mathcal{E}$ is the set of synapses between neurons; $\mathbf{V}$ is the state space for neuron dynamics; $\mathbf{E}$ is the state space for synaptic weights; $\mathbf{U}$ is the set of external control inputs, that is, stimuli from the environment; $\mathbf{T}$ is the set of continuous times in $\mathbb{R}$; Φ is the mapping of both neuronal and synaptic weight dynamics; $\mathcal{D}$ is the set of possible neural network configurations; Ω is the set of operators describing the possible types of structural evolution of the neural network (for example, the addition or removal of neurons or synapses); I are inputs from the current dynamics of the neural network and external factors that might be present during development, such as chemical patterning; and τ is the evolutionary plan describing how the stochastic process of network growth and development occurs, including the coupling with the underlying dynamics and the external inputs through I.

The major theme of investigating the dynamics of neuronal and neuronal-like Kuramoto networks with time-varying coupling structure and/or strength is addressed by four papers in this special issue.

3.2. Contributions to the special issue

Lehnertz et al. [44] present methodologies for inferring brain networks from empirical time series, aiming at improving inference and characterization of human epileptic brain networks. They report on findings obtained for evolving epileptic networks on time scales ranging from a few seconds to days or even weeks. This effort provides evidence for properties of evolving brain networks during epileptic seizures. It describes the evolution of the network structure as it transitions from a random to a more regular topology and back. Notably, the authors find the highest resilience and least stability of the synchronous state when the more regular topology is active. These results may provide clues as to how seizures self-terminate and how to control epileptic networks.

So, Luke, and Barreto [45] study a large heterogeneous network of coupled theta neurons that interact globally, via pulse-like synapses, and whose excitability parameter varies in time. They demonstrate that such variations can lead to the emergence of macroscopic chaos, multi-stability, and final-state uncertainty in the collective behavior of the neuronal network. Analytical techniques are used to identify the asymptotic behavior of the macroscopic mean field dynamics of the network. Finite size network effects and rudimentary control via an accessible macroscopic network parameter are also investigated in this paper.

Skardal, Taylor, and Restrepo [46] analyze complex macroscopic behavior in a network of coupled phase oscillators, which arise when the coupling between oscillators slowly evolves as a function of either the macroscopic or the local system properties. They find macroscopic excitability and intermittent synchrony in a system of time-delayed Kuramoto oscillators with Hebbian and anti-Hebbian learning. Transitions between macroscopic incoherent and synchronized states in response to one or more slowly changing coupling strengths are studied in three networks of increasing complexity, including time-delayed oscillators, with adaptive network structure and community interaction.

Finally, Chandrasekar et al. [47] demonstrate the occurrence of multi-stable states in a system of phase oscillators, induced by coupling plasticity. The multi-stable state is comprised of a two-cluster synchronization state, where the clusters are in anti-phase with each other, and a desynchronization state. Through analysis and numerics, the authors demonstrate that the effect of the coupling time scale is not only to introduce a two-cluster state but also to change the number of oscillators in the two clusters.

4. Adaptive and switching networks

4.1. Example: stochastically blinking networks

Other important examples of EDNs are those in which the individual network nodes interact only sporadically via short on–off interactions. Packet switched networks and event-triggered distributed control strategies for multi-agent systems are relevant examples of such systems [62,63]. To describe the dynamics of realistic networks with intermittent connections, a class of evolving dynamical networks with fast on–off connections, called ''blinking'' networks, has been extensively studied [17–19,25–28,30,40,41].

These networks are composed of dynamical systems with connections that switch on and off randomly; switching is fast and occurs stochastically and independently for different time intervals. The rapidity of the switching process and its lack of time memory allow for a rigorous analysis of the problem, whereby the evolving graph is replaced by an average topology controlling the network dynamics [17–19,25–30,40,41].

The ''blinking'' network was originally introduced in [17] in the context of network synchronization. The network consists of N oscillators interconnected pairwise via a stochastic information network, whether directed [27] or undirected [17]:

$$\dot{x}_i(t) = f(x_i(t)) - \varepsilon \sum_{j=1}^{N} L_{ij}(t) h(x_j(t)), \quad i = 1, \dots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state of oscillator i, $f : \mathbb{R}^n \to \mathbb{R}^n$ describes the oscillators' individual dynamics, $h(x_j(t))$ models coupling between agents, $\varepsilon > 0$ is a control parameter that partially assigns the strength of coupling between oscillators, and scalars $L_{ij}(t)$ are the elements of the time-varying graph Laplacian $L(t)$.

The network topology, defined by the graph Laplacian $L(t)$, switches at a series of time instants $\{Tk | k \in \mathbb{Z}^+\}$, where T is a fixed period. The existence of an edge from vertex j to vertex $i \neq j$ is determined randomly and independently of other edges with a given probability. Thus, every switch in the network is operated independently, according to a similar probability law, and each switch opens and closes in different time intervals independently. During each time interval $[kT, (k+1)T)$, the communication network $G(t)$ is constant, and if all possible edges L_{ij} are allowed to switch on and off, the network equals the well known Erdös–Rényi graph [64]. The network model (1) allows for other specific network configurations, including small-world graphs [1] and scale-free graphs [2]. A similar concept to that of a blinking network is that of a system of so-called conspecific agents [65–67]. This concept has been introduced to describe information sharing in animal groups, by assuming that the cardinality of an agent's neighbor set and the weights assigned to its links are controlled by two jointly distributed random variables and the neighbors of an agent are selected with equal probability.

Many practical networks can be modeled as stochastically blinking networks [17,25–28,68]. Examples include cellular neural networks [17,18], multivehicle teams [69], epidemic spreading [19], mobile ad hoc networks [70], opinion dynamics [71], and power converters [72].

The network (1) has two characteristic time scales, namely the characteristic time of the individual dynamical system and the characteristic time scale of the stochastic switching. If the stochastic switching is fast enough [17,18,25–30,40,41], the stochastically blinking system (1) behaves like an averaged system where the dynamics are controlled by the expectation of the stochastic variables. This corresponds to replacing the stochastic time-varying $L(t)$ with its average $E[L]$, corresponding to a static averaged network (see Fig. 2).

The relationship between the dynamics of the stochastically blinking network and its average is a non-trivial problem and many efforts have focused on investigating this issue [17,18,25–28,30,40,41]. For example, in [17,25,28] it was proven that switching networks (1) can synchronize even if the network is insufficiently coupled to support synchronization at every instant of time.

Four papers in this special issue represent this second theme of adaptive and switching networks.

4.2. Contributions to the special issue

Sun and Bollt [48] study information transfer in the dynamics of small-scale coupled oscillator networks, including switching/blinking networks. Several examples are used to show that the causal relationships, inferred from the transfer entropy, are often misleading when the underlying system contains indirect connections, dominance of neighboring dynamics, or anticipatory couplings. To account for these effects, a measure called causation entropy is introduced and it is demonstrated that its application can lead to reliable identification of true couplings and indirect influences, even in an evolving network.

Motivated by the observation of the lack of a unified framework for describing evolving complex networks in general, Joseph and Chen [49] present the concept of composite centrality for evaluating node and edge centralities, based on a set of several graph measures. The composite centrality measure for general weighted and directed complex networks is based on measure standardization and invariant statistical inheritance schemes. Two real-world cases of the world trade web and the world migration web, both during a time span of 40 years, are used to demonstrate the remarkable normative power and accuracy of the proposed measure.

Demirel et al. [50] investigate the performance of moment-closure approximations for discrete adaptive networks. The adap-

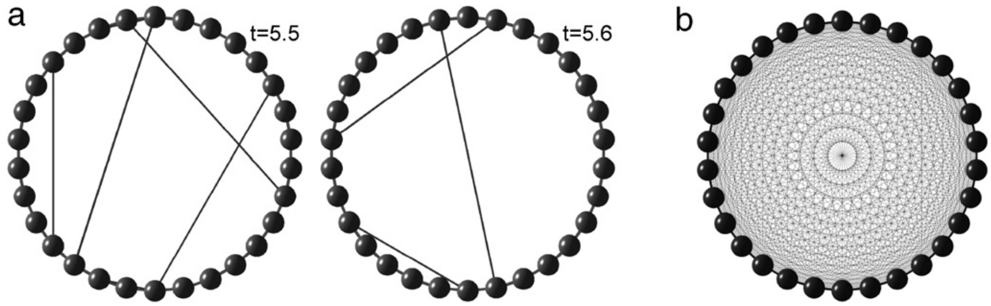

Fig. 2. (a) Example of a stochastic network with local static connections and blinking shortcuts [17]. The probability of switching is $p = 0.01$. (b) The averaged network; blinking connections of strength ε are replaced with static all-to-all connections of strength $p\varepsilon$.

tive voter model is used as a benchmark model to assess different approaches. Comparisons with agent-based simulations reveal that both homogeneous and heterogeneous moment-closure approximations capture qualitative properties of the fragmentation transition, but fail to provide good quantitative estimates close to the fragmentation point. Remarkably, even very sophisticated heterogeneous approaches can produce results that are less good than those from simple homogeneous schemes. This paper concludes that even the evolution of problematic models can be captured if an expansion is used that is specifically tailored to the system at hand.

Pais and Leonard [51] study the evolution of leadership in migratory populations, which depends not only on costs and benefits of leadership investments but also on the opportunities for individuals to rely on cues from others through social interactions. This work presents an analytically tractable adaptive dynamical network model of collective migration with fast time scale migration dynamics and slow time scale adaptive dynamics of individual leadership investment and social interaction. The analysis shows how the topology of the underlying social interaction network influences the emergence and location of leaders in the adaptive system.

5. Monitoring, control, and optimization in evolving networks

5.1. Example: pinning control of networks

Pinning control problems entail the design of control laws for taming the dynamics of a complex network of nonlinear systems onto a common desired reference trajectory by exerting control actions on a relatively small fraction of network nodes [73–86]. The reference trajectory is commonly generated by a reference system, that acts as a master for the rest of the network. The effects of the control inputs applied at the so-called pinned sites are spread to the rest of the nodes in the network through their mutual coupling.

Notably, several efforts have demonstrated that pinning control efficiency can benefit from the adaptation of the edge dynamics and evolution of the network structure [34,73,74,87–91,93]. For example, in [73,74], a simple control strategy termed node-to-node pinning control is presented, to reduce the number of pinning sites and optimize the pinning effectiveness. This approach consists in randomly pinning all the network nodes at a fast switching rate and at a uniform gain level. Under fast switching conditions, the node-to-node pinning scheme is virtually equivalent to simultaneously pinning all the network nodes with a homogeneous gain level. An alternative strategy is proposed in [91], where the structure of the connections between the master and the nodes, that will be controlled, self-evolves in time through so-called "edge-snapping". The broader problem of how to control a network of dynamical systems via real-time evolution of its structure remains a pressing open challenge.

The third theme of monitoring, control, and optimization in evolving engineering networks is represented by four papers in this special issue.

5.2. Contributions to the special issue

Bezzo et al. [52] present a decentralized framework for tracking variations of the network topology for a set of coupled mobile agents. The network may evolve in time due to both the relative motion of the mobile robots and unknown environmental conditions, such as the presence of obstacles in the environment. Each robotic agent is equipped with a chaotic oscillator whose state is propagated to the other robots through wireless communication, with the goal of synchronizing the oscillators. This paper proposes an adaptive technique based on synchronization of chaotic oscillators which, by obtaining limited information about the local network connectivity, is shown to be effective in synchronizing the oscillators and providing information on the local connectivity of the oscillators.

Shi et al. [53] consider an informed-agents selection problem for tracking control of first-order multi-agent systems. In this framework, n follower nodes are tasked with tracking a static leader, and only k of them can be connected to it. These followers that can communicate with the leader are called informed agents. The weights of the arcs are normalized, so the optimal choice of the selected informed agents leads to a structure optimization problem. The paper shows that the optimal selection of the k informed followers can be approximately obtained by minimizing this maximal distance, which corresponds to a metric k-center problem in combinatorial optimization.

Qiao and Sipahi [54] propose a delay-dependent coupling design on a multi-agent consensus system with homogeneous inter-agent delays. The coupling among the agents is designed as an explicit parameter of the delays, allowing couplings to autonomously adapt on the basis of the delay value. This process seeks to guarantee stability and a certain degree of robustness in the network despite the destabilizing effect of the delays. Design procedures, as well as analysis on the speed of consensus, are presented in connection with Laplacian eigenvalues.

Delvenne and Sandberg [55] identify a class of time-varying port-Hamiltonian systems that are able to modify their internal structure and their interconnection with the environment in time. This work shows how to use linear control theory to optimally extract work from a single heat source over a finite time interval. The optimal controller is a time-varying port-Hamiltonian system, which can be physically implemented as a variable linear capacitor and transformer. This paper provides a unique example for the feasibility of integrating established control and information theoretical tools, such as Kalman filtering, port-Hamiltonian theory, and passivity theory, with statistical physics, to explore the fundamental limits of work extraction, actuation, measurement, or computation.

6. Open questions and future challenges

The area of evolving dynamical networks is ripe with open problems and challenges, and we hope that the collection of papers included in this special issue will encourage and motivate junior readers to enter this exciting field of research. In addition to the open questions and problems highlighted in the papers included in the issue, we list some fundamental questions on the analysis and control of evolving networks, which we believe are central to this research domain.

An important research problem is to develop a rigorous theory for understanding and controlling the evolution of dynamical networks *beyond fast switching*. In fact, many practical switching networks, whether continuous-time or discrete-time, operate at lower frequencies than those required for implementing fast switching arguments. Synchronization of computer clocks is a representative example, whereby the clocks that generate the local time for the computer need to be synchronized throughout the network [17]. In addition, the independence of the switching events leveraged in fast switching arguments is sometimes difficult to guarantee and efforts should be devoted to studying adaptive dynamical networks where switching is a Markov process, instead of sequences of independent random variables, that could depend on the state of the dynamical systems.

A crucial challenge in evolving dynamical networks is not only to understand the interplay between the evolution of the network structure and the overall network dynamics but also to exploit these mechanisms for synchronization, pattern formation, and control. In fact, in many natural networks, agents are observed to rewire or form new interconnections in order to perform a certain function, such as escaping from a predator in the case of biological groups. With respect to such real-world networks, a central problem is the identification of the critical feedback mechanisms linking the adaptation and dynamics of real-world networks and often preserving synchronization performance when parts of the individual systems or links are destroyed. In engineering design, a key open problem is to devise distributed feedback strategies for network evolution to support the achievement of prescribed global objectives. Such strategies can be based on decentralized rewiring to form new connections that are based on local measurements or state observations. This evolution could in principle regulate the controllability and observability of the network and, as a result, afford pinning control schemes that would not be possible otherwise. As suggested in [91,35], pinning control strategies can be synthesized to allow for the master node to self-select the number and location of the nodes to control in order to achieve synchronization onto a reference trajectory or even induce the formation of select community structures. In the context of switching networks, a very relevant question entails the optimization of the switching frequency and the design of rewiring strategies for desired performance objectives.

In general, solving these problems requires overcoming a number of technically challenging issues and even developing entirely new analytical methods. More importantly, it demands a truly interdisciplinary approach that integrates tools and techniques from different disciplines, ranging from dynamical systems and control to statistical physics, biology, and sociology. We hope that this special issue will contribute to further igniting interest in this area and promoting interdisciplinary collaborations.

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