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WHAT WAS EINSTEIN'S "FATEFUL PREJUDICE"?

In the later pages of the notebook, as Einstein let general covariance slip away, he devised and abandoned a new proposal for his gravitational field equations. This same proposal, revived nearly three years later, opened passage to his final theory. In abandoning it in the notebook, Einstein had all but lost his last chance of deliverance. This chapter reports and develops our group's accounts of this decision. Einstein's later accounts of this decision blame it upon what he called the "fateful prejudice" of misinterpreting the Christoffel symbols. We suggest that Einstein's aberrant use and understanding of coordinate systems and coordinate conditions was as important as another fateful prejudice.

INTRODUCTION

Under a decade of analysis, discussion and reflection, Einstein's Zurich notebook has yielded. Strategies that were once enigmatic and pages that were once obscure have become familiar. For the great part, we understand the problems Einstein approached, how he sought to solve them, when these efforts succeeded, when they failed and even the hesitations behind the smallest markings. In other parts we may follow a calculation line by line but our view of his hopes and plans remain distant. Or he may abandon a calculation with just a few symbols surviving on the page. They can be deciphered only through luck or clairvoyance.

The boundary that fences in the clear from the obscure has grown so that less and less escapes it. The intriguing puzzles of the notebook remain at this boundary. They cannot be solved with the assurance that the weight of evidence admits no alternative. But they are not so distant that we must despair of any solution. We understand just enough of these puzzles to sense that a complete solution lies within our grasp. We may even articulate one or more candidates that are both plausible and attractive. Yet the evidence we cull from the notebook and elsewhere remains sufficient to encourage us, but insufficient to enable a final decision.

My purpose in this chapter is to review two of these problems. I will draw heavily on ideas that have circulated freely in our group and have grown, mutated and contracted as they passed between us.[1] This chapter will report on these communal ideas,

1 I gratefully acknowledge thoughtful discussion and responses on this chapter and its proposals from the members of this group (who are also co-authors in this volume) and also from Jeroen van Dongen.

Jürgen Renn (ed.). *The Genesis of General Relativity*
Vol. 2: *General Relativity in the Making*: *Einstein's Zurich Notebook*. *Part II*.

while it gives my own particular viewpoint on them and adds a conjecture. Many of the ideas in this chapter are not reflected in our joint commentary because my viewpoints and conjecture represent a minority opinion. At the boundary, where categorical evidence is elusive, our intuitions and sensibilities decide. They differ as we pass through the group. We do not all know the same Einstein.

Two Puzzles

The problems meet on page 22R of the notebook. There we find Einstein generating the very same gravitational field equations of near general covariance that will reappear briefly in his publication of November 4, 1915, when he ruefully returned to general covariance. This supplies our first puzzle:

- Why were these field equations rejected in the notebook, when they were deemed admissible in November, 1915?

These equations did not employ the Ricci tensor as gravitation tensor, as would the source free field equations of Einstein's final theory. Famously, Einstein and Grossmann had mentioned but discarded this possibility in their joint "Entwurf" paper. The equations on page 22R employ a different gravitation tensor, which we have come to call informally the "November tensor." It was carefully and apparently successfully contrived to avoid exactly the problems they imagined for the Ricci tensor.

The calculations on page 22R differ in no essential way from those Einstein would publish in 1915. The calculations on the surrounding pages do differ. The absolute differential calculus makes it easy to write down expressions that are generally covariant; they hold in all coordinate systems. In the modern literature we routinely restrict these expressions to specialized coordinate systems by imposing freely chosen coordinate conditions. As Einstein's calculations in the notebook progressed, he became quite adept at the purely mathematical aspects of applying these conditions. Careful analysis of the pages show that his use of these conditions came to differ considerably from the modern usage and possibly with fatal consequences. Our second puzzle is to understand these differences:

- Did Einstein *choose* to use coordinate conditions in an idiosyncratic way later in the notebook? Or was he unaware of the modern usage?

In solving these puzzles, more is at stake than merely deciphering a few pages of a notebook that may not have long occupied Einstein. These pages mark Einstein's all but last chance to rescue himself from the misconceptions that led him to his "Entwurf" theory and to more than two years of distress as his greatest discovery eluded him. A solution to these puzzles will tell us if Einstein's final slide into the abyss rested on simple blunders, lack of imagination or creative misunderstandings that have yet to be appreciated in the historical literature.

Four Parts

This chapter is divided into four parts. In the first, I will review the circumstances that induced Einstein to the proposal of the "November tensor" T^{x}_{il} as gravitation tensor. Its rejection in the notebook will be explained partially by drawing on a proposal of Jürgen Renn's. At the time of the notebook, as Einstein later recalled, he failed to see that the Christoffel symbols were the natural expression for the components of the gravitational field, his "fateful prejudice." As a result, he was unable to see how to recover a stress-energy tensor for the gravitational field and the associated conservation laws from the "November tensor." The calculation just proved too complicated. This problem was resolved in November 1915 when Einstein had developed more powerful mathematical methods.

The second part outlines the puzzle surrounding Einstein's use of coordinates. I will distinguish the standard way in which coordinate conditions are used from the way that Einstein came to use them later in the notebook. It is so different that our group labels coordinate conditions used this way as "coordinate restrictions." This non-standard use of coordinate restrictions can aid us in explaining the notebook rejection of the "November tensor," if in addition we assume that Einstein was unaware that the same mathematical manipulations could be used in the modern manner as coordinate conditions. The evidence available to us admits no final decision over Einstein's awareness of this usage. I will suggest however that there are sufficient indications to make his supposed lack of awareness implausible and that page 22R of the notebook might well mark a transition from the use of coordinate conditions to coordinate restrictions.

The third part develops a conjecture on what might lie behind Einstein's idiosyncratic use of coordinate conditions in the notebook. In his later hole argument, Einstein erred in tacitly according an independent reality to coordinate systems. It is now speculated he may have committed this same error within the notebook while using coordinate conditions to extract the Newtonian limit from the "November tensor." The outcome would be that his theory overall would seem to gain no added covariance from the use of coordinate conditions rather than coordinate restrictions, to which Einstein reverted for their greater simplicity. Once again, the available evidence admits no final decision. I will suggest however that the conjecture is plausible since it requires us to suppose no additional errors by Einstein; he merely needs to follow through consistently on the misapprehensions we know he harbored in the context of the hole argument.

The fourth part offers a summary conclusion.

1. THE PUZZLE OF THE GRAVITATION TENSORS

Why did Einstein abandon the gravitational field equations in the notebook on page 22R that he later deemed suitable for publication on November 4, 1915? This is our first puzzle. In the first section of this part I will review the essential background to this puzzle. In the pages preceding page 22R, Einstein considered and rejected the natural candidate for a gravitation tensor, the Ricci tensor. It fell to misconceptions about static fields and the form of gravitational field equations in the case of weak fields. In the second section of this part I will describe how the proposal of page 22R was contrived ingeniously to circumvent the illusory flaws he had imagined for the Ricci tensor. In the third section I will review Einstein's later recollections concerning the notebook rejection and the central role that, as I shall call it, "{} prejudice" played in them. Drawing on a proposal by Jürgen Renn, I will advance what I believe is a plausible account of its significance. The difficulty was the recovery of an expression for the stress-energy tensor of the gravitational field and its associated conservation law. Because Einstein did not recognize that the Christoffel symbols are the natural structure for representing the components of the gravitational field, he thought this recovery required the algebraic expansion of the Christoffel symbols. That yielded such a surfeit of terms that Einstein despaired of completing the calculation. This difficulty, along with others to be reviewed in later parts of this chapter, led Einstein to abandon the proposed gravitation tensor. In 1914, in the course of his work on the "Entwurf" theory, Einstein developed more powerful variational methods. These enabled him to complete the calculation and to see the significance of the Christoffel symbols.

1.1 Background: The Rejection of the Ricci Tensor

The "Entwurf" Papers

In the "Entwurf" paper, Einstein and Grossmann famously report their failure to find generally covariant gravitational field equations. Their search had focused on finding a gravitation tensor, $G_{\mu\nu}$, constructed from the metric tensor and its derivatives, to be used in the gravitational field equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{1}$$

where $T_{\mu\nu}$ is the stress-energy tensor and κ is a constant. The absolute differential calculus of Ricci and Levi-Civita supplied the natural structure from which generally covariant gravitation tensors can readily be constructed. It is the Riemann tensor, written in Einstein's paper of November 4, 1915 (Einstein 1915a) as[2]

$$(ik,lm) = \frac{\partial}{\partial x_m}\begin{Bmatrix} il \\ k \end{Bmatrix} - \frac{\partial}{\partial x_1}\begin{Bmatrix} im \\ k \end{Bmatrix} + \sum_{\rho}\begin{Bmatrix} il \\ \rho \end{Bmatrix}\begin{Bmatrix} \rho m \\ k \end{Bmatrix} - \begin{Bmatrix} im \\ k \end{Bmatrix}\begin{Bmatrix} \rho l \\ k \end{Bmatrix}, \tag{2}$$

where the Christoffel symbols of the second kind are

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2 My policy with notation will be to follow the conventions used at the time of the work discussed. In November 1915, Einstein indicated contravariant and covariant components of a tensor by raised and lowered indices. Summation over repeated indices was *not* implied. The notation for the Riemann tensor and Christoffel symbols do not respect this raising and lowering convention.

$$\begin{Bmatrix} il \\ k \end{Bmatrix} = \frac{1}{2}\sum_r g^{kr}\begin{bmatrix} il \\ r \end{bmatrix} = \frac{1}{2}\sum_r g^{kr}\left(\frac{\partial g_{ir}}{\partial x_k} + \frac{\partial g_{rl}}{\partial x_k} - \frac{\partial g_{il}}{\partial x_r}\right).$$

(The term $[il/r]$ is the Christoffel symbol of the first kind and is defined implicitly in this expression.) The Ricci tensor is the first nontrivial contraction, unique up to sign, of the Riemann tensor, written by Einstein as

$$G_{im} = \sum_{kl}(ik,lm). \tag{3}$$

Einstein later chose this tensor as the gravitation tensor in the source free case.

Einstein and Grossmann had revealed that they had considered this candidate for the gravitation tensor in preparing the "Entwurf" paper. They explained (Einstein and Grossmann 1913, 256–57), in Grossmann's words, "...it turns out that this tensor does *not* reduce to the [Newtonian] expression $\Delta\phi$ in the special case of an infinitely weak, static gravitational field." Einstein and Grossmann's explanation proved all too brief. It did not even mention the use of the coordinate conditions that are expected by the modern reader and that must be stipulated to restrict the coordinate systems of a generally covariant theory to those coordinate systems in which Newton's equations can hold. This omission even led to the supposition in the early history of this episode that Einstein was unaware of his freedom to apply these coordinate conditions.

With its earliest analyses,[3] we learned from the Zurich notebook that Einstein understood all too well how to reduce generally covariant gravitational field equations to a Newtonian form by restricting the coordinate systems under consideration. In particular, he knew how to select what we now call "harmonic coordinates" to reduce the Ricci tensor to an expression analogous to the Newtonian $\Delta\phi$. With deeper analysis as developed in our commentary, the notebook provides a detailed account of how Einstein tested the Ricci tensor against his other expectations and how he was led to reject it.

Two Misconceptions: The Static Field…

What precluded acceptance of the Ricci tensor as the gravitation tensor were two interrelated expectations that proved to be incompatible with Einstein's final theory. On the basis of several apparently sound arguments, Einstein expected that static gravitational fields would be represented by a spatially flat metric, whose coefficients in a suitable coordinate system would be

3 See (Norton 1984) and also (Stachel 1980).

$$g^{STAT}_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & c^2(x,y,z) \end{bmatrix}, \tag{4}$$

where the g^{STAT}_{44} component $c^2(x,y,z)$ is some function of the three spatial coordinates $(x,y,z) = (x_1,x_2,x_3)$. The spatial flatness is represented by the constant value -1 for the other non-zero components, g^{STAT}_{11}, g^{STAT}_{22}, and g^{STAT}_{33}. This spatial flatness is not realized in general in the final theory.

We can understand exactly why Einstein would fail to anticipate this lack of spatial flatness. His explorations were based on the principle of equivalence, which asserted that a transformation to uniform acceleration in a Minkowski spacetime yielded a homogenous gravitational field (see Norton 1985). The Minkowski metric in standard coordinates is given by

$$g^{SR}_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & c^2 \end{bmatrix} \tag{5}$$

for c now a constant interpreted as the speed a light. If one transforms to a coordinate system in uniform acceleration, the metric reverts to a form Einstein associated with a homogeneous gravitational field, $g^{HG}_{\mu\nu}$, which has the form of $g^{STAT}_{\mu\nu}$, but in which c is a linear function of the spatial coordinates, x_1, x_2, x_3. If the acceleration is in the direction of the $x_1 = x$ coordinate, for example, then $c = c_0 + ax$, for c_0 and a arbitrary constants whose values are set by the particulars of the transformation, so that

$$g^{HG}_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & (c_0 + ax)^2 \end{bmatrix} \tag{6}$$

Einstein's early strategy in his work on gravitation had been to recover the properties of arbitrary gravitational field by judiciously generalizing those of $g^{HG}_{\mu\nu}$. His mistake, in 1912 and 1913, was to fail to anticipate that the spatial flatness of $g^{HG}_{\mu\nu}$, was not a property of all static fields, but a very special peculiarity of $g^{HG}_{\mu\nu}$.

…and the Field Equations for Weak Fields

Einstein's second expectation concerned how the gravitational field equations (1) would reduce to the Newtonian limit. In the weak field case, one supposes that one can find coordinate systems in which the metric adopts the form

$$g_{\mu\nu} = g^{SR}_{\mu\nu} + \varepsilon_{\mu\nu} \tag{7}$$

The quantities $\varepsilon_{\mu\nu}$ are of first order of smallness. For this weak field, Einstein supposed that the gravitation tensor of (1) would reduce to[4]

$$\Gamma_{\mu\nu} = \sum_{\alpha\beta} \frac{\partial}{\partial x_\alpha}\left(\gamma_{\alpha\beta}\frac{\partial \gamma_{\mu\nu}}{\partial x_\beta}\right) + \left(\begin{array}{c}\text{terms of}\\ \text{second order}\\ \text{and higher}\end{array}\right). \tag{8}$$

If the gravitation tensor reduced to this form in the weak field, then all that would remain to first order is the first term of (8), so that the gravitational field equations would reduce to the near-Newtonian expression

$$\sum_{\alpha\beta} \gamma^{SR}_{\alpha\beta}\frac{\partial^2}{\partial x_\alpha \partial x_\beta} g_{\mu\nu} = \left(\frac{1}{c^2}\left(\frac{\partial}{\partial x_4}\right)^2 - \left(\frac{\partial}{\partial x_1}\right)^2 - \left(\frac{\partial}{\partial x_2}\right)^2 - \left(\frac{\partial}{\partial x_3}\right)^2\right) g_{\mu\nu} = \kappa T_{\mu\nu} \tag{9}$$

or more simply expressed

$$\Box g_{\mu\nu} = \kappa T_{\mu\nu}.$$

It turns out that these most natural of intermediates in the transition to Newton's law of gravitation are not realized by the final theory. In it, the weak field equations corresponding to (9) include an extra trace term. See (Einstein 1922, 87).

$$\Box g_{\mu\nu} = \kappa(T_{\mu\nu} - (1/2) g_{\mu\nu} T). \tag{9'}$$

These two expectations concerning the static field and weak field are closely connected. In particular, as Einstein showed in Einstein (Einstein 1913, 1259), one recovers a spatially flat static metric $g^{STAT}_{\mu\nu}$ if one solves the weak field equation (9) for the case a of a time independent field produced by a static, pressureless dust cloud.[5] This recovery of a spatially flat solution is blocked by the added trace term T in (9') in the final theory.

4 I revert to the notation of (Einstein and Grossmann 1913). Summation is *not* implied by repeated indices. All indices are written as subscript with the covariant and contravariant forms of a tensor represented by Latin and Greek letters respectively. Thus the modern $g_{\mu\nu}$ is written as $g_{\mu\nu}$, but the modern $g^{\mu\nu}$ is written as $\gamma_{\mu\nu}$. Coordinate indices are written as subscript as well.

5 The prediction of spatial flatness is almost immediate. The stress energy tensor $T_{\mu\nu}$ for this static dust cloud will satisfy $T_{\mu\nu} = 0$ excepting T_{44}. Thus we have immediately for all values of μ, ν excepting $4,4$, that $\Delta g_{\mu\nu} = 0$ for all spacetime. With finite values at spatial infinity as a boundary condition, these last equations solve to yield $g_{\mu\nu} = \text{constant}$ for all μ, ν excepting $4,4$, as required by $g^{STAT}_{\mu\nu}$ of (4).

That Prove Fatal

On page 19L of the notebook, Einstein showed that he knew how to reduce the Ricci tensor to the weak field form required by (9). Using a standard device in the literature, he simply restricted his coordinate systems to those in which the harmonic condition

$$\sum_{\kappa l} \gamma_{\kappa l} \begin{bmatrix} \kappa l \\ i \end{bmatrix} = 0 \tag{10}$$

is satisfied. He immediately found that he could eliminate all the second derivative terms that were not required by the operator (8) for the Newtonian limit. Disaster ensued over the pages 19R–21R for this promising combination of Ricci tensor as gravitation tensor and harmonic coordinate systems. Einstein sought to bring this combination into accord with his expectations (4) for static fields and for the weak field equations (9). He failed and inevitably so. The coordinate systems used to bring the static field into the form of $g^{STAT}_{\mu\nu}$ in (4) are not harmonic. That coordinate system does, however, satisfy a formally similar coordinate condition

$$\sum_{\kappa} \frac{\partial \gamma_{\kappa\alpha}}{\partial x_{\kappa}} = 0. \tag{11}$$

(We call this "Hertz condition" in this volume since it is mentioned by Einstein in a letter to Paul Hertz of August 22, 1915 (CPAE 8, Doc. 111).) What makes this condition attractive is that it entails the weak field form of the energy momentum conservation law[6]

$$\sum_{\nu} \frac{\partial \Theta_{\kappa\alpha}}{\partial x_{\nu}} = 0. \tag{12}$$

Einstein even realized that he could retain this form of the energy conservation law and the harmonic condition if he added the trace term in T in (9'), but the modified field equations were no longer compatible with his expectations for the weak static field $g^{STAT}{}_{\mu\nu}$, so they could not stand. Harmonic coordinate systems no longer appear in the notebook.

1.2 The "November Tensor"

The outcome of Einstein's investigations of the Ricci tensor was disappointing. But his creative energies were far from spent. He then turned immediately to another pro-

6 $\Theta_{\mu\nu}$ is the contravariant form of the stress-energy tensor $T_{\mu\nu}$. The condition (11) combined with the field equation (9) yields the weak field form of the energy conservation law through

$$\kappa \sum_{\nu} \frac{\partial \Theta_{\mu\nu}}{\partial x_{\nu}} = \sum_{\nu} \frac{\partial}{\partial x_{\nu}} \left(\sum_{\alpha\beta} \gamma_{\alpha\beta}{}^{SR} \frac{\partial^2}{\partial x_{\alpha} \partial x_{\beta}} \gamma_{\mu\nu} \right) = \sum_{\nu\alpha\beta} \gamma_{\alpha\beta}{}^{SR} \frac{\partial^2}{\partial x_{\alpha} \partial x_{\beta}} \left(\frac{\partial \gamma_{\mu\nu}}{\partial x_{\nu}} \right) = 0.$$

posal for a gravitation tensor, the one he would publish on November 4, 1915, upon his return to general covariance. It is laid out on page 22R of the notebook. Einstein shows how it is possible to split off a part of the Ricci tensor that is not a generally covariant tensor, but at least transforms tensorially under unimodular transformations.

Unimodular Transformations

The class of unimodular transformations has a simple defining property. A coordinate transformation $x_\alpha \to x'_\beta$ is fully specified by the associated matrix of differential coefficients $\partial x'_\beta / \partial x_\alpha$. A transformation is unimodular if the determinant of this matrix is unity:

$$\mathrm{Det}\left(\frac{\partial x'_\beta}{\partial x_\alpha}\right) = 1. \tag{13}$$

Unimodular transformations preclude transformations that uniformly expand the coordinate system, such as $x'_\alpha = 2x_\alpha$. They are volume preserving in spacetime.[7]

The coefficients of the metric tensor transform according to

$$g'_{\mu\nu} = \left(\frac{\partial x_\alpha}{\partial x'_\mu}\right)\left(\frac{\partial x_\beta}{\partial x'_\nu}\right) g_{\alpha\beta}.$$

Taking the determinants of these quantities we find that the (positive valued)[8] determinant G transforms according to

$$\sqrt{G'} = \mathrm{Det}\left(\frac{\partial x_\alpha}{\partial x'_\mu}\right)\sqrt{G}.$$

It now follows immediately that $\sqrt{G'} = \sqrt{G}$ for unimodular transformations, that is, when (13) holds. This equality tells us that $\sqrt{G}$ transforms as a scalar under unimodular transformation, as do functions of it such as $\log\sqrt{G}$. We can easily form unimodular vectors from this quantity. The coordinate derivative $\partial\phi/\partial x_i$ of a generally covariant scalar ϕ is a generally covariant vector. Similarly, the coordinate derivative of a unimodular scalar is a unimodular vector. Therefore $\dfrac{\partial \log\sqrt{G}}{\partial x_i}$ is a unimodular vector. This result is the key to Einstein's plan.

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7 They are volume preserving in the coordinate space. A volume element $dx_1 dx_2 dx_3 dx_4$ for a region bounded by the four coordinate differentials dx_α in coordinate space is preserved since it transforms according to the rule $dx'_1 dx'_2 dx'_3 dx'_4 = \mathrm{Det}\,(\partial x'_\beta / \partial x'_\alpha) dx_1 dx_2 dx_3 dx_4$. The invariant volume element of a metrical spacetime, $\sqrt{G} dx_1 dx_2 dx_3 dx_4$ is also preserved since $\sqrt{G}$ is an invariant under unimodular transformation.

8 In other places, it is written as $-g$.

Proposal: A Unimodular Tensor....

On page 22R of the notebook, Einstein took the Ricci tensor T_{il} and expressed it as the sum of two parts. He wrote

$$T_{il} = \underbrace{\left(\frac{\partial T_i}{\partial x_l} - \sum \begin{Bmatrix} il \\ \lambda \end{Bmatrix} T_\lambda\right)}_{\text{tensor 2nd rank}} - \underbrace{\sum_{\kappa l} \left(\frac{\partial \begin{Bmatrix} il \\ \kappa \end{Bmatrix}}{\partial x_\kappa} - \begin{Bmatrix} i\kappa \\ \lambda \end{Bmatrix} \begin{Bmatrix} l\lambda \\ \kappa \end{Bmatrix} \right)}_{\text{presumed gravitation tensor } T^x_{il}.} \tag{14}$$

His purpose is quite clear. And if there were any doubt, the proposal is explained in detail in (Einstein 1915a). The first term of T_{il} is a just the covariant derivative of the unimodular vector

$$T_i = \frac{\partial \log \sqrt{G}}{\partial x_i}$$

and therefore a tensor under unimodular transformations. Since the Ricci tensor T_{il} transforms as a tensor under all transformations, Einstein could infer that the second term of (14) must also transform as a tensor under unimodular transformations.[9] This second term, denoted as T^x_{il}, is chosen by Einstein as a candidate gravitation tensor. Because of its reappearance in November 1915, we have labeled it the “November tensor” in this volume. Its selection is compatible with Einstein’s ambitions for extending the principle of relativity to acceleration. While not supplying general covariance, covariance under unimodular transformations is sufficiently expansive to capture transformations between inertial and accelerated coordinate systems. As Einstein shows in (Einstein 1915a, 786), these acceleration transformations include ones that set the spatial coordinate axes into rotation as well as ones that accelerate its spatial origin without rotation.[10]

…that Gives the Newtonian Limit and Energy Conservation

The remainder of the page explains why Einstein was attracted to this new candidate. He had been unable to reduce the entire Ricci tensor to the form (8) without employing a coordinate condition, the harmonic condition, that brought fatal problems. Einstein now showed that he could reduce the tensor T^x_{il} to the form (8) if he considered coordinate systems which satisfied the coordinate condition (11) introduced above.

As Einstein proceeded to show, with the assumption of this condition, the candidate gravitation tensor T^x_{il} reduced to

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9 The result is automatic. The quantity T^x_{il} can be expressed as a difference of two quantities, each of which are tensors at least under unimodular transformations.

$$-2T^x_{il} = \sum_{\alpha\beta} \gamma_{\alpha\beta} \frac{\partial^2 g_{il}}{\partial x_\alpha \partial x_\beta} + \left(\text{terms quadratic in } \frac{\partial g_{\mu\nu}}{\partial x_\alpha}\right). \tag{15}$$

In the weak field of (7), the terms quadratic in

$$\frac{\partial g_{\mu\nu}}{\partial x_\alpha}$$

will all be of order ε^2 and thus of the second order of smallness; the first term of (15) agrees with the first term of (8) in quantities of first order. The candidate tensor T^x_{il} has been reduced to the requisite form (8). In addition, the reduction has been effected by just the condition (11) needed to enforce energy conservation in the weak field. As Einstein had already found, that coordinate condition, in conjunction with the weak field equations (9) entailed energy conservation in the weak field form (12).

…Or Does It?

Einstein could hardly hope for a more satisfactory outcome. He was burdened by strict and unforgiving requirements on static fields and the weak field limit. Yet he found gravitational field equations of very broad covariance compatible with both. So satisfactory is this resolution that Einstein published it in November 1915 upon his return to general covariance. T^x_{il} is the gravitation tensor he proposed in his communication

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10 This last compatibility is not straightforward. The choice of T^x_{il} as gravitation tensor is not compatible with Einstein's favorite examples of a field produced by uniform, rectilinear acceleration in Minkowski spacetime, the static, homogeneous field, $g^{HG}_{\mu\nu}$, given as (6). One finds by explicit calculation that $g^{HG}_{\mu\nu}$ is not a solution of the source free field equations $T^x_{il} = 0$. This failure is already suggested by that fact that T^x_{il} is only a tensor under unimodular transformations and that the transformation from $g^{SR}_{\mu\nu}$ to $g^{HG}_{\mu\nu}$ is not unimodular. (Unimodular transformations preserve the determinant of the metric. But $\text{Det}\,(g^{SR}_{\mu\nu}) = c^2 = \text{constant}$, whereas $\text{Det}(g^{HG}_{\mu\nu}) = (c_0 + ax)^2 \neq \text{constant}$.) Now $g^{SR}_{\mu\nu}$ is obviously a solution of the source free field equations $T^x_{il} = 0$. So we cannot infer from the covariance properties of T^x_{il} that $g^{HG}_{\mu\nu}$ is also a solution.
If Einstein was aware of this problem, he did not find it immediately fatal to T^x_{il} as gravitation tensor. The problem should have been apparent as soon as Einstein contemplated a gravitation tensor covariant only under unimodular transformations. Yet he proceeded on page 22R with the elaborate recovery of the Newtonian limit. Again there is no trace of a concern over the homogeneous field, $g^{HG}_{\mu\nu}$, in the pages surrounding and following. (The concern is directed towards the coordinate restriction (11) and the rotation field $g^{ROT}_{\mu\nu}$ defined below.) The failure amounts to a failure of his principle of equivalence. But Einstein had already reconciled himself to such a failure in his theories of 1912 and it arose again in his "Entwurf" theory. See (Norton 1985, §4.3).
In the text I have explained his apparent indifference by assuming that he adopted the position expressed later in (Einstein 1915a, 786). Employing the same gravitation tensor T^x_{il}, the theory of that paper was also covariant under unimodular transformations. In order to affirm that the theory satisfied the relativity of motion, he observed (in part) that the coordinate transformation $x' = x - \tau_1$, $y' = y - \tau_2$, $z' = z - \tau_3$, $t' = t$, with τ_1, τ_2 and τ_3 arbitrary functions of t, is unimodular. We might note that this transformation corresponds to a large class of unidirectional accelerations. While the class does not include the transformation from $g^{SR}_{\mu\nu}$ to $g^{HG}_{\mu\nu}$, Einstein may well have simply assumed that it did include related transformation of comparable physical significance.

of November 4 to the Prussian Academy (Einstein 1915a). On November 4, he had little choice. The natural gravitation tensors, the Ricci tensor and then the Einstein tensor, were still unavailable to him. He was still bewitched by his early, mistaken expectations concerning static fields and the weak field limit. These expectations were dispelled after that communication, over the course of that November. A rapid series of communications first brought him his successful explanation of the anomalous motion of Mercury and then his final, generally covariant field equations.

In the November 4 communication, Einstein paused to explain the transient charms of the "November tensor" T^x_{il}. He closed the communication of November 4 by showing (§4) that the coordinate condition (11), in the case of weak fields, reduces his field equations to the form (9).

Yet Einstein's achievement on page 22R of the Zurich notebook proves to be as puzzling as it is impressive. For the proposal disappears as rapidly as it appeared; it receives no further serious consideration in the notebook.[11] The difficulties that led to its dismissal cannot be those that defeated the combination of the Ricci tensor and the choice of harmonic coordinate systems. These were the misleading expectations about static fields and the weak field limit. The gravitation tensor T^x_{il} was compatible with both. Why did Einstein so rapidly discard this promising candidate for his gravitation tensor? What changed to make it acceptable in November 1915?

1.3 The { } Prejudice

We have fragments of evidence that allow us to answer these questions. Some come from the pages of the notebook surrounding page 22R. The most important come in Einstein's later recollections.

A Letter to Sommerfeld of November 28, 1915

Einstein's most complete account comes in all too brief remarks in this letter (CPAE 8, Doc. 153). Having recounted the final field equations of his theory, Einstein continued:

> Of course it is easy to write down these generally covariant equations. But it is hard to see that they are the generalization of Poisson's [Newtonian] equations and not easy to see that they allow satisfaction of the conservation laws.
>
> Now one can simplify the whole theory significantly by choosing the reference system so that $\sqrt{-g} = 1$. Then the equations take on the form

11 We shall see below in section 3.7 that T^x_{il} is reevaluated on the following page 23L, but now with the coordinate restriction (11) replaced by another. The candidate gravitation tensor reappears briefly on page 25L in an incomplete attempt to compute the stress energy tensor of the gravitational field associated with this gravitation tensor.

$$-\sum_{l}\frac{\partial\begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l}+\sum_{\alpha\beta}\begin{Bmatrix} i\alpha \\ \beta \end{Bmatrix}\begin{Bmatrix} m\beta \\ \alpha \end{Bmatrix} = -\kappa\left(T_{im}-\frac{1}{2}g_{im}T\right).$$

> I had already considered these equations 3 years ago with Grossmann up to the second term on the right hand side, but had then arrived at the result that they did not yield Newton's approximation, which was erroneous. What supplied the key to this solution was the realization that it is not
>
> $$\sum g^{l\alpha}\frac{\partial g_{\alpha i}}{\partial x_m}$$
>
> but the associated Christoffel symbols $\begin{Bmatrix} im \\ l \end{Bmatrix}$ that are to be looked upon as the natural expression for the "components" of the gravitational field. If one sees this, then the above equation becomes simplest conceivable, since one is not tempted to transform it by multiplying out [*Ausrechnen*] the symbols for the sake of general interpretation.

Which equations had he considered three years before? "…[T]hese equations…up to the second term on the right hand side…," that is, excluding the trace term in T, coincide with the choice of the "November tensor" T_{il}^{x} as gravitation tensor on page 22R. Einstein tells Sommerfeld that he had considered these equations with Grossmann and that detail is affirmed by the appearance of Grossmann's name on the top of page 22R.[12]

The Fateful Prejudice

The elements of the account Einstein laid out to Sommerfeld reappear in other places in Einstein's writing. In his publication, (Einstein 1915a, 1056), he also recounted his misidentification of the "'components' of the gravitational field." He recalled how he had reformulated the energy conservation law in his earlier work, (Einstein 1914c). In the absence of non-gravitational forces, the law is just the vanishing of the covariant divergence of the stress-energy tensor $T_{\mu\nu}$. It could be re-expressed as[13]

$$\sum_{\nu}\frac{\partial\mathfrak{T}_{\nu}^{\sigma}}{\partial x_{\nu}}-\begin{Bmatrix} \nu\sigma \\ \tau \end{Bmatrix}\mathfrak{T}_{\tau}^{\nu} = \sum_{\nu}\frac{\partial\mathfrak{T}_{\nu}^{\sigma}}{\partial x_{\nu}}-\frac{1}{2}g^{\tau\mu}\frac{\partial g_{\mu\nu}}{\partial x_{\sigma}}\mathfrak{T}_{\tau}^{\nu} = 0,$$

where the tensor density $\mathfrak{T}_{\nu}^{\sigma} = \sqrt{-g}T_{\nu}^{\sigma}$. Einstein now reflected upon his earlier error:

> This conservation law had earlier induced me to view the quantities

---

12 Presumably Einstein alludes to his earlier recovery of these equations in the introduction to his paper of November 4, 1915. Einstein (Einstein 1915a, 778) recalls his work three years earlier with Grossmann and then claims: "In fact we had already then come quite close to the solution of the problem given in the following."

13 Einstein refers back to the results in (Einstein 1914c). There the energy conservation law was written in terms of the covariant divergence of $T_{\mu\nu}$. In his paper of November 4, 1915, Einstein had discarded a term in $\sqrt{-g}$ to simplify the result at the expense of reducing its covariance to unimodular transformations only.

$$\frac{1}{2}\sum_{\mu} g^{\tau\mu}\frac{\partial g_{\mu\nu}}{\partial x_{\sigma}}$$

> as the natural expression for the components of the gravitational field, even though it is obvious, in view of the formulae of the absolute differential calculus, to introduce the Christoffel symbols
>
> $$\begin{Bmatrix} \nu\sigma \\ \tau \end{Bmatrix}$$
>
> instead of those quantities. This was a fateful prejudice.

Einstein continues to argue for the naturalness of this choice. The Christoffel symbols are symmetric in the indices ν and σ and they reappear in the geodesic equation. However he does not explain precisely how this "prejudice" led him astray. For convenience, I will call this the "$\{\}$ prejudice."

A letter written to Lorentz the following January 1, 1916, repeats and slightly clarifies the role of the $\{\}$ prejudice. (CPAE 8, Doc. 177.)

> I had already considered the present equations [of the final theory, not of November 4] in their essentials 3 years ago with Grossmann, who had made me aware of Riemann's tensor. But since I had not recognized the formal meaning of the $\{\}$, I could achieve no overview and could not prove the conservation laws.

The Problems Collected

If we assemble the clues, we find Einstein giving two reasons for his rejection of the "November tensor" T_{il}^{x} when he worked with Grossmann:[14]

- He was unable to recover the Newtonian limit.
- The $\{\}$ prejudice precluded recognition of the inherent simplicity of the equations and the recovery of the energy conservation law.

Both elements of Einstein's account are puzzling. A straightforward reading of page 22R shows Einstein recovering the Newtonian limit in exactly the same way as in his later publication of November 4, 1915. A more careful analysis will be needed, but that will be postponed to later parts of this chapter. Einstein's remarks about the $\{\}$ prejudice are also puzzling at first. Einstein had a perfectly acceptable expression for the energy conservation law. It is just the vanishing of the covariant divergence of $T_{\mu\nu}$ and was introduced by Einstein on page 5R of the notebook. I believe that these last remarks admit a fairly simple explication.

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14 Since recollections are not infallible, there is always a possibility that the first difficulty with the Newtonian limit was misremembered and really pertained only to his problem with the Ricci tensor. We need not have such concerns for the $\{\}$ prejudice. Since it was published on November 4, 1915, the notion was clearly formulated before Einstein had realized the problems with the Newtonian limit associated with his assumptions about the static field and the weak field equations.

Recovering Energy Conservation

To understand why this prejudice was fateful, we need to recall a major difference between Einstein's work in the notebook and in November 1915. Here I draw heavily on the insights of Jürgen Renn and Michel Janssen (Janssen and Renn 2004). By 1915 Einstein had developed techniques of considerably greater sophistication for recovering energy conservation than he had used in 1913. Also, when Einstein talks of proving the conservation laws, we must understand him to mean a little more than merely recovering the standard result that the covariant divergence of $T_{\mu\nu}$ vanishes. We must understand an important part of the recovery to be the identification of a stress-energy tensor for the gravitational field, $t_{\mu\nu}$, that will figure in an alternate form of the energy conservation law (as given in Einstein and Grossmann 1913, 17)

$$\sum_{\mu\nu} \frac{\partial}{\partial x_\nu}\{\sqrt{-g}\ \gamma_{\sigma\mu}(T_{\mu\nu} + t_{\mu\nu})\} = 0.$$

At the time of the "Entwurf" theory, Einstein employed a simple device for generating this stress-energy tensor. It had been used on pages 19R, 20L and 21L of the notebook in the weak field, while Einstein weighed the fate of the Ricci tensor as gravitation tensor. Einstein took the expression for the gravitational force density in the weak field (7),

$$\frac{1}{2}\sum_{\mu\nu} \frac{\partial g_{\mu\nu}}{\partial x_\sigma}\Theta_{\mu\nu},$$

where $\Theta_{\mu\nu}$ is the contravariant form of the stress-energy tensor $T_{\mu\nu}$. He then substituted for $\Theta_{\mu\nu}$ using the gravitational field equation for the weak field (9). A simple manipulation that preserved only terms of lowest order in the derivatives of $g_{\mu\nu}$ allowed this force density to be rewritten as the divergence of a tensor $t_{\alpha\sigma}$: [15]

$$\frac{1}{2}\sum_{\mu\nu\alpha\beta} \frac{\partial g_{\mu\nu}}{\partial x_\sigma}\left(\gamma_{\alpha\beta}\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\gamma_{\mu\nu}\right)$$
$$= \frac{1}{2}\sum_{\mu\nu\alpha\beta\rho\tau} \frac{\partial}{\partial x_\beta}\left(\gamma_{\alpha\beta}\frac{\partial g_{\mu\nu}}{\partial x_\sigma}\frac{\partial \gamma_{\mu\nu}}{\partial x_\alpha} - \frac{1}{2}\delta_{\beta\sigma}\gamma_{\rho\tau}\frac{\partial g_{\mu\nu}}{\partial x_\rho}\frac{\partial \gamma_{\mu\nu}}{\partial x_\tau}\right) = \frac{1}{2}\gamma_{\alpha\beta}\frac{\partial t_{\alpha\sigma}}{\partial x_\beta}$$

$$\frac{1}{2}\sum_{\mu\nu\alpha\beta} \frac{\partial g_{\mu\nu}}{\partial x_\sigma}\left(\gamma_{\alpha\beta}\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\gamma_{\mu\nu}\right)$$
$$= \frac{1}{2}\sum_{\mu\nu\alpha\beta\rho\tau} \frac{\partial}{\partial x_\beta}\left(\gamma_{\alpha\beta}\frac{\partial g_{\mu\nu}}{\partial x_\sigma}\frac{\partial \gamma_{\mu\nu}}{\partial x_\alpha} - \frac{1}{2}\delta_{\beta\sigma}\gamma_{\rho\tau}\frac{\partial g_{\mu\nu}}{\partial x_\rho}\frac{\partial \gamma_{\mu\nu}}{\partial x_\tau}\right) = \frac{1}{2}\frac{\partial}{\partial x_\beta}(\gamma_{\alpha\beta}t_{\alpha\sigma}).$$

Einstein identified that tensor with the stress-energy tensor of the gravitational field.

---

15 The symbol $\delta_{\beta\sigma}$, where $\delta_{\beta\sigma} = 1$ when $\beta = \sigma$ and zero otherwise, was not then used by Einstein, but is introduced here for simplicity.

This equation holds only for quantities of second order of smallness (ε^2) in the metric tensor of (7) of the weak field. The major part of Einstein's strategy for deriving his "Entwurf" field equations was to determine what quantities must be added to the gravitation tensor of the weak field equations (9) to make the relation between force density and the divergence of $t_{\alpha\sigma}$ exact, that is, true for all orders. This strategy reappears after page 22R, on pages 24R and 25R, and then in the full derivation of the "Entwurf" gravitational field equations by this strategy on pages 26L–26R.

Why the $\{\}$ *Prejudice Was Fateful*

Now we can understand why the $\{\}$ prejudice was fateful as Einstein inspected the candidate gravitation tensor T_{il}^{x} on page 22R. On his account, he was unable to see how to recover the Newtonian limit, a problem we shall return to. He also needed to assure himself that the gravitation tensor was compatible with energy conservation and that included admission of a well-defined stress-energy tensor $t_{\alpha\sigma}$ for the gravitational field. Following his standard practice, that would mean that he must be able to reformulate the expression for gravitational force density as a divergence. We can immediately see the problem Einstein would face. The tensor T_{il}^{x} is expressed fully in terms of Christoffel symbols, with each representing a sum of three terms in

$$\frac{\partial g_{\mu\nu}}{\partial x_{\tau}}.$$

The product of two Christoffel symbols would yield nine of these derivative terms.[16] Faced with so many terms, we could well imagine Einstein's sense that he had no overview (as he wrote to Lorentz above) or that this was certainly not the simplest conceivable equation (as he wrote to Sommerfeld above). We could well imagine that this difficulty, along with failure of the Newtonian limit, would be sufficient grounds for him to abandon the candidate tensor.

What changed by November 1915? In the course of 1914, Einstein developed powerful variational methods for recovering energy conservation and expressions for the stress-energy tensor of the gravitational field. (Einstein and Grossmann 1914; Einstein 1914c). He applied those to his field equations of November 4, 1915, and found that the expressions took on just about the simplest form one could expect—as long as all quantities were expressed in terms of the Christoffel symbols. His field Lagrangian was just

$$\sum_{\sigma\tau\alpha\beta} g^{\sigma\tau}\begin{Bmatrix}\sigma\beta\\ \alpha\end{Bmatrix}\begin{Bmatrix}\tau\alpha\\ \beta\end{Bmatrix}.$$

16 Einstein's "Entwurf" gravitation tensor has one second derivative term and three first derivative terms. Unless there are duplications, the November tensor would have three second derivative terms and nine first derivative terms.

It is one of the simplest fully contracted expressions quadratic in the Christoffel symbols. (The Lagrangian must be quadratic if it is to return field equations linear in the second derivatives of the metric tensor.) His expression for the canonical stress-energy tensor of the gravitational field was scarcely more complicated.

Einstein's analysis in the notebook began with a force density

$$\frac{1}{2}\sum_{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x_\sigma}\Theta_{\mu\nu}$$

expressed in terms of the derivatives of the metric tensor. It overwhelmed him and he abandoned it. Einstein's analysis in November 1915 retained the Christoffel symbols and, using his more powerful methods, yielded just about the simplest expressions he could expect. In hindsight, Einstein diagnosed the error to lie in his starting point. Had he not misidentified the components of the gravitational field, would he have resisted the temptation to expand the Christoffel symbols? Would he have come to see that he had the right equations before him?

Einstein used the term "prejudice"—a belief not properly grounded in evidence. It is a fitting label for the error we reconstruct. He was not assured that energy conservation would fail for this tensor in the notebook. He had no firm proof, no result around which to maneuver. He merely balked at a very complicated calculation that could have, in principle, been completed. He had no good reason to abandon the tensor other than the hunch that the true way could not be that complicated. And he later found that it was not at all complicated when viewed from another perspective.

2. COORDINATE CONDITIONS AND COORDINATE RESTRICTIONS

On page 22R of the notebook, Einstein shows how to use the coordinate condition (11) to reduce the gravitation tensor T_{il}^{x} to the requisite Newtonian form (8). Why does he report to Sommerfeld that he and Grossmann had originally thought the resulting gravitational field equations incompatible with the Newtonian limit? In this part of the chapter and in the part to follow, I will describe two explanations, both requiring that Einstein did not use coordinate conditions in the modern way. The explanation to be developed in this part is the simpler of the two. It asserts that Einstein understood field equations to be compatible with the Newtonian limit if they had the form (8) not just in some specialized coordinate systems, but in all coordinate systems. A cursory inspection would reveal that T_{il}^{x} does not have this form (8), rendering it incompatible with the Newtonian limit.

For this account to be tenable, we must now explain why page 22R displays the apparently successful reduction of the tensor T_{il}^{x} to the Newtonian form (8) using coordinate condition (11). This part will supply that explanation by suggesting that Einstein did not use the coordinate condition (11) in the standard way, as it was later in Einstein's paper of November 4, 1915. It was not a condition just to be invoked in the case of the Newtonian limit. It was a postulate to be used universally. In part one of this section, I will review the these two ways of using conditions such as (11). In this volume, we reserve the term "coordinate condition" for the standard usage and "coordinate restriction" for the other usage suggested here. This notion of coordinate restriction was introduced by Jürgen Renn and Tilman Sauer (Renn and Sauer 2004). We will see in the second section of this part that there is clear evidence that Einstein used coordinate restrictions in the notebook on page 22 and afterwards.

In the third section of this part, I will describe how we can use the notion of coordinate restrictions to explain why Einstein abandoned the gravitation tensor T_{il}^{x}. To do so, we need a further assumption. Einstein did not just use (11) as a coordinate restriction on page 22. We must assume that he was unaware of the other possibility of using (11) as a coordinate condition. Then his rejection of T_{il}^{x} as gravitation tensor is automatic; it does not have the Newtonian form (8). This account is the majority viewpoint within our group.

The account depends upon the assumption that Einstein was unaware of the possibility of using (11) as a coordinate condition. In the fourth section of this part, I will explain why I do not believe the assumption. There is no single piece of evidence that allows us to decide either way on the assumption. It lies on the boundary. However I believe that there are so many indications that speak against it, that their combined weight makes the assumption untenable. The most plausible account, I believe, is that page 22 of the notebook marks a turning point. Prior to it, Einstein used coordinate conditions; after he reverted to the use of coordinate restrictions.

2.1 Two Uses of One Equation

Four Equations Select a Coordinate System…

The equations of a generally covariant spacetime theory hold in arbitrary coordinate systems. In applying the theory, we may pick the coordinate system freely. The four coordinates are just four real valued functions x_{α} defined on the spacetime manifold. Therefore a coordinate system can be chosen with four conditions $x_{\alpha}(p) = f_{\alpha}(p)$, where the $f_{\alpha}(p)$ are four arbitrary real valued functions of suitable differentiability defined over events p. Thus four arbitrary conditions are associated with the choice of a coordinate system.

This simple fact about coordinate systems is often rendered as the much looser idea that there are four degrees of freedom in a generally covariant theory associated with the freedom of choice of the coordinate system.[17] These four degrees of free-

17 This slogan—four degrees of freedom associated with the choice of coordinates—must be approached with some caution. It does not mean we can adjoin any four equations we like to our theory under the guise of choosing the coordinate system. Adding the single equation $R = 0$, where R is the curvature scalar, does a great deal more than restrict the coordinate system. One might imagine that restricting the equations to first order derivatives in $g_{\mu\nu}$ would protect us from these problems, since, at any event in spacetime, we can always transform such derivatives to zero. But it does not. Imagine that we have 100 such conditions, $C_1 = 0$, $C_2 = 0$, …, $C_{100} = 0$ that more than exhaust the freedom to choose coordinates. They can be disguised as a single equation $(C_1)^2 + ... + (C_{100})^2 = 0$.

dom are more usually exploited indirectly by specifying four differential conditions on quantities defined in spacetime. Examples are the harmonic condition (10) and the condition (11) used on page 22R. They do not fully exhaust the freedom. Since they are differential conditions on the metric, they do not force a unique choice of coordinate system; differential equations admit many solutions according to the choice of boundary conditions. So each of (10) and (11) admit many coordinate systems. For example, if one admits a coordinate system x_α, it also admits any coordinate system linearly related to it. This follows immediately from the covariance of (10) and (11) under linear coordinate transformations.

In the case of the harmonic condition (10), the relation between the different forms of the condition can be made more explicit. We can define the natural, generally covariant analog of the d'Alembertian operator $\Box$ used in (9) as follows. If φ is a scalar, we take its covariant derivative twice and contract with $\gamma_{\mu\nu}$ over the two resulting indices. In Einstein's notation of 1913, this gives

$$\Box\varphi = \sum_{\alpha\beta\rho\sigma} \gamma_{\alpha\beta}\frac{\partial^2\varphi}{\partial x_\alpha \partial x_\beta} - \gamma_{\alpha\beta}\begin{bmatrix}\alpha\beta\\ \rho\end{bmatrix}\gamma_{\rho\sigma}\frac{\partial\varphi}{\partial x_\sigma}.$$

If we now form $\Box x_\tau$ for each of the four coordinates, we quickly see that the harmonic condition (10) is equivalent to[18]

$$\Box x_\tau = \sum_{\alpha\beta\rho\sigma} \gamma_{\alpha\beta}\frac{\partial^2 x_\tau}{\partial x_a \partial x_\beta} - \gamma_{\alpha\beta}\begin{bmatrix}\alpha\beta\\ \rho\end{bmatrix}\gamma_{\rho\sigma}\frac{\partial x_\tau}{\partial x_\sigma} = 0. \tag{10'}$$

One sees from this equation that the harmonic condition cannot fix the coordinate system uniquely.[19] If the condition is satisfied by x_τ, it will also be satisfied by any linear transform of it. Other transforms will also be admissible. The condition cannot fix the coordinate system up to linear transformation unless one invokes further restrictive conditions (see Fock 1959, §93).

We know that Einstein was aware of this form (10') of the harmonic condition on coordinate systems, then routinely available in the literature on infinitesimal geometry as the "isothermal" coordinates. At the bottom of page 19L on which he introduced the condition in form (10), he wrote "…Holds for coordinates which satisfy the eq[uation] ($\Delta\varphi = 0$)."

We see how equations (10) and (11) allow us choose a restricted set of coordinates. There are two ways relevant to our present interests that these equations may be used: as coordinate conditions and as coordinate restrictions.

18 To see the equivalent, notice that the first term of (10') vanishes for any coordinate system. The second term vanishes if (10) holds. So (10) entails (10'). Conversely, if (10') holds, its second term must vanish, which immediately entails (10).

19 Notice that the operator $\sum_{\alpha\beta\rho\sigma} \gamma_{\alpha\beta}\begin{bmatrix}\alpha\beta\\ \rho\end{bmatrix}\gamma_{\rho\sigma}\frac{\partial}{\partial x_\sigma}$ is invariant under linear transformation.

…As Coordinate Conditions

Einstein later used a standard procedure for recovering the Newtonian limit from his generally covariant general theory of relativity. See for example, (Einstein 1922, 86–87). That theory must revert to Newton's theory of gravitation in the special circumstance of weak static fields, that is, under the assumption that the metric has the form (7) and is static. In addition, Newton's theory is not generally covariant, but is covariant under Galilean transformation only.[20] Therefore the covariance of Einstein's theory must be restricted if Newton's theory is to be recovered.

That covariance is already restricted in part by the presumption that the metric have the form (7). That form is not preserved under arbitrary transformations. The restriction to the weak field metric (7) is not, however, sufficient to reduce the covariance of the theory to the Galilean covariance of Newton's theory. That form is preserved by any transformation which introduces small changes of order of the $\varepsilon_{\mu\nu}$ to the coefficients of the metric. This last freedom is eliminated by imposing a coordinate condition, such as the harmonic condition (10). We have already seen the direct effect of this condition. It eliminates all second derivative terms from the Ricci tensor beyond those in the Newtonian like form (8). In so far as Einstein expected his "Entwurf" theory to have broad covariance, he must have believed the restriction of the metric to the weak field form (7) was sufficient restriction on its covariance for the recovery of the weak field limit.

A coordinate condition is used only in the special circumstances of the Newtonian limit; it is not imposed universally on the theory.

…As Coordinate Restrictions

The same equation (10) and (11) can be used in a different way. Einstein's goal in the notebook is a theory with sufficient covariance to satisfy a generalized principle of relativity. General covariance supplies more covariance than he needs; it includes covariance under transformations not associated with changes of states of motion, such as the transformation from Cartesian spatial coordinates to radial coordinates. So Einstein can afford to use the generally covariant expressions of the Ricci Levi Civita calculus merely as intermediates. If those expressions are not themselves suitable for his theory, then he can simplify them to generate others of somewhat less covariance that are. The generation of the November tensor T_{il}^{x} on page 22R is an example. The Ricci tensor itself appeared unsuitable as a gravitation tensor. There proved to be a way of splitting the tensor into two parts each of which is a tensor under unimodular transformations. Since Einstein was willing to accept unimodular covariance instead of general covariance, he could select one of these parts as his gravitation tensor.

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20 At least, this is the way it seemed to Einstein in the 1910s. Cartan and Friedrichs later showed that Newtonian theory could be given a generally covariant formulation, so that the problem of recovering the Newtonian limit from Einstein's theory takes on a different cast. See (Havas 1964) and also (Stachel 2004).

The equations (10) and (11) could be used in the same way. If the Ricci tensor or the tensor T_{il}^{x} proved unsuitable as a gravitation tensor, why not sacrifice a little more covariance to produce expressions that are suitable? Conjoining (10) or (11) to their associated tensors produces simpler expressions. The tensor T_{il}^{x}, for example is reduced to (15). If Einstein selected this reduced form as his gravitation tensor, then he assured recovery of the Newtonian limit. The gravitation tensor has the required form (8).

The cost of using equations (10) and (11) in this way is a further sacrifice of covariance. The final equations will have no more covariance than the coordinate restrictions (10) and (11). Whether these have sufficient covariance to support an extension of the principle of relativity cannot easily be read by inspecting equations (10) and (11). It is a matter of computation.

2.2 The Evidence for Einstein's Use of Coordinate Restrictions

There is strong evidence that Einstein used equation (11) and another similar restriction as a coordinate restriction, that is, as a universal restriction not limited to the special case of the Newtonian limit.

The Non-Linear Transformation of Equation (11)

The first piece of evidence is on page 22L. There Einstein undertakes a simple calculation. He writes down two equations. The second is $|p_{\mu\nu}| = 1$. Since, in Einstein's notation,

$$|p_{\mu\nu}| = \frac{\partial x'_{\mu}}{\partial x_{\nu}},$$

this is just the condition that the transformation $x_{\nu} \rightarrow x'_{\mu}$ be unimodular. The first is equation (11) in the primed coordinate system. Einstein then expands this equation in terms of unprimed quantities and the coefficients of the transformation $p_{\mu\nu}$ and their inverses.

The calculation is incomplete and its outcome obscure. Its purpose is not obscure and that is all that matters here. Einstein is checking the covariance of equation (11) within the domain of unimodular transformations. If Einstein intended (11) to be a coordinate condition, it is hard to see why he would concern himself with its transformation properties. The role of equation (11) as a coordinate condition is merely to assist in reducing the covariance of the theory to enable recovery of the Newtonian limit. Galilean covariance only is required in that Newtonian limit. Einstein can be assured of this much covariance. Galilean transformations are a subset of the linear coordinate transformations. Einstein can determine rapidly that equation (11), used as a coordinate condition, will give him that much covariance. The calculation is trivial. It merely requires noticing that the coefficients

$$|p_{\mu\nu}| = \frac{\partial x'_\mu}{\partial x_\nu}$$

and their inverses

$$\pi_{\mu\nu} = \frac{\partial x_\nu}{\partial x'_\mu}$$

are constants under linear transformation. Therefore the quantity in equation (11) transforms as a vector under linear transformation since

$$\sum_{\kappa} \frac{\partial \gamma'_{\kappa\alpha}}{\partial x'_\kappa} = \sum_{\kappa\rho\sigma\tau} \pi_{\kappa\rho} \frac{\partial (p_{\kappa\sigma} p_{\alpha\tau} \gamma_{\sigma\tau})}{\partial x_\rho} = \sum_{\kappa\rho\sigma\tau} p_{\kappa\sigma} p_{\alpha\tau} \pi_{\kappa\rho} \frac{\partial \gamma_{\sigma\tau}}{\partial x_\rho} = \sum_{\sigma\tau} p_{\alpha\tau} \frac{\partial \gamma_{\sigma\tau}}{\partial x_\sigma},$$

where we use $\sum_{\kappa} P_{\kappa\sigma} \pi_{\kappa\rho} = \delta_{\sigma\rho}$. Hence, if this quantity vanishes in one coordinate system as (11) requires, it will vanish in any coordinate system to which one transforms with a linear transformation.

Einstein cannot have had this simple linear case in mind on page 22L. For the calculation there clearly allows non-constancy of the coefficients $p_{\mu\nu}$; he does not eliminate the derivative terms

$$\frac{\partial P_{\mu\alpha}}{\partial x_i},$$

which vanish automatically for linear transformations. This concern is unintelligible if equation (11) is being used as a coordinate condition. The concern is explained quite simply if that equation is being used as a coordinate restriction. The quantity in (15), the tensor T_{il}^{x} after reduction by coordinate restriction (11), is his gravitation tensor. He is computing its covariance the easy way. By its construction, this candidate gravitation tensor will transform as a tensor under unimodular transformations that leave equation (11) unchanged. If the candidate gravitation tensor is to allow a generalization of the principle of relativity, its covariance must include non-linear transformations.

The Theta Requirement

The result of the calculation on page 22L cannot have been encouraging for the combination of the tensor T_{il}^{x} and condition (11) receive no further serious attention in the notebook. Instead, on page 23L, Einstein introduced another way of recovering the Newtonian like expression (8) from T_{il}^{x} that did not require use of equation (11). That it not be required was apparently of some importance since, in a document of pure calculation with essentially no explanatory text at all, Einstein went to the trouble to explain in writing

"$\sum \frac{\partial \gamma_{\kappa\alpha}}{\partial x_\kappa} = 0$ is not necessary."

In its place Einstein introduced a coordinate restriction of another type. He constructed the quantity

$$\vartheta_{i\kappa\lambda} = \frac{1}{2}\left(\frac{\partial g_{i\kappa}}{\partial x_\lambda} + \frac{\partial g_{\kappa\lambda}}{\partial x_i} + \frac{\partial g_{\lambda i}}{\partial x_\kappa}\right) \tag{16}$$

and required that transformations between coordinates be so restricted that this quantity $\theta_{i\kappa\lambda}$ transform as a tensor. (We shall call this the "theta requirement," the "theta condition" or the "theta restriction" according to its interpretation.) He then proceeded to show by adding and discarding terms in $\theta_{i\kappa\lambda}$ from T_{il}^{x} how one could construct a quantity

$$\sum \frac{\partial}{\partial x_\kappa}\left(\gamma_{\kappa\alpha}\frac{\partial g_{il}}{\partial x_\alpha}\right) + \sum \gamma_{\rho\alpha}\gamma_{\kappa\beta}\frac{\partial g_{i\kappa}}{\partial x_\alpha}\frac{\partial g_{l\rho}}{\partial x_\beta} \tag{17}$$

that is a tensor under unimodular transformations for which $\theta_{i\kappa\lambda}$ transforms tensorially. Einstein's efforts have produced another expression in the form of (8), apparently yet another candidate for the gravitation tensor, at least in the Newtonian limit.

Its Relation to Rotational Covariance…

Through another part of the notebook we also learn what apparently interested Einstein in the requirement that $\theta_{i\kappa\lambda}$ transforms tensorially. The simplest requirement of this type would be to ask that the quantity $\partial g_{i\kappa}/\partial x_\lambda$ transform as a tensor. But that, perhaps, was an excessively restrictive. It is easy to see that this quantity transforms as a tensor only under linear transformations of the coordinate systems. If one sought a natural weakening of this requirement, the simplest weakening is to consider just the symmetric part of $\partial g_{i\kappa}/\partial x_\lambda$, which is (up to multiplicative factor) the quantity $\theta_{i\kappa\lambda}$.[21] One might hope that the weakened requirement would now admit other interesting transformations, such as those to coordinate systems in uniform rotation. More explicitly, these are the transformations that take the coordinates $x_\alpha = (x, y, z, t)$ to a new coordinate system $x'_\alpha = (x', y', z', t')$ in uniform rotation at angular velocity ω about the z axis

$$x' = x\cos\omega t + y\sin\omega t \quad y' = -x\sin\omega t + y\cos\omega t \quad z' = z \quad t' = t. \tag{18}$$

That this is Einstein's hope is revealed, apparently, by calculations on pages 42L–42R of the notebook. Einstein sets up and solves the following problem: what are all the metrics of unit determinant that satisfy the conditions[22]

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21 Another advantage is that the symmetrized form $\theta_{\iota\kappa\lambda}$ of $\partial g_{i\kappa}/\partial x_\lambda$ is very similar in structure to the Christoffel symbols, so that the Christoffel symbols in T_{il}^{x} can readily be rewritten in terms of $\theta_{i\kappa\lambda}$, easing the course of the calculations.

22 Einstein also suppresses the x_3 coordinate.

$$\frac{\partial g_{ik}}{\partial x_4} = 0 \text{ and } \vartheta_{i\kappa\lambda} = \frac{1}{2}\left(\frac{\partial g_{i\kappa}}{\partial x_\lambda} + \frac{\partial g_{\kappa\lambda}}{\partial x_i} + \frac{\partial g_{\lambda i}}{\partial x_\kappa}\right) = 0? \tag{19}$$

The problem posed by Einstein is a reformulation of this more interesting problem: assume we start from the metric $g^{SR}_{\mu\nu}$. To which time (= x_4 coordinate) independent metrics can we transform by means of unimodular coordinate transformations for which $\theta_{i\kappa\lambda}$ transforms as a tensor? Since $g^{SR}_{\mu\nu}$ has constant coefficients, we have $\theta_{i\kappa\lambda} = 0$, so that the requirement that $\theta_{i\kappa\lambda}$ transforms as a tensor reduces to the requirement that $\theta_{i\kappa\lambda}$ remain the zero tensor. Thus the metrics to which we can transform must satisfy (19). Apparently Einstein hoped that these transformations would include the unimodular transformation (18), so that this class of metrics would include what we can call a "rotation field", the form of the metric $g^{SR}_{\mu\nu}$ that results when it is transformed by the rotation transformation (18)

$$g^{ROT}_{\ \ \mu\nu} = \begin{bmatrix} -1 & 0 & 0 & \omega y' \\ 0 & -1 & 0 & -\omega x' \\ 0 & 0 & -1 & 0 \\ \omega y' & -\omega x' & 0 & c^2 - \omega^2(x'^2 + y'^2) \end{bmatrix}. \tag{20}$$

…Is Not Close Enough

And Einstein's hopes were almost realized. The result of his calculation was that the two conditions (19) were satisfied by a metric whose coefficients in its *covariant* form equaled those of the *contravariant* form of $g^{ROT}_{\ \ \mu\nu}$ that is $\gamma^{ROT}_{\ \ \mu\nu}$. This was close to showing that the transformations under consideration would allow the transformation from $g^{SR}_{\mu\nu}$ to $g^{ROT}_{\ \ \mu\nu}$. But it is not good enough for a mathematical result such as this to be close. It either succeeds or fails—and this one failed. Einstein revealed his frustration by remarking in one of the few textual comments in the notebook of calculations, "Schema of γ for a rotating body identical with the adjacent g- schema!" The exclamation remark is very unusual and flags Einstein's surprise and, probably, disappointment.[23]

The Theta Requirement is Not a Coordinate Condition

We can reconstruct the content of these calculations fairly confidently. But their purpose is quite mysterious if we assume that the theta requirement is simply a coordinate condition being used to reduce T^x_{il} to the Newtonian form (8) for the case of the

23 That this result proved fatal to the proposal of the theta restriction is confirmed by the calculations that follow on page 43L. There Einstein attempts to define a contravariant form of $\theta_{i\kappa\lambda}$ and begins to check whether it might be able to reduce the tensor T^x_{il} if used in the same way as $\theta_{i\kappa\lambda}$ in the original theta restriction. Presumably Einstein chose a contravariant form of $\theta_{i\kappa\lambda}$ as a replacement of the failed $\theta_{i\kappa\lambda}$ in the hope that a calculation analogous to that on pages 42L–42R would yield the correct covariant form of $g^{ROT}_{\ \ \mu\nu}$.

Newtonian limit. There are two problems. First, if the theta requirement has this purpose, then there is no need to investigate its covariance under rotation transformations (18) or, for that matter, to contrive the condition to have this covariance. Linear covariance is sufficient for the Newtonian limit and it is obvious without calculation that the theta condition has that much covariance.[24] Nonetheless, lack of rotational covariance seems to have been fatal to the proposal of the theta condition.

The second problem is that the reduction of T^x_{il} to (17) is not the calculation that would be undertaken if the theta condition were a coordinate condition. In that case, one would merely seek the expression to which T^x_{il} reduced in coordinate systems compatible with the condition. Expression (17) is not that expression. In generating it, Einstein freely added terms in $\theta_{i\kappa\lambda}$ so contrived as not to disturb the covariance of the resulting expression under these transformations. One cannot revert to T^x_{il} merely by relaxing the constraint of the theta restriction. In short, (17) is guaranteed to transform tensorially under these restricted transformations, but it is not the quantity one would seek if one chose T^x_{il} as the gravitation tensor and sought its Newtonian limit through a coordinate condition.

Both these problems are resolved immediately if we assume that Einstein is using the theta requirement as a coordinate restriction. The expression (17) is his candidate gravitation tensor. It can have no more covariance than the theta condition, so an investigation of the latter's covariance is, indirectly, an investigation of the covariance of the candidate gravitation tensor. Moreover T^x_{il} is merely an intermediate used in the construction of the candidate gravitation tensor (17). There is no need to ensure that this latter expression be a form of T^x_{il} in a restricted class of coordinates. Einstein's goal is merely a quantity of Newtonian form (8) with as much covariance as the theta condition. Einstein can add terms in theta freely if they allow a simpler final result, for these terms do not compromise the covariance of the result.

2.3 The Problem of the Newtonian Limit

How can the notion of coordinate restriction help us understand why Einstein rejected T^x_{il} as a candidate gravitation tensor in the notebook? In particular, how can it help us to understand Einstein's remark to Sommerfeld that the tensor did not yield the Newtonian limit when page 22R of the notebook appears to contain the calculation that shows how to recover the Newtonian limit? That is, it shows how to use equation (11) to reduce T^x_{il} to a Newtonian form, just as Einstein would in his paper of November 4, 1915.

The answer is simple. The expression T^x_{il} does not have the Newtonian form (8) and that may already be sufficient to explain Einstein's remark. Indeed, in addition to problems of energy conservation, Einstein may also have succumbed at this point to

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24 The deep concern with the covariance of the theta condition is also evident on the page facing the one on which the theta restriction is introduced. That facing page, 23R, is given over to computation of the transformation behavior of $\theta_{i\kappa\lambda}$ under infinitesimal transformations.

the temptation to multiply out the Christoffel symbols in an effort to get closer to an expression of the Newtonian form (8). If equation (11) is being used as a coordinate restriction in this effort, then T_{il}^{x} has ceased to be Einstein's candidate gravitation tensor. The new gravitation tensor is its reduced form, expression (15). While the formal manipulation of the reduction to expression (15) is the same in the notebook and in the November 4, 1915, paper, their interpretations would be very different. In 1915, the calculation shows how to recover the Newtonian limit from T_{il}^{x}. In the notebook, the calculation merely used T_{il}^{x} as an intermediate to generate a new candidate gravitation tensor, expression (15).

What was the fate of this new candidate gravitation tensor? It does not survive beyond page 22R. The notion of coordinate restriction can help us surmise its fate. If expression (15) is to be used as a gravitation tensor, it is of the greatest importance to determine its covariance. As we have seen, that is determined indirectly by investigating the covariance of the coordinate restriction (11). Presumably this was Einstein's goal on the facing page 22L when he probed the covariance of equation (11). We do not know how far Einstein went in these investigations. But we do know the results he would have found had he persisted. It is not too hard to see that coordinate restriction (11) is not covariant under rotation transformation (18). The simplest way to see this is to substitute $g^{ROT}_{\mu\nu}$ directly into (11). Since (11) vanishes for $g^{SR}_{\mu\nu}$, if it is covariant under rotation transformation (18), then it must also vanish for $g^{ROT}_{\mu\nu}$. But it does not. We have

$$\sum_{\kappa} \frac{\partial \gamma^{ROT}_{\kappa\alpha}}{\partial x'_{\kappa}} = \left(-\frac{\omega^2}{c^2}x', -\frac{\omega^2}{c^2}y', 0, 0\right) \neq 0.$$

We know that the rotation transformation (18) and the rotation field $g^{ROT}_{\mu\nu}$ became a topic of continued concern to Einstein on the pages following page 22. The rotation field enters indirectly on page 23L through the connection of the theta condition to the rotation field on pages 42L–42R. The rotation field is explicitly the subject of pages 24L, 24R and 25L.

It is natural to suppose that Einstein somehow came to see that his coordinate restriction (11) lacked rotational covariance, although we cannot identify a particular calculation in the notebook that unequivocally returns the result. The supposition that he had found the result would explain the strategy of the introduction of the theta condition on page 23L. Having found that his coordinate restriction (11) fails to satisfy rotational covariance, Einstein would respond by introducing a new coordinate restriction explicitly contrived to have rotational covariance. The theta condition is formulated directly as a covariance condition—Einstein will consider coordinate systems for which $\theta_{i\kappa\lambda}$ transforms as a tensor. As we saw above, the quantity $\theta_{i\kappa\lambda}$ was plausibly chosen exactly because it might yield covariance under rotation transformation (18). And we saw that Einstein remarked with evident satisfaction on page 23L that equation (11) was not needed, affirming his goal of replacing it with the theta condition.

This account of the failure of T_{il}^{x} as a gravitation tensor in the notebook is both simple and appealing. It depends crucially on one assumption: *Einstein was unaware* *of how to use conditions like (11) as a coordinate condition at the time of the writing of the notebook*. Without this assumption, we cannot use the notion of coordinate restrictions to explain Einstein's remark that the candidate gravitation tensor T_{il}^{x} does not yield the Newtonian limit. For, if he then understood the use of coordinate conditions, the calculation of page 22R supplied everything needed for recovery of the Newtonian limit. We must assume that he was unaware of the use to which his formal manipulation could be put.

2.4 Was Einstein Unaware of Coordinate Conditions?

I know of no evidence that decisively answers this question. So my final assessment is that we just do not know. There are weak indications, however, that point in both directions and I will try to assess them here. My view is that the case for the negative is stronger; that is, I find it most credible that Einstein was aware of possibility of using coordinate conditions.

In the Notebook

Requirements that may be either coordinate conditions or coordinate restrictions play a major role in the notebook on pages 19–23. The harmonic condition/restriction persists on pages 19–21, the requirement (11) on page 22 and then the theta requirement on page 23. The theta requirement was used as coordinate restriction and Einstein's calculation admitted no alternative interpretation of its use as a coordinate condition because of the way he added terms in $\theta_{i\kappa\lambda}$ in the course of his calculation. The calculation that used requirement (11) on page 22R is compatible with the requirement being used as either coordinate condition or coordinate restriction *or both*; the interest in the non-linear transformation of (11) on page 22L suggests its use as a coordinate restriction. There seems to be no indication that lets us decide whether the harmonic condition is used as a coordinate condition or restriction.[25] In particular, if it were used as a coordinate restriction, we might expect Einstein at some point to check its covariance in the way that he checked the covariance of requirement (11) and the theta restriction. The pages 19–21 contain no such check. Was that because he was using the requirement as a coordinate condition so that it needed no such check? Or was is that he was too preoccupied with the ultimately fatal difficulty of recovering the Newtonian limit and energy conservation to proceed to a test of covariance?

While Einstein certainly used coordinate restrictions in the notebook, nothing in the above precludes his awareness of coordinate conditions and that he may have *also* thought of using the harmonic requirement and equation (11) as coordinate restrictions.

"Presumed Gravitation Tensor"

Of the fragments of relevant evidence in the notebook, the most important is Einstein's labeling on page 22R. There, as we saw above in expression (14), Einstein splits the Ricci tensor into two parts. The first is easily seen to be a tensor under unimodular transformation. Therefore the second is also such a tensor. Einstein labels this second quantity “Vermutlicher Gravitationstensor T_{il}^{x}” —“presumed gravitation tensor. T_{il}^{x}”.

If Einstein is unaware of the use of coordinate conditions, then the identification of T_{il}^{x} as a gravitation tensor is very hard to understand. It does not have the Newtonian form (8). The derivative of the Christoffel symbol will immediately contribute three second derivative terms in the metric tensor, two more than (8) allows. This failure is not difficult to see. A Christoffel symbol is the sum of three first derivative terms. Its derivative will contain three second derivative terms in the metric tensor. Perhaps a novice in these calculations might overlook it. But Einstein is not a novice in these calculations at this stage in the notebook. In the pages preceding in the notebook he has become increasingly adept at more and more elaborate calculations involving the expansion of Christoffel symbols. On the following page 23L he devises the theta requirement. It depends on the recognition that the quantity $\theta_{i\kappa\lambda}$ and a Christoffel symbol have very similar structures so that the latter could be re-expressed profitably in terms of the former.

Perhaps this was just an oversight by Einstein. Perhaps it was haste that led him to label a manifestly inadmissible term as his presumed gravitation tensor. This supposition of haste becomes harder and harder to reconcile with what we know. At least the top half of page 22R is fairly neatly written and compact in argument, suggesting that it is not a live calculation but the record of deliberations elsewhere. Perhaps they record the outcome of a meeting with Grossmann—this is suggested by Grossmann’s name on the top of the page and Einstein’s later report to Sommerfeld of November 28, 1915, that he and Grossmann together had considered the gravitation tensor of this page. Einstein’s failure to notice the two additional second derivative terms would have to survive whatever deliberation or meeting that produced the result and its transcription.

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25 We might clutch at straws. If the harmonic requirement is used as a coordinate condition merely for the Newtonian limit, one needs to recover only the second derivative terms in the metric tensor and not the full reduced expression with first derivative terms, as Einstein does on page 19L. Or is this just Einstein being thorough and carrying a simple computation through to completion, wondering, perhaps, if the full result has an especially simple form? If the harmonic requirement were used as a coordinate restriction, then the full result would be needed, but that would still not preclude the possibility that Einstein weighed the use of the harmonic requirement as both coordinate condition and coordinate restriction. At the top of page 19R Einstein decomposes the harmonic requirement in the weak field into two equations comprising five conditions in all. That is one more than is allowed for a coordinate condition but admissible for coordinate restrictions. But since one of the new equation sets is just energy momentum conservation in the weak field, the decomposition is not necessarily an illegitimate strengthening of a coordinate condition as supplementing it with a physical requirement he demands on other grounds. Alternately but in the same spirit, that same condition, which is just equation (11) in the weak field, is a differential condition that must be satisfied by any static metric of form (4), as Einstein has already found earlier on page 39R of the notebook. On this same page 19R, he calls the harmonic requirement a “Nebenbedingung”—a “supplementary condition.” If the requirement is a coordinate restriction, that is an odd label for what is as much a physical postulate as the original gravitational field equations they modify. But then perhaps that is the right way to view their action—as a universal supplement to those equations.

Yet more curious is the success of the equation (11) in reducing the tensor T_{il}^{x} to the Newtonian form (8). If Einstein chose T_{il}^{x} as a candidate gravitation tensor in haste, what sublime good fortune came with the equation (11). It just happens to be a restriction compatible both with the form he demanded for the static field and with energy conservation in the weak field, the problems that proved fatal to the harmonic requirement. And it just happened to the one that rescued his poor choice of T_{il}^{x} as gravitation tensor and allowed him to use it as an intermediate on the way to a better choice. On the supposition that Einstein was unaware of the use of coordinate conditions, we cannot presume that he already knew that equation (11) would effect this reduction. For if he already knew that, he would not label T_{il}^{x} his presumed gravitation tensor. It would just be an intermediate as the Ricci tensor itself is.

The supposition of haste and unanticipated good fortune seems necessary to make the page compatible with a lack of awareness of the use of coordinate conditions. I find this supposition incredible. I find it much more credible that Einstein simply wrote what he meant. He chose T_{il}^{x} as his gravitation tensor, fully aware of the surplus second derivative terms and fully aware, by the time of the writing of page 22R, that they could be eliminated by the condition (11). That condition (11) has this power need longer be a fortuitous coincidence. After the failure of the harmonic requirement, we may suppose that Einstein sought a tensor that could be reduced to the Newtonian form by equation (11), for that was the requirement that was manifestly compatible with energy conservation in the weak field. Surely what attracted Einstein to the gravitation tensor T_{il}^{x} was exactly the fact that condition (11) allowed its reduction to the Newtonian form (8) and its selection as a presumed gravitation tensor resulted from working backwards from this result.

If we accept this last version of the story, then we accept that Einstein intended to use requirement (11) on page 22R as a coordinate condition and only later considered using it as a coordinate restriction.

Einstein's Later Discussion and Treatment of Coordinate Conditions

If the content of the notebook allows no final decision, we might look for evidence elsewhere. If Einstein were unaware of the use of coordinate conditions and this played some role in his failure, we might expect some trace of it in his later recollections and writings. He would have failed to see what later became his standard method for recovery of the Newtonian limit. Many of the errors of the notebook and "Entwurf" theory are mentioned later. He remarks both in correspondence and in his publications on his surprise that static fields turn out not to be spatially flat, (see Norton 1984, §8). He eventually also puts some effort into explaining to his correspondents how he erred in the "hole argument" and an enduring trace of this correction was his publication of the "point-coincidence argument," see section 3.2 below and (Norton 1987). I know of no place in which Einstein directly allows that he was unaware of the use of coordinate conditions when he devised the "Entwurf" theory.

What were the errors he corrected when he returned to the tensor T_{il}^{x}? A problem with the Newtonian limit accrues a brief mention in his letter to Sommerfeld. The real force of Einstein's correction in that letter lies in his confession of the $\{\}$ prejudice. That he regarded this error as decisive is affirmed by the fact that it also is discussed at some length in the text of the paper of November 4, 1915, as we say above. In stark contrast, the use of coordinate conditions gets no mention in this correspondence. In the November 4 paper, the correct use of coordinate conditions is introduced with an indifference that suggests he thinks their use entirely obvious.[26] His *complete* discussion is merely (p. 786):

> [Through this previous equation] the coordinate system is still not determined, in that four equations are needed for their determination. Therefore we may arbitrarily stipulate for the first approximation

$$\sum_{\alpha\beta} \frac{\partial g^{\alpha\beta}}{\partial x_\beta} = 0. \qquad [(11)]$$

If Einstein had suffered a failure to see that equation (11) could be used this way for almost three years, would he not offer some elaboration if only to assure readers that the manipulation is admissible? Or should we assume that Einstein was feeling too vulnerable at this crucial time in his theory's development to admit all his former errors?[27]

The Entwurf Theory

What is striking about the "Entwurf" theory is that it does not require coordinate conditions for the recovery of the Newtonian limit. Its gravitation tensor already has the Newtonian form (8). So merely presuming a weak field of form (7) indirectly introduces enough of a restriction on the coordinate system to allow recovery of the Newtonian limit.

This striking feature of the "Entwurf" theory and Einstein's silence on coordinate conditions would be explained quite simply by the supposition that Einstein was then unaware of the use of coordinate conditions. But both could also be explained in another way. If he decided in favor of the "Entwurf" field equations for other reasons,

26 This nonchalant attitude persisted into his review article (Einstein 1916, E§21), where the recovery of the Newtonian limit is formally incomplete exactly because Einstein neglects to invoke a coordinate condition. Einstein considers just the first term of the tensor T_{il}^{x} as part of his recovery of the Newtonian limit. He observes correctly that from it one recovers Poisson's equation of Newtonian theory, $\Delta g_{44} = \kappa\rho$ (where Δ is the Laplacian, κ a constant and ρ the mass density) by considering just the 44 component in the case of a time (x_4) independent field. That observation is insufficient for the recovery of the Newtonian limit. One must also establish that the remaining components of the field equations do not impose further conditions that restrict Poisson's equation in a way incompatible with the Newtonian limit. This further step requires a coordinate condition and that Einstein simply neglects to introduce or even mention. Einstein's later textbook exposition (Einstein 1922, 87) does give a serviceable account of how coordinate conditions are used to reduce the gravitational field equations to a Newtonian form, but without any special care that would suggest he thought the matter delicate.

27 Einstein did not explain in this paper where his "hole argument" against general covariance had erred. Below (see section 3.7) I will suggest that this reticence may have reflected a difficulty in seeing clearly what the problem was and this difficult will be a part of the account developed there.

then he might well never mention the use of coordinate conditions simply because he had no occasion to. Indeed, even in his later theory which did require them, he seemed quite reluctant to say anything more than the absolute minimum about them.

One thing that we cannot infer from the "Entwurf" theory and his writings associated with it is that Einstein was somehow naive about coordinate systems and how one might introduce a specialized coordinate system. We shall see below in section 3.6 that Einstein explained both in print and correspondence that he understood that equations of restricted covariance must correspond to generally covariant equations if they are to be anything more than just a restriction on the choice of coordinate system. He also made quite clear that he understood the subtleties of introducing specialized coordinate systems. That is, they might be introduced in two ways. In one way, one merely chooses to consider a restricted class of the coordinate systems already available; this decision does not alter the geometry of the spacetime. In the second way, one demands that this geometry must be such that it admits a coordinate system of a particular type; this demand indirectly applies a further and often profound restriction to the geometry of the spacetime.

If Einstein was unaware of the possibility of using coordinate conditions, it was not part of a broader blindness or lack of sophistication concerning coordinate systems.

What is More Plausible?

Since none of these items of evidence is decisive, we should also ask after the plausibility of different answers. Here our personal Einsteins speak as much as evidence. One might be comfortable with an Einstein unaware of the possibility of coordinate conditions. They never appear unequivocally in the notebook—although the labeling of T_{il}^{x} on page 22R as the "presumed gravitation tensor" is, in my view, very hard to explain if the initial intent was not to use a coordinate condition. So perhaps, on a principle of parsimony, we attribute the least knowledge we need to Einstein.

I find the supposed lack of awareness quite implausible. Coordinate systems and covariance requirements are Einstein's great strength and the locus of his deepest reflection. As we shall recall below in section 3.4, the essential goal in devising his general theory of relativity was the elimination of the preferred inertial coordinate systems of Newtonian theory and special relativity, which is in turn sustained by their limited covariance. It is fundamental to his entire research project that his final theory not harbor them. So how then is Einstein to recover the Newtonian limit from his theory? He must introduce specialized assumptions that obtain only in the case of the Newtonian limit and restores the characteristically Newtonian elements. One assumption is that the metric field have the specialized weak form of (7). He must also reduce the covariance of theory and thereby reintroduce exactly the preferred coordinate systems he had labored so hard to eliminate. Einstein's knew how to restrict covariance. It is done partly in the coordinate dependence of the metric given as (7). It is done explicitly through a set of four equations such as the harmonic requirement or equation (11). But is it really possible that Einstein would fail to notice that he need only impose these covariance restricting requirements in the circumstances of the Newtonian limit? He would see that a specialized form of the metric is admissible in these special circumstances. But he must somehow overlook that a restriction of covariance in these special circumstances is also admissible.

Mistakes and oversights are all too common in science. We enter them into the historical record readily when we have evidence for them. This is one for which we have no unequivocal evidence and we have indications that speak against it. It must happen in Einstein's area of greatest expertise and concern. And it must not be a momentary lapse. It must persist[28] into the development of the "Entwurf" at least up to the development of his general arguments against general covariance later in 1913.

A Transition from Coordinate Conditions to Coordinate Restrictions?

Our evidence on Einstein's awareness or lack of awareness of the use of coordinate conditions remains incomplete. Yet all these considerations make it most credible that Einstein was aware of their use and could have considered using requirements such as the harmonic and equation (11) as both coordinate conditions and coordinate restrictions. Let us go a little further. If we had to choose a single narrative that would fit best with the progression of calculations in the notebook, it would be this.

When the harmonic requirement is introduced on page 19L, it is used as a coordinate condition, with Einstein perhaps reserving the possibility of using it as a coordinate restriction if that should prove viable and simpler. On page 22R, requirement (11) is introduced as a coordinate condition with T_{il}^{x}, chosen as the gravitation tensor. However he is unable to see how to use T_{il}^{x} as his gravitation tensor. So he decides he must look for simpler expressions. He reverts to use of requirement (11) as a coordinate restriction so that he can use the simpler gravitation tensor (15), the reduced form of T_{il}^{x}. That also proves inadmissible, presumably because of its restricted covariance. So, on the following page, Einstein introduces the theta restriction, which can only be a coordinate restriction. It is especially contrived to have the covariance that requirement (11) lacked.

What makes it credible that page 22R is the turning point is Einstein's labeling of T_{il}^{x} as the "presumed gravitation tensor" when he must have known already that equation (11) would reduce it to the Newtonian form. That suggests that equation (11) was first introduced as a coordinate condition. The investigation of its covariance properties on page 22L marks the decision to treat the requirement as a coordinate restriction.[29] In the earlier pages 19–21, the harmonic requirement could have been either coordinate condition or restriction. Nothing in the calculations would have committed Einstein to either. The lack of interest in the covariance properties of the harmonic requirement suggests that Einstein had less interest in its use as a coordinate restriction.

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28 Thoughts of the use of condition (11) did not leave Einstein after the "Entwurf" theory was completed. As late as August 1915, he recalled in a letter to Paul Hertz how he had struggled with this condition, (Einstein to P. Hertz, August 22, 1915, (CPAE 8, Doc. 111)). Presumably this continued presence facilitated revival of T_{il}^{x} in November 1915.

If these suppositions are correct, then they bear directly on the "mathematical" and "physical strategy" we describe Einstein as using elsewhere in these volumes. The use of coordinate conditions would be associated with the mathematical strategy in its purest form. If recovery of the Newtonian limit will be through the harmonic condition, for example, then Einstein is able to use the full Ricci tensor as his gravitation tensor and not a simpler reduced form. With his failure to see that the Ricci tensor or that T_{il}^{x} are viable gravitation tensors, Einstein begins to withdraw from the mathematical strategy towards the physical strategy. The use of coordinate restrictions represents an intermediate stage in that withdrawal. He is still trying to use the gravitation tensors naturally suggested by the mathematical formalism, but now in reduced form. The failure of these last efforts leads him to revert to the physical strategy.

3. A CONJECTURE: THE HOLE ARGUMENT AND THE INDEPENDENT REALITY OF COORDINATE SYSTEMS

The Puzzle Continues

If we accept that Einstein knew about the possibility of using coordinate conditions, page 22R once again presents us with a troubling puzzle. In his later recollection to Sommerfeld, Einstein reports that he had been unable to recover the Newtonian limit from the gravitation tensor T_{il}^{x}. But page 22R contains just the calculation that seems to do this. As we saw in section 2, the remark could be explained using the notion of coordinate restrictions. But that explanation fails if we accept that Einstein was aware of the use of coordinate conditions. So how can we reconcile his later recollection with the content of the notebook?

There is a further aspect of page 22R that bears cautious reflection. Page 22R should have been a great triumph for Einstein. He had labored since page 14L through calculations of great complexity in an effort to recover a gravitation tensor from the Riemann tensor. The problem seemed to yield on page 19L with the introduction of the harmonic condition and the easy reduction of the Ricci tensor to a quantity of Newtonian form (8). But the success faded over the following pages in the face of a final hitch that grew to be fatal. He could not see how to reconcile the harmonic condition with the form he expected for the static field, the weak field equa-

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29 Is there a trace of two stages of calculation on page 22R? The calculations there are divided by a horizontal rule. The calculations above the rule deal only with the term in T_{il}^{x} that contains second derivatives of the metric tensor and its reduction by equation (11) to the Newtonian form (8). Those below deal with expansion of the terms quadratic in the Christoffel symbols in T_{il}^{x}. The calculations above the rule are the ones needed if equation (11) is to be used as a coordinate condition; in the Newtonian limit all that matters are the terms in the second derivatives of the metric. The ones below are needed in addition if (11) is used as a coordinate restriction; they give the full expression for the reduced form of T_{il}^{x}. The calculations above the rule are neater and, as I suggested earlier, may just report discussions and calculations conducted elsewhere. Those below the rule are massively corrected and have the look of live calculations. Were they undertaken later after Einstein had decided to revert from the use of (11) as a coordinate condition to a coordinate restriction?

tions and energy conservation in the weak field. On page 22R he finally had the answer to that last hitch. By choosing T_{il}^{x} as his gravitation tensor, he could replace the harmonic condition with condition (11) and this new condition resolved all the earlier problems, since it was both compatible with the form expected for static fields and with energy conservation in the weak field. The solution was so unobjectionable that he published it upon his return to general covariance in November 1915. But in the notebook, that successful solution is just abandoned and apparently quite hastily. His later recollections explain this decision in terms of the $\{\}$ prejudice. Just when he had everything else working, he gave up because, on the best reconstruction, he could not see how to extract an energy-momentum tensor for the gravitational field from the tensor. He gave up more than just the gravitation tensor T_{il}^{x}. He seems to have given up the use of coordinate conditions entirely and with it the easy access to the gravitation tensors of broad covariance naturally suggested by the mathematical formalism. If the $\{\}$ prejudice was all there was to it, Einstein had lost his customary tenacity and become fickle or faint hearted or both.

Might there have been a further difficulty that compromised the recovery of the Newtonian limit and that he did not report?

Another Error?

Might we find another error or misconception that Einstein may have committed that would give answers to both the above questions? Of course it is always possible to invent hidden errors varying from the trivial slip to the profound confusion, tailor made to fit this or that aberration. The real difficulty is to establish that the error was really committed.

If there is such an error, we would expect it to be somehow associated with the use and understanding of coordinate systems. We do know of a serious misconception concerning coordinate systems that drove Einstein away from general covariance during the years of the "Entwurf" theory. This was the misconception that supported the hole argument. Months after the completion of the "Entwurf" theory, Einstein introduced this argument as a way of showing that the achievement of general covariance in his gravitation theory would be physically uninteresting. After he had returned to general covariance Einstein explained the error of the hole argument. He had unwittingly accorded an independent reality to spacetime coordinate systems and this had compromised his understanding of what is represented physically in a transformation of the fields of his theory.[30] In our histories to date, this error affected Einstein only through the hole argument and thus well after Einstein's turn away from

30 More precisely stated: A particular set of coordinate values in a coordinate system will designate a definite physical event in spacetime. In Einstein's later view and our modern view, the physical event designated depends on the metric field; an alteration of the metric field changes which physical event is designated by these coordinate values. Einstein initially believed, however, that these same coordinate values could continue to pick out the same physical event even though the metrical field in that coordinate system was changed. That is, the coordinate system's power to pick out events is independent of the metrical field.

general covariance in 1912 and 1913. However Einstein's theory was, on his own report, dependent intimately and fundamentally on the transformation of fields and spacetime coordinates. Is it possible that Einstein's misconception on the independent reality of coordinate systems had earlier damaging effects?

The Conjecture

The conjecture to be advanced here is that Einstein's misconception about the independent reality of coordinate systems did not just exert its harmful influence with Einstein's discovery of the hole argument, well after the "Entwurf" theory was in place. Rather I shall urge that it decisively misdirected Einstein's investigations at a much earlier stage, the time of the calculation of the notebook. I believe that it can explain why Einstein abandoned the use of coordinate conditions so precipitously, why he would have judged the calculation concerning the Newtonian limit of page 22R to be a failure and why he acquiesced so readily to the gravely restricted covariance of the "Entwurf" theory. Einstein failed to see this error until 1915. Until then it precluded his use of coordinate conditions. It led him to expect that any coordinate condition must have sufficient covariance to support an extension of the principle of relativity to acceleration.

More specifically, I will suggest that when Einstein applied a coordinate condition such as (11), he unwittingly accorded an independent existence to the coordinate systems picked out by the condition. Then, merely by repeating the same way of thinking about transformations as used in the hole argument, he would end up entertaining extraordinary expectations for each of these special coordinate systems. If some metric field $g_{\mu\nu}$ is a solution of his field equations in one of these special coordinate systems, then he would expect all its transforms (in a sense I will make clear below) $g'_{\mu\nu}$ also to be realizable as solutions in this coordinate system. A failure of the coordinate system to admit these transforms would appear as an objectionable, absolute property of the coordinate system. Such properties are just the type that Einstein had denounced in the inertial systems of classical physics and special relativity and which he promised his new theory would eliminate. Now the transforms $g'_{\mu\nu}$ will only be admissible in the special coordinate system if they are compatible with the coordinate condition that defines the special coordinate system. Thus the covariance of theory as a whole had effectively been reduced to the covariance of the coordinate condition used in extracting the Newtonian limit. That condition had to be of sufficient covariance to support Einstein's hopes for a generalization of the principle of relativity to acceleration. In spite of proposals of great ingenuity in his preparation for the "Entwurf" theory, Einstein could find no combination of gravitational field equations of broad covariance and coordinate condition that satisfied these extraordinary demands.

The effect of the misconception conjectured is that coordinate conditions would lose their appeal. If a coordinate condition was used to extract the Newtonian limit, the covariance of the theory as a whole would now be reduced to that of the coordinate condition. As a result, Einstein would acquire no additional covariance for his theory in using a requirement as a coordinate condition rather than a coordinate restriction. The advantage of the latter, however, is that it delivers a gravitation tensor of considerably simpler form. Therefore I suggest that Einstein's recognition of this outcome, quite plausibly on page 22R itself, would explain why he so precipitously abandoned coordinate conditions in the notebook. The extraction of the Newtonian limit from tensor T_{il}^{x} via equation (11), whether it is construed as a coordinate condition or restriction, would fail for the same reason. Equation (11) would fail to have sufficient covariance.

Its Tacit Character

In the hole argument, the independent reality of the coordinate systems has a tacit, hidden character. Indeed Einstein found it hard to express explicitly what he meant. Even something as simple as the exact steps of his construction really only became clear with publication of the fourth version of the argument. It was not until after his return to covariance and possibly some prompting from his correspondents that he seemed able to give a clear account of where the argument erred. We must surely presume that, at the time of the hole argument, Einstein was simply not aware that his manipulations presumed an independent reality for his coordinate systems. It is an essential part of the present conjecture that he was not aware of the corresponding presumption earlier at the time of the calculations of the notebook. The hole argument was first offered in a hasty, ill-digested form that still led to a powerful conclusion, the inadmissibility of general covariance. The same would be true in the notebook. A similarly hasty check of the covariance of the coordinate condition would suffice to convince Einstein that disaster had struck. Its haste would allow him to overlook that his conclusion depended upon an assumption about the independent reality of coordinate systems that he would surely never endorse if it were articulated clearly.

In the Sections to Follow…

I will layout the background, context and elaboration of the conjecture. In section 3.1, I will describe the hole argument and, in section 3.2, how Einstein later diagnosed his error as the improper attribution of an independent reality to coordinate systems. In section 3.3 I will lay out the content of the conjecture in greater detail. Einstein's treatment of coordinate systems founders since it ends up ascribing absolute properties to certain coordinate systems. In section 3.4, I will review Einstein's insistence on the inadmissibility of such absolute properties, for that inadmissibility is what defeats his use of coordinate conditions. In section 3.5, I will review Einstein's early remarks on the restricted covariance of his "Entwurf" theory and his recognition that the restricted equations must correspond to generally covariant equations. I will use Einstein's mistaken attitude to the independent reality of the coordinate systems to explain his evident indifference towards finding those equations. During the reign of the "Entwurf" theory, Einstein gave several accounts of the introduction of specialized coordinate systems. In section 3.6, I will review these remarks to show that they are compatible with the present conjecture concerning Einstein's attitude to coordinate systems. Finally in section 3.7, I will review our evidence concerning the conjecture. I will conclude that we have neither decisive evidence in favor of it or against it, but weaker indications that both benefit and harm it.

3.1 The Hole Argument

Its Fullest Statement

Einstein and Grossmann's "Entwurf" paper was published mid 1913 as a separatum by Teubner (Einstein and Grossmann 1913).[31] There they reported their failure to find acceptable, generally covariant gravitational field equations. By late 1913, Einstein had found what soon became his favored vehicle for excusing this lack of general covariance, the "hole argument," which purported to show that all generally covariant gravitational field equations would be physically uninteresting.[32] Of its four presentations, the clearest is the final version of November 1914 (Einstein 1914c, 1067); Einstein's emphasis:

> *Proof of the necessity of a restriction on the choice of coordinates.*
>
> We consider a finite region of the continuum Σ, in which no material process takes place. Physical occurrences in Σ are then fully determined, if the quantities $g_{\mu\nu}$ are given as functions of the x_ν in relation to a coordinate system K used for description. The totality of these functions will be symbolically denoted by $G(x)$.
>
> Let a new coordinate system K' be introduced, which agrees with K outside Σ, but deviates from it inside Σ in such a way that the $g'_{\mu\nu}$ related to the K' are continuous everywhere like the $g_{\mu\nu}$ (together with their derivatives). We denote the totality of the $g'_{\mu\nu}$ symbolically with $G'(x')$. $G'(x')$ and $G(x)$ describe the same gravitational field. In the functions $g'_{\mu\nu}$ we replace the coordinates x'_ν with the coordinates x_ν, i.e. we form $G'(x)$. Then, likewise, $G'(x)$ describes a gravitational field with respect to K, which however does not agree with the real (or originally given) gravitational field.

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31 In a letter of May 28, 1913 to Paul Ehrenfest, Einstein promises that paper will appear "in at least a few weeks" (CPAE 5, Doc. 441).

32 The earliest written and unambiguously dated mention of the hole argument is in a letter of Nov. 2, 1913, from Einstein to Ludwig Hopf, (CPAE 5, Doc. 480). Einstein is not likely to have had the hole argument in hand much earlier than this. The hole argument supplanted another means of exonerating his theory's lack of general covariance, a consideration based on the law of conservation of energy-momentum. We know from a letter of Aug. 16, 1913, from Einstein to Lorentz that Einstein only hit upon this earlier consideration on August 15 (CPAE 5, Doc. 470). For further discussion see (Norton 1984, §5). (The hole argument is also mentioned in the printed version of a lecture delivered on Sept. 9 to the 96th annual meeting of the Schweizerische Naturforschende Gesellschaft in Frauenfeld (Einstein 1914b, 289). But we cannot be assured that Einstein had the hole argument at the time of the lecture since the printed version of the lecture was published many months later on March 16, 1914, see (CPAE 4, 484). Also the hole argument is not mentioned in another, briefer, printed version of the talk (Einstein 1913). That briefer version does call for a restriction on the basis of the conservation laws. It is curious that the mention of the hole argument in the printed version of (Einstein 1914b) appears in the context of the discussion of the conservation laws. In this longer and presumably later version, did Einstein strike out the consideration based on the conservation laws and write in a mention of the hole argument?)

> We now assume that the differential equations of the gravitational field are generally covariant. Then they are satisfied by $G'(x')$ (relative to K'), if they are satisfied by $G(x)$ relative to K. Then they are also satisfied by $G'(x)$ relative to K. Then relative to K there exist the solutions $G(x)$ and $G'(x)$, which are different from one another, although both solutions agree in the boundary region, *i.e. occurrences in the gravitational field cannot be uniquely determined by generally covariant differential equations for the gravitational field*.

A Notational Convenience

The content, interpretation and significance of the hole argument has been examined extensively elsewhere.[33] Thus I will concentrate on those aspects of importance to the present conjecture. The argument depends on exploiting the defining property of a covariance group to generate new solutions of the gravitational field equations from old solutions. Assume that a transformation from coordinate system x^μ to x'^μ is within the covariance group of the gravitational field equation and that a metric field $g_{\mu\nu}(x_\alpha)$ in the coordinate system x_μ satisfies the field equations. It follows that the metric $g'_{\mu\nu}(x'_\alpha)$ in the coordinate system x'_μ, defined by the tensor transformation law

$$g'_{\mu\nu} = \frac{\partial x_\alpha}{\partial x'_\mu}\frac{\partial x_\beta}{\partial x'_\nu}g_{\alpha\beta} \tag{21}$$

is also a solution of the field equations. These two solutions of the field equations are merely representations in different coordinate systems of the same gravitational field; it is represented by $g_{\mu\nu}(x_\alpha)$ in the coordinate system x_α and by $g'_{\mu\nu}(x'_\alpha)$ in the coordinate system x'_α. In an attempt to reduce distracting notational complications, Einstein represented the two metrics as "$G(x)$" and "$G'(x')$". His point was to draw attention to the functional dependence of the $g_{\mu\nu}$ on the coordinates x_α with the latter considered as variables; and the functional dependence of the $g'_{\mu\nu}$ on x'_α. The device is helpful, since it suppresses the various indices that play no role in Einstein's argument. I will use it below but with lower case g instead of upper case G:

$$\begin{gathered} g_{\mu\nu}(x_\alpha) \text{ is represented by } g(x) \\ g_{\mu\nu}(x'_\alpha) \text{ is represented by } g(x') \\ g'_{\mu\nu}(x'_\alpha) \text{ is represented by } g'(x'). \end{gathered} \tag{22}$$

How the Argument Works

This functional dependence allows Einstein to generate a further solution of the gravitational field equations that is, apparently, physically distinct from the original field described by $g(x)$ and $g'(x')$. It is constructed by considering the solution $g'(x')$ as a set of ten functions of the four independent variables comprising the coordinate sys-

33 See, for example, (Stachel 1980, §§3–4; Norton 1984, §5; 1987).

tem x'. One then replaces the independent variables x' by x, so that Einstein recovers a new field in the original coordinate system x, which is $g'(x)$. Now $g'(x')$ is a solution of the gravitational field equations not because of any special properties of the coordinate system x' but merely because of the functional dependence of the $g'(x')$ on the independent variables x'. That functional form is all that generally covariant gravitational field equations consider in determining whether $g'(x')$ is admissible. By construction, $g'(x)$ shares exactly the same functional dependence on its independent variables as $g'(x')$. Thus if $g'(x')$ is a solution of the field equations so is $g'(x)$.[34]

Einstein can now complete his argument. He has two solutions of his gravitational field equations $g(x)$ and $g'(x)$, both in the *same* coordinate system x. These two solutions were constructed from the transformation x to x'. This transformation had a special property. By supposition the transformation is the identity everywhere but inside a matter free region of spacetime Σ (the "hole"), where it comes smoothly to differ from the identity. This special property confers a corresponding property on the two solutions $g(x)$ and $g'(x)$: they agree outside the hole, but they come smoothly to disagree within, for the g and g' are different functions within that hole. And since they are defined in the same coordinate system, this difference entails, Einstein urged, that they represent physically distinct gravitational fields. The result is a violation of determinism. The metric field and matter distribution outside the matter free hole fails to determine how the metric field will extend into the hole; it may extend as $g(x)$ or $g'(x)$. Einstein deemed this circumstance sufficiently troublesome to warrant rejection of all generally covariant gravitational field equations, for all generally covariant field equations will admit solution pairs $g(x)$ and $g'(x)$.

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34 An example illustrates the reasoning. The metric

$$g'(x') = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & (x'^1)^2 \end{bmatrix}$$

happens to be a solution of the generally covariant gravitational field equations $R_{\mu\nu} = 0$, where $R_{\mu\nu}$ is the Ricci tensor, in a coordinate system x'. What makes this g' a solution is the way that each coefficient of g' depends functionally on the coordinates x'. All coefficients are 0 or -1 excepting g'_{44} which is the square of the coordinate x'^1. We find $R_{\mu\nu}$ vanishes if we compute it for a g' with this functional dependence. It now follows immediately that the metric

$$g'(x) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & (x^1)^2 \end{bmatrix}$$

in the coordinate system x is also a solution since it shares this same functional dependence.

The Hole Construction

For our purposes what is important is that Einstein saw in the general covariance of the gravitational field equations an immediate license to construct the field $g'(x)$ from $g(x)$. This construction will be the focus of our attention, so I will restate it:

> *If*
>
> (a) a transformation x to x' is within the covariance group of the gravitational field equations and
>
> (b) a metric field $g(x)$ in the coordinate system x satisfies the field equations,
>
> *then*
>
> the metric field $g'(x)$ is also a solution of the gravitational field equations in the *original* coordinate system x, where the functions g' are defined by the tensor transformation law (21).

Einstein's Difficulty in Expressing the Argument

Einstein found it very hard to make clear that his hole argument depended essentially on the use of the hole construction. Rather, the three earlier versions of the hole argument seemed to depend on merely noticing that the two coordinate representations $g(x)$ and $g'(x')$ of the same gravitational field employed different functions. In that case the hole argument becomes the elementary blunder of failing to notice that the one gravitational field has different representations in different coordinate systems. I take this as evidence that, in his own work, Einstein did not clearly distinguish the two types of transformations $g(x)$ to $g'(x')$ and $g(x)$ to $g'(x)$. His invocation of the transformation law (21) could refer to either, without the need for explanation or apology. As we shall see below, Einstein's early presentations of the hole argument merely invoked (21) and Einstein must have presumed that readers would follow him and understand the transformation under consideration to be $g(x)$ to $g'(x)$.

The hole argument appears in Einstein's 1914 addendum to (Einstein and Grossmann 1913) where the crucial passage reads "...one can obtain $\gamma'_{\mu\nu} \neq \gamma_{\mu\nu}$ [for the metric field $\gamma_{\mu\nu}$] for at least a part of [the hole] L...it follows...that more than one system of the $\gamma_{\mu\nu}$ is associated with the same [matter distribution]."[35] In a later version (Einstein and Grossmann 1914, 218), the corresponding passage reads "at least for a part of [the hole] L $\gamma_{\mu\nu}' \neq \gamma_{\mu\nu}$ ... so that more than one system of $\gamma_{\mu\nu}$ is associated with the same system of [stress-energy tensor] $\Theta_{\mu\nu}$ ... " Again, in the version of the hole argument of Einstein in (Einstein 1914a, 178), the corresponding passage reads "...even though we do have $\mathfrak{T}_{\sigma\nu}' = \mathfrak{T}_{\sigma\nu}$ everywhere [for stress-energy tensor density $\mathfrak{T}_{\sigma\nu}$], the equations $g_{\mu\nu}' = g_{\mu\nu}$ are certainly not all satisfied in the interior of [the hole] Φ. This proves the assertion."[36] Fortunately Einstein gave a cryptic but sufficient clue in this last instance that he intended the failure of the equality

35 Translation from (CPAE 4E, 289).

36 Translated in (CPAE 4E, 285).

$g_{\mu\nu}' = g_{\mu\nu}$ to be understood in the manner of the hole construction above, for he appended a footnote to the sentence containing the inequality $g_{\mu\nu}' = g_{\mu\nu}$ that read: "The equations are to be understood in such a way that the same numerical values are always assigned to the independent variables x'_ν on the left sides as to the variables x_ν on the right sides."

These presentations were sufficiently ambiguous to confuse the early historical literature on the hole argument. It interpreted Einstein as believing that the two coordinate representations of the same field, $g(x)$ and $g'(x')$, somehow represented physically distinct fields. One of the achievements of Stachel in his path-breaking paper (Stachel 1980) was to demonstrate that Einstein was not guilty of this novice blunder.[37]

3.2 The Error of the Hole Argument: The Independent Reality of Coordinate Systems

Why the Hole Argument Fails

Of course the hole argument fails to establish that all generally covariant gravitational field equations are physically uninteresting. The standard resolution allows that the two fields $g(x)$ and $g'(x)$ are mathematically distinct but counters that they represent the same physical field. Thus the hole argument does not show that the field and matter distribution outside the hole leave the field within underdetermined. It just shows that the mathematical description of the field within the hole is undetermined. After his return to general covariance, Einstein argued for the physical equivalence of the fields $g(x)$ and $g'(x)$ with his "point-coincidence argument;" the two fields are equivalent since they must agree on the disposition of all coincidences, such as the intersections of the world lines of particles.[38] Alternatively, following the approach favored in Göttingen by the Hilbert school, we could argue for the equivalence of the two fields by noting that they agree on all invariant properties.[39]

Letters to Ehrenfest and Besso Explain Einstein's Error

The point coincidence argument explains how we should understand the system described in the hole argument. But it does not diagnose the error of thought that lured Einstein to interpret the system differently prior to November 1915. That diagnosis came in Einstein's letters in late 1915 and early 1916 when he explained to his

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37 As Stachel showed, the transformation from $g(x)$ to $g'(x')$ corresponded to what we now call a passive transformation in which the coordinate system changes but not the field. The transformation of the hole construction from $g(x)$ to $g'(x)$ corresponds to an active transformation in which the coordinate system remains unchanged but the field alters. See (Norton 1987; 1989, §2). However, as I argue in (Norton 1989, §5), it is possible to remain faithful to Einstein's purpose and wording without explicitly introducing the notions of active and passive transformations.

38 See (Norton 1987; Howard and Norton 1993, §7) for a proposal on the origin of the point-coincidence argument.

39 See (Howard and Norton 1993) for the proposal of a premature communication of this viewpoint to an unreceptive Einstein by Paul Hertz in the late summer of 1915.

correspondents how he had erred in the hole argument. In preparing his correspondent Paul Ehrenfest for the point coincidence argument, Einstein explained in a letter of December 26, 1915 (CPAE 8, Doc. 173), Einstein's emphasis:[40]

> In §12 of my work of last year, everything is correct (in the first three paragraphs) up to the italics at the end of the third paragraph. One can deduce no contradiction at all with the uniqueness of occurrences from the fact that both systems $G(x)$ and $G'(x)$, related to the same reference system, satisfy the conditions of the grav. field. The apparent force of this consideration is lost immediately if one considers that
>
> (1) the reference system signifies nothing real
>
> (2) that the (simultaneous) realization of the two different g- systems (better said, two different gravitational fields) in the same region of the continuum is impossible according to the nature of the theory.
>
> In the place of §12 steps the following consideration. The physical reality of world occurrences (in opposition to that dependent on the choice of reference system) consists *in spacetime coincidences*...

He wrote an essentially identical explanation to his friend Michele Besso a little over a week later on January 3, 1916 (CPAE 8, Doc. 178), Einstein's emphasis:

> Everything was correct in the hole argument up to the last conclusion. There is no physical content in two different solutions $G(x)$ and $G'(x)$ existing with respect to the *same* coordinate system K. To imagine two solutions simultaneously in the same manifold has no meaning and the system K has no physical reality. In place of the hole argument we have the following. *Reality* is nothing but the totality of space-time point coincidences...

Ehrenfest proved difficult to convince of the correctness of Einstein's new way of thinking over the hole argument and Einstein needed to enter into a more detailed exchange that centered on the example of light from a star passing through an aperture onto a photographic plate.[41] In his letter of Jan. 5, 1916 (CPAE 8, Doc. 180), Einstein noted the instinctive attractiveness of the notion of the reality of the coordinate system:

> I cannot blame you that you still have not seen the admissibility of generally covariant equations, since I myself needed so long to achieve full clarity on this point. Your problem has its root in that you instinctively treat the reference system as something "real."

Surely we are to read in this that Einstein too was misled by this instinct.

On Being Real

We learn from these letters that Einstein was under the influence of a deep-seated prejudice at the time of formulation of the hole argument: he improperly accorded a physical reality to coordinate systems. It can often be difficult to decipher precisely what is meant by an attribution of reality or non-reality—one need only recall the

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40 Adjusted translation from (Norton 1987, 169).

41 For details, see (Norton 1987, §4).

extended debates over realism in philosophy of science to be reminded of these difficulties. But in this case the attribution of reality has quite precise consequences. When Einstein accords physical reality to a coordinate system x, this entails that the coordinate system can support two distinct fields, $g(x)$ and $g'(x)$. In particular, Einstein is committed to the x in each system of metrical coefficients representing the *same* coordinate system. This sameness entails that the two mathematical structures, $g(x)$ and $g'(x)$, represent different physical fields. Some particular set of coordinate values, such as $x^{\alpha} = (0,0,0,0)$, will pick out the same point of spacetime in each field. But, since the g and g' are different functions of the same coordinates in a neighborhood of the point, they will each attribute different properties to that point, revealing that they represent different physical fields.

In Einstein's later view it no longer makes sense to say that x represents the same coordinate system in each structure $g(x)$ and $g'(x)$. Thus we can no longer conclude that some particular set of coordinate values picks out the same point in each field and the inference to their physical distinctness is blocked.

Einstein's misconception about the independent reality of coordinate systems was clearly firmly in place towards the end of 1913, the time of his creation of the hole argument.[42] Nothing we have seen indicates that this misconception arose at that time. Rather his description of its "instinctive" character suggests that Einstein had tacitly harbored this misconception beforehand. Might this misconception have misdirected Einstein's work on his "Entwurf" theory at an earlier stage? In the following I will conjecture that it did in a quite precise way.

3.3 The Conjecture: How the Independent Reality of Coordinate Systems Defeats the Use of Coordinate Conditions

I have urged that Einstein knew about the possibility of coordinate conditions, that he used them in the notebook and then abandoned them in favor of the use of coordinate restrictions. I have even suggested that this transition may have taken place on page 22R of the notebook, in which the same requirement (11) might have been used first as a coordinate condition and then as a coordinate restriction. I now conjecture that Einstein abandoned the use of coordinate conditions because of the same error committed in the context of the hole argument. Einstein unwittingly attributed an independent reality to the coordinate systems introduced by coordinate conditions. The effect was that he mistakenly believed that the covariance of his entire theory was reduced to that of the coordinate condition. The reversion to coordinate restrictions is

42 In describing Einstein's earlier misconception I will speak of his belief that the coordinate system has "*independent* reality," which is to be understood as asserting reality independent of the metrical field. This is because Einstein's later denial of the physical reality of the coordinate system can only apply to a reality independent of the metric. For it is entirely compatible with Einstein's later views that a coordinate system can reflect an element of reality, but only if it does so indirectly by virtue of its relation to the metric defined on the spacetime. For example, the possibility of a coordinate system in which the metrical coefficients are all constant, reflects a real property of the spacetime, its flatness.

now natural. He mistakenly thought that using coordinate conditions to recover the Newtonian limit provided no greater covariance for this theory and the use of coordinate restrictions had the advantage of simplifying the equations of his theory.

The Example of T_{il}^{x}

To see how this notion of the independent reality of coordinate systems would defeat the use of coordinate conditions, we will look at the example of coordinate condition (11) applied to the candidate gravitation tensor T_{il}^{x}. The example illustrates clearly the general argument. It is also of interest in itself since I believe Einstein may well have fallen into the general mistake outlined while considering this very example.

Einstein's essential purpose in considering a structure as complicated at T_{il}^{x} is to achieve the broadest covariance possible for his gravitational field equations. By construction, T_{il}^{x} is covariant under unimodular transformations. We have seen that one particular unimodular transformation comes to special prominence in the pages immediately following the proposal of the gravitation tensor T_{il}^{x}. That is the transformation (18) to uniformly rotating coordinates that brings a rotation field g^{SR} (20) into being in a Minkowski spacetime.[43]

The simple reading of this covariance of the gravitational field equations in the case of a Minkowski spacetime is that it admits the transformation of $g^{SR}(x)$ to the rotation field $g^{ROT}(x')$ under the coordinate transformation (18). They are just the same Minkowski spacetime represented in two coordinate systems x and x'. However we have already seen that when Einstein speaks of such a simple transformation he may actually be referring to a more complicated transformation. In the context of the hole argument, as we saw above, when Einstein wrote about the transformation of a metric g under the transformation of the coordinates x to x', he did not just refer to the transformation of $g(x)$ to $g'(x')$. He also tacitly referred to construction of a new solution of the field equations $g'(x)$ in the original coordinate system x. Indeed Einstein seemed to treat the construction of the new field $g'(x)$ as an automatic consequence of the covariance of the gravitational field equations—so much so that, in three of four presentations of the hole argument, Einstein appears just to refer to the transformation $g(x)$ to $g'(x')$ whereas he intended to refer to the construction of the new field $g'(x)$. Thus Einstein would read the covariance of his gravitational field equations under transformation (18) as the license to take the solution $g^{SR}(x)$ of these equations and construct a new solution $g^{ROT}(x)$, both in the same coordinate system x.

Applying the Hole Construction

Einstein would see this construction as an automatic part of the covariance of his field equations, although its construction requires some manipulation as codified in what I called the "hole construction" above. We may pause here for a moment to affirm that the construction of the new solution $g^{ROT}(x)$ follows directly from the hole con-

43 I shall continue to use the abbreviation (22), so that $g^{SR}(x)$ stands for $g^{SR}_{\mu\nu}(x_{\alpha})$.

struction, although Einstein would surely not have resorted to such a labored development. The two antecedent conditions (a) and (b) are satisfied as:

a) If T^x_{il} is chosen as the gravitation tensor, then the gravitational field equations are covariant under the transformation (18) from inertial to uniformly rotating coordinates, for this is a unimodular transformation.

b) $g^{SR}(x)$ is a solution of the source free field equations $T^x_{il} = 0$.[44]

It now follows that $g^{ROT}(x)$ will also be a solution of the source free field equations in the original coordinate system x.

The Independent Reality of the Coordinate Systems x^{LIM} of the Newtonian Limit…

In his evaluation of T^x_{il}, Einstein would have a particular class of coordinate systems in mind as admitting g^{SR} as a solution. These are the coordinate systems in which the candidate gravitation tensor T^x_{il} reduces to (8) in preparation for recovery of the Newtonian limit. Let us label one of these coordinate systems x^{LIM}. Thus Einstein's field equations must admit both $g^{SR}(x^{LIM})$ and $g^{ROT}(x^{LIM})$ as solutions of the source free field equations in the same coordinate system x^{LIM}.

While these results follow from a straightforward application of Einstein's 1913 understanding of covariance and coordinate systems, they have brought us close to disaster for the candidate gravitation tensor T^x_{il}. To complete the journey to disaster we now must now ask what it would mean to say that these source free field equations must admit both $g^{SR}(x^{LIM})$ and $g^{ROT}(x^{LIM})$ as solutions. In Einstein's *later* view (and the modern view), this could mean nothing more than the following: there exists coordinate systems x in which $g^{SR}(x)$ solves the source free field equations; and there exists coordinate systems y in which $g^{ROT}(y)$ solves the source free field equations. But there can be no physical sense in the notion that the coordinate systems x and y are the same coordinate systems. Yet the Einstein of 1912 and 1913 would be committed to the notion that the coordinate systems x^{LIM} appearing in each solution are the same coordinate systems.

There is only one resource available to give meaning to this sameness. The coordinate systems x^{LIM} of the Newtonian limit are introduced and identified in calculation by satisfaction of the coordinate condition (11). If it is really the same coordinate systems x^{LIM} appearing in each of $g^{SR}(x^{LIM})$ and $g^{ROT}(x^{LIM})$, then coordinate condition (11) must be satisfied by both $g^{SR}(x^{LIM})$ and $g^{ROT}(x^{LIM})$. In hindsight, we know that this demand is excessive. But, I conjecture, the Einstein of 1912 and 1913 did not realize this. There is a natural robustness to the application of coordinate conditions such as (11) in the modern sense that is easily mistaken for the troublesome use of the condition that I attribute to Einstein. It was legitimate in 1912 and 1913 and remains legitimate today to use the same coordinate condition to pick out the coordinate systems for the Newtonian limit in a diverse array of distinct physical situations:

44 $g^{SR}_{\mu\nu}$ has all constant coefficients; so all its derivatives vanish and T^x_{il} along with them.

in the source free case, in the case of static fields, in the case of fields with propagating gravitational waves, in the case of a field produced by a single mass or in the case of a field produced by distributed matter; and in many more cases. Now we might use a condition such as the harmonic coordinate condition rather than Einstein's (11) but that difference is inessential to the point. In using the same harmonic condition in each of these distinct physical cases, we routinely say that we choose harmonic coordinates. Are we always aware that the harmonic coordinates of a Minkowski spacetime are not the same in any physical sense as the harmonic coordinates of a Minkowski spacetime perturbed ever so slightly by the most minute of gravitational waves? Proceeding with the tacit assumption of the independent reality of coordinate systems, Einstein could easily overlook this subtlety. It would surely be quite natural for him to presume that his coordinate condition (11) would pick out the same coordinate systems x^{LIM} in all these cases and also in the case of g^{SR} and g^{ROT}.

Treated this way, the coordinate condition (11) becomes a physical postulate that picks out a real entity, the class of coordinate systems x^{LIM}, much as the gravitational field equations pick out the gravitational fields that can be realized physically. This character of the coordinate condition (11) does not compromise our freedom to stipulate the coordinate systems that we will use in describing our fields. We are still free to choose which coordinate systems we will use and that choice can be made by accepting or rejecting a coordinate condition such as (11). But that choice is among entities that enjoy some physical reality.

…Brings Disaster and Explains Why Einstein Would Check the Covariance of His Coordinate Condition

Thus I infer that the Einstein of 1912 and 1913 would expect that the condition (11) picks out the same coordinate systems x^{LIM} in the cases of the solutions $g^{SR}(x^{LIM})$ and $g^{ROT}(x^{LIM})$. This is the disastrous conclusion. While the coordinate condition (11) holds for $g^{SR}(x^{LIM})$, we saw above that it fails for $g^{ROT}(x^{LIM})$. Einstein has arrived at a contradiction that serves as a *reductio ad absurdum* of his choice of T_{il}^{x} as gravitation tensor and the expectation that his theory is covariant under all unimodular transformations. If the theory has that degree of covariance, $g^{ROT}(x^{LIM})$ must be a solution of its source free field equations in the coordinate system x^{LIM}. But it is not. The proposed gravitation tensor has failed.

This is a failure of coordinate condition (11) to have sufficient covariance. Under the normal understanding of coordinate conditions, Einstein would have no reason to check the covariance of (11). But if Einstein accords independent reality to the coordinate system x^{LIM}, then the natural outcome is to check its covariance. If the present conjecture is correct, this explains why Einstein checked the covariance of condition (11) on page 22L, the one facing the page on which condition (11) is used to reduce T_{il}^{x} to a Newtonian form.

This contradiction between the expected and actual covariance of Einstein's theory would appear to have a particular character to Einstein, a conflict between the covariance of his theory and the ability to recover the Newtonian limit. Upon choosing T^x_{il} as the gravitation tensor, his entire gravitation theory would be covariant at least under unimodular transformations. That is, the gravitational field equations are covariant under unimodular transformations and the remaining equations governing energy-momentum conservation, the motion of particles and the electromagnetic field are generally covariant. However if the theory admits coordinate systems in which the Newtonian limit can be realized, then the theory loses its broad covariance. In particular, it loses covariance under transformations to uniform rotation, so that Einstein could no longer conceive of uniform rotation as a rest state, in contradiction with his requirement of a generalized principle of relativity.

The Problem Generalized

The power attributed to the coordinate condition (11) does not depend on any specific properties of the gravitation tensor T^x_{il} or the coordinate condition (11). The arguments rehearsed here would proceed equally with any candidate gravitation tensor of suitably broad covariance and any coordinate condition able to reduce that gravitation tensor to the form (8). Again, the argument does not require that the transformation be a rotation transformation (18). Any transformation in the covariance group of the gravitational field equations could be used. Thus, if the conjecture is correct, Einstein must have held very restrictive expectations for the covariance of his emerging general theory of relativity, whatever its gravitation tensor might be.

To find these expectations, we generalize the argument above for any gravitation tensor, any transformation in the covariance group of the resulting field equations and any coordinate condition that reduces the gravitation tensor to the form (8). For the case of the gravitation tenor T^x_{il}, the coordinate condition (11) picks out the class of coordinate systems x^{LIM} in which the Newtonian limit obtains and the gravitation tensor has form (6). Correspondingly for some gravitation tensor $G_{\mu\nu}$ of broad covariance, a coordinate condition $C_\alpha = 0$ will pick out the coordinate systems in which the Newtonian limit obtains and the gravitation tensor reduces to form (8). Since rotation transformation (18) is in the covariance group of T^x_{il}, Einstein would expect through the hole construction that the two metrics g^{SR} and g^{ROT}, related by this transformation, are admissible as solutions in this coordinate system x^{LIM}. But this can only obtain if the coordinate condition (11) is covariant under rotation transformation (18). Correspondingly, if g and g' are solutions of the (source free) gravitational field equations based on the gravitation tensor $G_{\mu\nu}$, Einstein would expect, through the hole construction, that they are solutions of the reduced gravitational field equations in the limit coordinate system. But this can only obtain if the coordinate condition $C_\alpha = 0$ is covariant under the transformation that takes g to g'. That is, Einstein would expect the following results (C1), (C2) and (C3), to obtain:

> (C1) The covariance of the theory as a whole is limited to the covariance of the coordinate condition used to pick out the coordinate systems in which the Newtonian limit is realized.

For the covariance of that coordinate condition delimits the transformations admissible for solution of the field equations in those coordinate systems.

> (C2) The covariance of the gravitational field equations, *after* they have been reduced by the coordinate condition to the form (8), defines the covariance of the theory as a whole.

For these reduced gravitational field equations just result from the conjunction of the unreduced gravitational field equations and the coordinate condition so that their covariance is limited by the covariance of the coordinate condition. (In both (C1) and (C2), if the unreduced gravitational fields equations have restricted covariance, then these conditions also limit the covariance of the theory as a whole.)

> (C3) In a viable theory, the coordinate condition used and the resulting reduced gravitational field equations will still exhibit broad covariance, including covariance under the rotation transformations (18), so that they admit $g_{\mu\nu}^{ROT}$ as a solution.

If the covariance required in (C1) or (C2) does not include acceleration transformations, such as the rotation transformation (18), then the theory fails to meet the demands of a generalized principle of relativity. It harbors covariance restricting coordinate systems akin to the objectionable, absolute inertial systems of classical mechanics and special relativity (see below).

If the present conjecture is correct, Einstein would adopt (C1), (C2) and (C3). The immediate outcome would be that there is no gain is using a requirement like (11) as a coordinate condition rather than a coordinate restriction. In either use, the equation will impose the same restriction on his gravitation theory's covariance. But the advantage of using coordinate restrictions is that they allow for simpler gravitational field equations.

Moreover, let us suppose that Einstein came to see (C1), (C2) and (C3) as a part of his evaluation of the candidate gravitation tensor T_{il}^{x} on page 22R. Then his natural response would be to discontinue the use of coordinate conditions, as he does after page 22R. Indeed his construction of the theta condition on page 23L would be a natural next step. He abandons coordinate conditions in favor of coordinate restrictions, so he contrives a coordinate restriction specifically to have the rotational covariance lacked by (11).

3.4 The Problem of Absolute Coordinates

The cause of the difficulty is the coordinate systems x^{LIM}, essential for the recovery of the Newtonian limit. Throughout his scientific life Einstein had railed against the objectionable, absolute properties of inertial coordinate systems. The coordinate systems x^{LIM} had now adopted just those objectionable properties and Einstein could not tolerate their presence in his theory. Einstein had made quite clear that the fundamental goal of his general theory of relativity was to eliminate exactly these preferred systems of coordinates.

His Denunciations Persist from his Early Work…

Typical of his denunciations of such systems were his remarks in (Einstein 1914a, 176),[45] written in the early days of the "Entwurf" theory:[46]

> The theory presently called "the theory of relativity" [special relativity] is based on the assumption that there are somehow preexisting "privileged" reference systems K with respect to which the laws of nature take on an especially simple form, even though one raises in vain the question of what could bring about the privilegings of these reference systems K as compared with other (e.g., "rotating") reference systems K'. This constitutes, in my opinion, a serious deficiency of this theory.

The privileging of the reference system K in special relativity resides in the fact that only in K do free bodies move inertially (the "specially simple form" of the laws of motion of free bodies), whereas in K' they move under the influence of a rotation field. K and K' cannot switch roles. K cannot admit a rotation field while bodies move inertially in K'. Of course Einstein was not referring in these remarks to the special coordinate systems x^{LIM} introduced in the Zurich notebook. However, these special coordinate systems have exactly the properties that Einstein found objectionable in K: the coordinate systems x^{LIM} admit g^{SR} so that free bodies will move inertially in x^{LIM}. But x^{LIM} does not admit the rotation field g^{ROT}.

The presence of such absolute coordinate systems would cut Einstein to the quick. In the course of nearly half a century of writing on the general theory of relativity, Einstein found the need to reappraise much of what he wrote on the foundations of his theory. His vacillations on Mach's Principle are probably the best known instance. But he never wavered in his insistence that the absolute of the inertial system must be eliminated. These sentiments supported the need for a generalization of the principle of relativity to acceleration when Einstein wrote his explanatory texts, the popular (Einstein 1917, Ch.XXI) and the textbook (Einstein 1922). The latter read (page 55):

> All of the previous considerations have been based upon the assumption that all inertial systems are equivalent for the description of physical phenomena, but that they are preferred, for the formulation of the laws of nature, to spaces of reference in a different state of motion. We can think of no cause for this preference for definite states of motion to all others, according to our previous considerations, either in the perceptible bodies or in the concept of motion; on the contrary, it must be regarded as an independent property of the space-time continuum. The principle of inertia, in particular, seems to compel us to ascribe physically objective properties to the space-time continuum. Just as it was consistent from the Newtonian standpoint to make both the statements, *tempus est absolutum, spatium est absolutum,* so from the standpoint of the special theory of relativity we must say, *continuum spatii et temporis est absolutum*. In this latter statement *absolutum* means not only "physically real," but also "independent in its physical properties, having a physical effect, but not itself influenced by physical conditions."

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45 Translated in (CPAE 4E, 282).

46 Again writing at the time of the "Entwurf" theory, Einstein (Einstein 1913, 1260) expressed similar sentiments when he spoke of "...reference systems with respect to which freely moving mass points carry out rectilinear uniform motion (inertial systems). What is unsatisfactory is that it remains unexplained *how* the inertial systems can be privileged with respect to other systems." Translation in (CPAE 4E, 219), Einstein's parentheses and emphasis.

…To His Final Years

These sentiments persist essentially unchanged in the final years of his life. In a letter of December 28, 1950, Einstein explained to D. W. Sciama his concern over the latter's theory of restricted covariance; the equations held in coordinate systems in the set M but not in the forbidden set M^x: [47]

> We now ask: on what basis can natural laws hold with respect to M but not with respect to M^x? (Logically considered, both sets M and M^x are after all completely equivalent.) If one takes the theory really seriously, there is only one answer: the preference for M over M^x is an independent physical property of space, which must be added as a postulate to the field equations, so that the physical theory as a whole can have a clear meaning. Newton recognized this with complete clarity ("Spacium est absolutum"). In fact, each theory based on a subgroup introduces an "absolute space", only one that is "less absolute" than classical mechanics.
>
> It was first achieved in G. R., that a space with independent (absolute) properties is avoided. There first are the laws, as they are expressed through the field equations, *complete* and require no augmenting assumptions over physical space. "Space" subsists then only as the continuum property of the physical-real (field), not as a kind of container with independent existence, in which physical things are placed.

These same sentiments would apply to x^{LIM}. In resisting admission of g^{ROT}, the coordinate system would be restoring independent, absolute properties to spacetime, properties that went beyond what was given through the field equations. Einstein would shortly characterize just such behavior as a reversion to the flawed viewpoints of Antiquity. To George Jaffé on January 19, 1954, he wrote (EA 13 405):

> You consider the transition to the special theory of relativity as the most essential of all the ideas of the theory of relativity, but not the transition to the general theory of relativity. I hold the reverse to be true. I see the essential in the conquest of the inertial system, a thing that acts on all processes but experiences no reaction from them. This concept is in principle no better than the central point of the world in Aristotelian physics.

3.5 The Structure and Program of the Entwurf Theory

Explaining Einstein's Indifference to General Covariance

According to the accounts developed in this volume, at the time of the creation of his "Entwurf" theory, Einstein thought rather differently from his later views on coordinate systems. There appears to be a trace of this difference in his early discussion of the limited covariance of his "Entwurf" theory. That is, he was curiously indifferent about discovering the generally covariant gravitational field equations that he believed must correspond to his "Entwurf" equations. Once Einstein has developed general arguments against the admissibility of general covariance, we need not search

47 The typescript of the letter is EA 20–469. The autograph manuscript, EA 20 470, contains an extra sentence given in parentheses here as the second sentence, ("Logically considered..."). (EA 20–469 denotes the item with control number 20–469 in the Einstein Archive.)

for a reason for this indifference. But these arguments emerged only later in 1913, after the "Entwurf" was published. We need some explanation for this indifference in the intervening months.

The accounts discussed in this paper supply them. The "Entwurf" equations would be recovered from generally covariant equations by application of a coordinate condition. So, if Einstein accorded an independent reality to the coordinate systems so introduced, then his indifference would be explained by the misapprehension that his theory overall would gain no added covariance from the transition to these generally covariant equations. Or, more simply, if Einstein was just unaware of the use of coordinate conditions, then he would be unaware of how to retain the "Entwurf" gravitational field equations for the essential case of the Newtonian limit, so the generally covariant equations would appear unusable within his theory.

The Restricted Covariance of the "Entwurf" theory

Einstein's exploration of T_{il}^{x} and the theta restriction are some of his final efforts in the Zurich notebook to recover gravitational field equations from covariance considerations. These efforts halt decisively on pages 26L–26R, where Einstein laid out in capsule the derivation of the gravitational field equations of the "Entwurf" theory. This derivation uses no covariance considerations at all. It is based essentially on the demand of the Newtonian limit and energy-momentum conservation. Einstein and Grossmann arrived at a gravitation tensor of form (8)

$$\sum_{\alpha\beta}\frac{1}{\sqrt{-g}}\cdot\frac{\partial}{\partial x_{\alpha}}\left(\gamma_{\alpha\beta}\sqrt{-g}\cdot\frac{\partial\gamma_{\mu\nu}}{\partial x_{\beta}}\right)-\sum_{\alpha\beta\tau\rho}\gamma_{\alpha\beta}g_{\tau\rho}\frac{\partial\gamma_{\mu\tau}}{\partial x_{\alpha}}\frac{\partial\gamma_{\nu\rho}}{\partial x_{\beta}}$$
$$+\frac{1}{2}\sum_{\alpha\beta\tau\rho}\gamma_{\alpha\mu}\gamma_{\beta\nu}\frac{\partial g_{\tau\rho}}{\partial x_{\alpha}}\frac{\partial\gamma_{\tau\rho}}{\partial x_{\beta}}-\frac{1}{4}\sum_{\alpha\beta\tau\rho}\gamma_{\mu\nu}\gamma_{\alpha\beta}\frac{\partial g_{\tau\rho}}{\partial x_{\alpha}}\frac{\partial\gamma_{\tau\rho}}{\partial x_{\beta}}.$$

Unfortunately Einstein and Grossmann were unable to specify the covariance group of the resulting gravitational field equations. They were able to assure the reader only of covariance under linear transformation. Of course Einstein was apologetic over their failure to discover the covariance of these equations. In closing his critique of any gravitation theory based on a scalar gravitation potential, Einstein candidly conceded how this omission had crippled Einstein's program (Einstein and Grossmann 1913, I§7):[48]

> Of course, I must admit that, for me, the most effective argument for the rejection of such a theory rests on the conviction that relativity holds not only with respect to orthogonal linear substitutions but also with respect to a much wider group of substitutions. But already the mere fact that we were not able to find the (most general) group of substitutions associated with our gravitational equations makes it unjustifiable for us to press this argument.

What is puzzling is that the deficiency could be set aside with such a simple disclaimer. The driving force of Einstein's program was the conviction that the relativity

48 Translation in (CPAE 4E, 170–71).

of motion must be extended to acceleration and that this would be realized by a theory covariant under non-linear coordinate transformations, for only the latter corresponded to transformations to accelerated states of motion.

To see just how puzzling this is, we need to recall two of Einstein's commitments at this time. First we are assured by Einstein's remarks in a letter to Lorentz of August 14, 1913 (CPAE 5, Doc. 467) of his continued commitment to a broader covariance and his alarm at his continued failure to affirm the broader covariance of the theory:

> *But the gravitational equations themselves unfortunately do not have the property of general covariance*. Only their covariance under *linear* transformations is assured. However the whole trust in the theory rests on the conviction that acceleration of the reference system is equivalent to a gravitational field. Therefore if all the systems of equations of the theory, thus also equation (18) [gravitational field equations], do not admit still other transformations aside from the linear, then the theory contradicts its own starting point; it's left hanging in the air (sie steht dann in der Luft). (Einstein's emphasis)

It is a measure of Einstein's frustration and desperation that the following day—August 15, 1913[49]—he fell into an embarrassing error. He thought that he could establish from the requirement of energy conservation that his gravitation theory could be at *most* covariant under linear transformations. He retracted this trivially flawed argument in a paper published the following May (Einstein and Grossmann 1914, 218), but not before the argument had appeared several times in print.[50]

Correspondence with Generally Covariant Equations

Second, Einstein expressed his belief that his "Entwurf" field equations must correspond to generally covariant equations. Having presented his "Entwurf" gravitational field equations, (Einstein 1914a, 179)[51] he continued:

> It is beyond doubt that there exists a number, even if only a small number, of generally covariant equations that correspond to the above equations, but their derivation is of no special interest either from a physical or from a logical point of view, as the arguments presented in point 8 clearly show.[52] However, the realization that generally covariant equations corresponding to [these gravitational field equations] must exist is important to us in principle. Because only in that case was it justified to demand the covariance of the rest of the equations of the theory with respect to arbitrary substitutions. On the other hand, the question arises whether those other equations might not undergo specialization owing to the specialization of the reference system. In general, this does not seem to be the case.

Although Einstein does not make explicit what the relation of correspondence is between the "Entwurf" equations and their generally covariant counterparts, it would

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49 The dating is derived from Einstein's report to Lorentz in a letter of August 16, 1913 (CPAE 5, Doc. 470).

50 For discussion see (Norton 1984, §6).

51 Translation in (CPAE 4E, 286).

52 In his point 8, Einstein had stated the hole argument and the argument against general covariance based on the conservation of energy-momentum.

surely be that the former are recovered from the latter by some kind of coordinate condition or restriction.

While these remarks come from a paper of January 1914, we have no reason to doubt that they reflected Einstein's feelings just a few months earlier at the time of completion of the "Entwurf" paper. They provide a natural interpretation of remarks made by Einstein in (Einstein and Grossmann 1913, I.§5)[53] when he reflected on their failure to find generally covariant gravitational field equations (Einstein's emphasis):[54]

> To be sure, it cannot be negated a priori that the final, exact equations of gravitation could be of higher than second order. Therefore there still exists the possibility that the perfectly exact differential equations of gravitation could be covariant with respect to *arbitrary* substitutions. But given the present state of our knowledge of the physical properties of the gravitational field, the attempt to discuss such possibilities would be premature. For that reason we have to confine ourselves to the second order, and we must therefore forgo setting up gravitational equations that are covariant with respect to arbitrary transformations.

Einstein cannot mean by this that the higher order equations are incompatible with the "Entwurf" equations. For then solutions of the "Entwurf" equations would not be solutions of the higher order equations, so that each would admit a different class of physical fields. In this case, the selection of the "Entwurf" equations is just the selection of the wrong equations. It is hard to imagine that Einstein would dismiss correcting such an outright error by calling the correction "premature." But the dismissal is more intelligible if these higher order equations are the generally covariant equations that reduce to the "Entwurf" equations with the application of a coordinate condition or restriction. For then all solutions of the "Entwurf" equations would be solutions of the higher order equations; transition to the higher order equations would merely admit more coordinate representations of the same physical fields into the theory.

The Incongruity of Einstein's Approach…

If Einstein held these two views at the time of publication of the "Entwurf" theory and he also held to an essentially modern view of coordinate systems and coordinate conditions, then his assessment of the theory's state and his further development of the theory is quite mysterious. For the sole effect of a coordinate condition, in this modern view, is to obscure the covariance of the theory. As long as the coordinate condition does not extend beyond the four equations routinely allowed, it does not preclude any physical field; it merely reduces the range of coordinate representations

53 Translation in (CPAE 4E, 160).

54 Michel Janssen has suggested an alternative interpretation: Einstein may merely mean that his "Entwurf" field equations might be good empirical approximations in the domain of weaker fields for some set of generally covariant gravitational field equations of higher order. If this interpretation is correct, we still have ample evidence from his other remarks that Einstein also expected the "Entwurf" field equations to be recoverable from generally covariant equations by means of a coordinate condition. See, for example, the remarks quoted in section 3.6.

of each physical field.[55] In so far as the field equations, after reduction by the coordinate condition, are intended to yield the Newtonian limit, they need only exhibit covariance under linear transformation. It might just happen that the reduced field equations exhibited greater covariance so that they might play a direct role in the generalization of the principle of relativity. But there is no reason to expect this. The only sure way to expand the covariance of the theory is to find the unreduced, generally covariant form of the gravitational field equations. That is the obvious and natural way to develop the "Entwurf" theory.

This was not Einstein's approach. Rather than seeking out these generally covariant equations, he let all his hopes hang on a slender thread: the "Entwurf" equation might just have sufficient covariance to support a generalized principle of relativity. So Einstein devoted his efforts to two tasks, both of which came to fruition after he had hit upon the hole argument. First he sought to discover the extent of the covariance of his "Entwurf" equations, describing this, in (Einstein and Grossmann 1913, I.§6)[56] as the most important problem to be solved in the context of this theory (Einstein's emphasis).[57]

> ...the equation of the gravitational field that we have set up do not possess this property [of general covariance]. For the equations of gravitation we have only been able to prove that they are covariant with respect to arbitrary *linear* transformations; but we do not know whether there exists a general group of transformations with respect to which the equation are covariant. The question as to the existence of such a group for the system of equations (18) and (21) [gravitational field equations] is the most important question connected with the considerations presented here.

These efforts culminated in the discovery with Grossmann (Einstein and Grossmann 1914) that the covariance of his theory extends to what they call "adapted coordinate systems;" that is, coordinate systems that satisfy

$$\sum_{\alpha\beta\mu\nu} \frac{\partial^2}{\partial x_\nu \partial x_\alpha}\left(\sqrt{-g}\gamma_{\alpha\beta} g_{\sigma\mu} \frac{\partial \gamma_{\mu\nu}}{\partial x_\beta}\right) = 0. \tag{23}$$

Second he threw himself into the task of establishing that whatever limited covariance the "Entwurf" theory may have is good enough, for further covariance would be physically uninteresting. Here Einstein had more success than his material warranted. He first showed in a trivially flawed and soon retracted argument that one can expect no more than linear covariance. Then the hole argument showed that generally cova-

55 For example, if our "field equation" is just a flatness requirement, the vanishing of Riemann-Christoffel curvature tensor, then one of its solutions is a Minkowski spacetime, whose coordinate representations include $g^{SR}_{\mu\nu}$ and $g^{ROT}_{\mu\nu}$. The effect of a coordinate condition such as (11) is not to eliminate a physical possibility such as this solution. It precludes the representation $g^{ROT}_{\mu\nu}$ with which is it incompatible; but it admits $g^{SR}_{\mu\nu}$.

56 Translation in (CPAE 4E, 167).

57 The continuation of the letter to Lorentz of August 14, 1913, quoted above describes some of his efforts to uncover these covariance properties.

riance would be physically uninteresting and his analyses of 1914 showed that the "Entwurf" theory has the maximum covariance compatible with the hole argument.[58]

…is Explained

While Einstein's approach is baffling if we assume that he had a modern understanding of coordinate systems and coordinate conditions, it becomes entirely reasonable in the light of the conjecture of this part. He believed his "Entwurf" field equations to result from some set of unknown generally covariant equations reduced by a coordinate condition, presumably what turned out to be the adapted coordinate condition (23). In accord with (C1) and (C2), Einstein would pay no penalty in using the reduced form of the field equations in his theory. The covariance of the theory as a whole is just the covariance of the reduced equations (or, equivalently, the covariance of the coordinate condition (23). So the reduced form of these equations is not obscuring the true covariance of the theory as a whole, contrary to the modern view. And, since the effect of a coordinate condition (23) is just to restrict the covariance of the generally covariant equations, the reduced equations are not eliminating any physical fields; the limitation is just that each physical field arises in the theory in fewer coordinate representations. Thus, with the completion of the "Entwurf" theory in mid 1913, Einstein could have entered into the search for the generally covariant equations that correspond to his "Entwurf" equations. But there would have been little to gain from finding them. Finding them would not alter the covariance of the theory as a whole and it would not admit into the theory any new physical fields.[59]

There was a more pressing problem that had to absorb his immediate attention. Einstein did not know the covariance of the "Entwurf" theory. According to (C3), Einstein hoped that this covariance would extend to include transformations representing acceleration, for otherwise Einstein's hopes of extending the principle of relativity to acceleration would not be met by his theory. More was at stake. Einstein believed that his "Entwurf" gravitational field equation were unique; that is, they were the only equations employing a gravitation tensor of form (8) compatible with energy-momentum conservation.[60] Thus if the "Entwurf" equations failed to have sufficient covariance, then Einstein's entire project would be called into doubt. He could not just reject the "Entwurf" field equations and seek a better alternative. He now believed that he had no option other than the "Entwurf" equations. Thus Einstein

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58 For discussion, see (Norton 1984, §6).

59 Or more simply, if Einstein was unaware of the use of coordinate conditions, the use of the generally covariant field equations, unsupplemented by adapted coordinate condition (23), would be incompatible with recovery of the Newtonian limit, since those equations would be unlikely to have the Newtonian form (8).

60 The uniqueness of these equations is suggested by the description of the identities (12) of Einstein and Grossmann (Einstein and Grossmann 1913, §5) used in the derivation of these equations as "uniquely determined" and then directly affirmed by Einstein (Einstein 1914b, 289). See (Norton 1984, §4) for discussion; the equations prove not to be unique, although this is not easy to see.

had to find the covariance of the "Entwurf" equations and, if his efforts to extend the principle of relativity were to succeed, it had to include acceleration transformations.

Thus the conjecture explains exactly the direction of Einstein's research on completion of the "Entwurf" theory.[61] He would gain nothing of significance from finding the generally covariant equations corresponding to his "Entwurf" equations. The problem urgently needing his attention was the discovery of the extent of the covariance of his "Entwurf" equations. These efforts of discovery soon transformed into the arguments that sought to established the need, in physical terms, for a restriction on covariance: that is, the arguments from the conservation laws and the hole argument. As Einstein's remarks from early 1914 quoted above indicate, these arguments establish that the quest for the generally covariant equations is of "no special interest"—a conclusion that I urge had already been forced implicitly by his according independent physical reality to the coordinate systems arising in the process of extracting the Newtonian limit.

3.6 Einstein's Pronouncements on the Selection of Specialized Coordinate Systems

The conjecture advanced here requires that Einstein's 1912–1915 understanding of coordinate systems in quite irregular. It is essential that this conjecture be compatible with Einstein's pronouncements on coordinate systems from this period. As it turns out, Einstein made few such pronouncements—so few, that it was initially thought in the history of science literature that Einstein was unaware of how to use four conditions to constrain the choice of coordinate systems. My purpose in this section is to review Einstein's most important pronouncements on the selection of specialized coordinate systems from this period and to show that they are quite compatible with the conjecture advanced here, although they neither speak for nor against it.

Two Ways to Introduce Specialized Coordinate Systems

Best known of these pronouncements is a distinction made in (Einstein 1914a, 177–178). Since this last pronouncement turns out to be a somewhat awkward statement of the same distinction explained more clearly in a later letter to Lorentz, I shall consider the later remarks first. In a letter of January 23, 1915, to Lorentz (CPAE 8, Doc. 47) Einstein sought to explain that his "choice of coordinates makes no assumption physically about the world." He used a "geometric comparison" to illustrate the possibilities:

> I have a surface of unknown kind upon which I want to carry out geometrical investigations. If I require that a coordinate system (p, q) on the surface can be so chosen that
>
> $$ds^2 = dp^2 + dq^2$$

61 A supposed lack of awareness of the use of coordinate conditions would also explain this direction.

> then I thereby assume that the surface can be developed onto a plane. However if I require only that the coordinates can be so chosen that
>
> $$ds^2 = A(p, q)dp^2 + B(p, q)dq^2$$
>
> i.e. that the coordinates are orthogonal, I thereby assume nothing about the nature of the surface; one can realize them on any surface.

Einstein's remark is a commonplace of differential geometry and applies equally in the geometry of two-dimensional surfaces and in the geometry of spacetimes. In presuming the existence of a particular coordinate system, we might be tacitly restricting the geometry of the space, or we might not. So, as in Einstein's first example, if we assume that there is a coordinate system in which the metrical coefficient $g_{\mu\nu}$ are constant, then we are assuming that the space is also metrically flat.[62] For constancy of the $g_{\mu\nu}$ is necessary and sufficient for metrical flatness. Other coordinate systems, however, can be realized in any space, so that the presumption of their existence does not restrict the geometric properties of the space.

To proceed to Einstein's (1914) remarks, we express the constraint that picks out a coordinate system in which the metrical coefficients are all constant as

$$\frac{\partial \gamma_{\alpha\beta}}{\partial x_\kappa} = 0. \tag{24}$$

This condition is equivalent to metrical flatness, which is a condition that can be given in invariant or generally covariant form, that is, as the vanishing of the Riemann-Christoffel curvature tensor

$$R^{\alpha}_{\beta\mu\nu} = 0. \tag{24'}$$

However the now familiar

$$\sum_{\kappa} \frac{\partial \gamma_{\kappa\alpha}}{\partial x_\kappa} = 0 \tag{11}$$

consumes just the four degrees of freedom available in selection of a coordinate system in any four-dimensional spacetime and thus places no restriction on its geometry. Whatever (11) states cannot be re-expressed by a non-vacuous invariant or generally covariant relation.

Working Backwards

This is the distinction that Einstein describes in (Einstein 1914a, 177–178).[63] The difference is that Einstein starts with an expression of restricted covariance and then

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62 That a surface "can be developed onto a plane" is synonymous with flatness.

63 Translation in (CPAE 4E, 284).

works backwards, asking if the expression came from a generally covariant expression by restriction of the coordinate system.[64]

> If we are given equations connecting any quantities whatsoever[65] that are valid only for a special choice of the coordinate system, then one has to distinguish between two cases:
>
> 1. To these equations there correspond generally covariant equations, i.e. equations valid with respect to arbitrary reference systems;
>
> 2. There are no generally covariant equations that can be deduced from the equations given for the specially chosen reference frame.
>
> In case 2, the equations say nothing about the things described by the quantities in question; they only restrict the choice of reference system. If the equations say anything at all about the things represented by the quantities, then we are dealing with case 1, i. e., in that case, there always exist generally covariant equations between the quantities.

The constraint (24) is an instance of a non-generally covariant equation of case 1. Its existence does restrict the quantities involved, for it entails the flatness of the metric. Thus there is a corresponding generally covariant relation (24'). The requirement (11), however, generates no restriction on these quantities and thus corresponds to no (non-vacuous) generally covariant requirement.

The distinction outlined here does not bear on the reading of coordinate restrictions I urge Einstein held in 1912–1915. The requirement (11), places no restriction on the geometric properties represented by the metric $g_{\mu\nu}$. That is an issue independent of how the requirement picks out particular coordinate systems. To parrot Einstein, the requirement "says nothing" about the metrical quantities, but it certainly "says something" about the coordinate systems, for it admits some and precludes others. Deciding just what it says about them is the issue that defeated Einstein in 1912–1915.

Specialized Coordinate Systems and Nordström's Theory of Gravitation

There is an important instance of case 1 in (Einstein and Fokker 1914), submitted for publication in February 1914, a month after the submission of (Einstein 1914a). Their work pertains to Nordström's latest theory of gravitation, which Einstein judged the most viable of the gravitation theories then in competition with the Einstein and Grossmann "Entwurf" theory.[66]

Nordström's theory had been developed by Nordström and Einstein as a Lorentz covariant theory of gravitation. With Fokker, Einstein now showed that the theory could be recovered in the generally covariant framework of the "Entwurf" theory, complete with its generally covariant energy conservation law. In place of the Einstein-

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64 Einstein's purpose is to assert that his non-generally covariant gravitational field equations of the "Entwurf" theory do make some assertion about the quantities involved. Thus they are an instance of case 1. and there must exist corresponding generally covariant equations.

65 Einstein's footnote: "Of course, the transformation properties of the quantities themselves must be considered here as being given for arbitrary transformations."

66 For an account of Nordström's theories, see (Norton 1992; 1993).

Grossmann gravitational field equations, Einstein and Fokker adopted the single field equation $R = kT$, where R is the Riemann curvature scalar, T the trace of the stress-energy tensor and k a constant. That single equation would be insufficient to fix the ten coefficients of the metric tensor, so additional constraints were needed. "It turns out," Einstein and Fokker observed in their introductory summary (page 321), "that one arrives at the Nordström theory instead of the Einstein-Grossmann theory, if one makes the sole assumption that it is possible to choose preferred coordinate systems in such a way that the principle of the constancy of the speed of light obtains." They interpreted the presumption of such a coordinate system as equivalent to assuming the existence of coordinate systems in which the spacetime's line element has the form[67]

$$ds^2 = \phi^2(dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2). \tag{25}$$

That a spacetime admits a line element of this form greatly restricts its geometry; it is equivalent to conformal flatness. As Einstein suggests, this restriction can be written in generally covariant form. It was later found to be equivalent to the vanishing of the Weyl conformal tensor.

Einstein does not mention the Nordström theory in remarks in a letter to Planck July 7, 1914 (CPAE 8, Doc. 18), written about six months after publication of Einstein and Fokker's paper. However his remarks describe exactly the specialized coordinate system introduced in the Einstein-Fokker formulation of the Nordström theory.

> There is a fundamental difference between that specialization of the reference system that classical mechanics or [special] relativity theory introduces and that which I apply in the theory of gravitation. That is, one can always introduce the latter, no matter how the $g_{\mu\nu}$ may be chosen. However the specialization introduced by the principle of the constancy of the speed of light presumes differential relations between the $g_{\mu\nu}$, and indeed relations whose physical interpretation would be very difficult. The satisfaction of these relations cannot be enforced for every given manifold through suitable choice of the reference system. According to the latter understanding, there are two heterogeneous conditions for the $g_{\mu\nu}$
>
> 1) the analog of Poisson's equation
>
> 2) the conditions that enable the introduction of a system of constant c.

These two "heterogeneous conditions" correspond exactly with the two laws of the Nordström theory. The first, the field equation $R = kT$, is the analog of Poisson's equation. The second is the presumption that we can introduce a coordinate system in which the line element takes the conformally flat form (25). Its introduction is enabled by further conditions, which were later found to be expressible as "differential relations between the $g_{\mu\nu}$," the vanishing of the Weyl conformal tensor.

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67 Then a light signal, for which $ds^2 = 0$, propagates with unit coordinate velocity. For example, if it propagates along the x_1 axis, the light signal satisfies $dx_1/dx_4 = \pm 1$.

3.7 Was Einstein Really Defeated by According an Independent Reality to Coordinate Systems?

That is, is the conjecture of this part true? In sum, the answer is similar to the one given in section 2.4 to the question of whether Einstein was aware of coordinate conditions. There is no decisive piece of evidence for or against, but there are indications that point in both directions. Again, our ultimate assessment depends in some significant measure on issues of plausibility. My view is that the latter favor the conjecture.

The Notebook and the "Entwurf" Theory

If we accept that Einstein was aware of the use of coordinate conditions in the notebook and later, then we have several incongruities to explain. Why does he abandon their use so precipitously? Why does his later correspondence discount a perfectly serviceable extraction of the Newtonian limit from the candidate gravitation tensor T_{il}^{x}? Why is his discussion of the "Entwurf" theory, prior to his discovery of general arguments against general covariance so indifferent to the recovery of the generally covariant gravitational field equations he allowed must exist? The conjecture of this part supplies an explanation that answers all of these questions.

Before we embrace that explanation, however, we should note that there is no direct evidence that Einstein did accord an independent reality to coordinate systems in the relevant context of the Newtonian limit. That is, we do not have unequivocal remarks by Einstein announcing it or a calculation whose only reasonable interpretation is that independence. It is hard to know how seriously to take this omission. Since Einstein was not using coordinate conditions to recover the Newtonian limit in his "Entwurf" theory, he had no occasion to undertake calculations that would unequivocally display an independent reality accorded his limit coordinate systems. What Einstein does give us are the manipulations of the hole argument. It is quite evident that he does there accord independent reality to the coordinate systems and his later admissions affirm this. Similarly, there were few occasions for Einstein to discuss how coordinate conditions could be used to recover the Newtonian limit, for this was not the construction he used in the "Entwurf" theory. On the few occasions in which he discussed general principles surrounding specialization of the coordinate system (see section 3.6 above), he makes no mention of an independent reality of the specialized coordinate systems. But then we would not expect him to. In section 3.1 we saw Einstein's difficulty in making explicit just how the manipulation of the hole argument depended on the independent reality of the coordinate system. If Einstein had such difficulty describing that independent reality when it was the essential point of the calculation, why should we expect him to express it clearer elsewhere?

Einstein's Later Discussion

Once Einstein had discovered his errors and returned to general covariance, he again had the opportunity to admit that he had accorded an independent reality to his coordinate systems. There were two prime occasions for such admission: his paper of November 4, 1915, and his letter to Sommerfeld of November 28, in which he explained his rejection of the candidate gravitation tensor T^x_{il}. In both places, however, he emphasized the $\{\}$ prejudice as the source of his mistake. What is odd about both sources is that neither seek to explain the most public conceptual error of his "Entwurf" theory, the hole argument. At the time of the November 4 paper, Einstein had not yet discovered his misconception about static fields. As far as we know, the hole argument was the only foundational error of principle in the "Entwurf" theory, short of the ultimate mistake of choosing the "Entwurf" equations of restricted covariance. Since the error of the hole argument and the conjectured misuse of coordinate conditions are closely related, hesitancy in discussing the one should be expected to accompany hesitancy in discussing the other. And there was great hesitancy.

There are early published remarks that amount to the briefest retraction of the hole argument. But they offer little to explain the error of the argument. They appear in Einstein's celebrated computation of the anomalous motion of Mercury, in a paper presented to the Berlin Academy on November 18, 1915, (Einstein 1915b, 832). There Einstein considers the gravitational field of a point mass at the origin of spatial coordinates, which he takes to be the sun. Solving for this case, even in lower order approximation, involves a system analogous to the hole of the hole argument. The field is constrained by Minkowskian boundary conditions at spatial infinity, just as the field in the hole is constrained by the surrounding matter distribution. In addition the field of the sun is constrained by the requirements that it be static and spatially symmetric about the origin. These additional requirements do not preclude all transformations; a spatial radial coordinate r could be arbitrarily transformed as long as the transformation does not disturb the limit at spatial infinity and preserves unit modulus by, say, corresponding adjustments elsewhere. Einstein remarked:

> We should however bear in mind that for a given solar mass the that the $g_{\mu\nu}$ are still not completely determined mathematically by the equations (1) and (3).[68] This follows since these equations are covariant with respect to arbitrary transformations of determinant 1. We may assume, however, that all these solutions can be reduced to one another through such transformations, so that they differ from one another only formally but not physically (for given boundary conditions). As a result of this conviction, I am satisfied for the present to derive a solution without being drawn into the question of whether it is the only possible [solution].

If the covariance of the field equations is to block determination of the field in this case, it must be through the hole construction, so we have many solutions mathematically in the one coordinate system. Einstein parries the threat by observing that these solutions "differ from one another only formally but not physically" and the same remark would serve as an escape from the hole argument. Only a quite attentive reader would see the connection and even then such a reader may well find the remark unconvincing. Certainly Ehrenfest needed a more elaborate account of the failure of the hole argument before he was satisfied.[69] Yet Einstein concluded by

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68 Einstein's equations are $T^x_{il} = 0$ and $|g_{\mu\nu}| = 1$, which are covariant under unimodular transformations.

explicitly disavowing any further discussion This neglect is striking in comparison to the careful self diagnosis elaborated as the { } prejudice.

Why might he be reluctant to discuss the error of the hole argument? He may just have been reluctant to relive a painful experience, especially if he saw no benefit from it. Or perhaps he had some difficulty formulating precisely what the error was, even after he knew of it. It was sufficient that he knew that the hole construction did not produce physically distinct fields. If he had suffered this difficulty it would explain why he delayed detailed discussion of the error of the hole argument for nearly two months after his public announcement of his return to general covariance. As far as we know from documents available to us, the first detailed discussion comes in his letter of December 26, 1915, to Ehrenfest (see section 3.2).

Whatever may have underpinned his reluctance to discuss the error of the hole argument, the same reason would surely induce a similar reluctance to discuss the closely related error conjectured here.

Einstein's Letter to de Sitter

According to the conjecture of this part, there is a close connection between two of Einstein's errors: the notebook rejection of the candidate gravitation tensor T_{il}^{x} and the hole argument. We would hope to see some trace of that connection. Such a trace may appear in a letter Einstein wrote to de Sitter on January 23, 1917.

To see how this letter can be interpreted, we must recall Einstein's return to general covariance in the Fall of 1915. In several places, Einstein listed the clues that forced him to accept the inadequacy of his "Entwurf" theory.[70] In particular, Einstein had erroneously convinced himself that the "Entwurf" theory was covariant under rotation transformation (18).[71] The discovery of this error cast Einstein into despair over his theory, as he confided to his astronomer colleague Erwin Freundlich in a letter of September 30, 1915 (CPAE 8, Doc 123). In it, he was reduced to a despondent plea for help. He was not frozen into inactivity, however. A little over a month later, on November 4, he announced his return to general covariance and the adoption of T_{il}^{x} as his gravitation tensor.

That one discovery of the lack of rotational covariance of the "Entwurf" theory seems to have been a powerful stimulus. Two things followed rapidly after it. He returned to general covariance (and therefore rejected the hole argument) and he readmitted the gravitation tensor T_{il}^{x} as gravitation tensor. If the original rejection of T_{il}^{x} had been due to improperly according independent reality to coordinate systems, then we may readily conceive natural scenarios that connect the two. For example,

69 See section 3.2 and (Norton 1987, §4).

70 See (Norton 1984, §7).

71 Janssen (Janssen 1999) supplies a fascinating chronicle of this episode. It includes display of calculations in Einstein's hand apparently from June 1913 in which Einstein erroneously affirms that $g_{\mu\nu}^{ROT}$ is a solution of the "Entwurf" gravitational field equations and then a repetition of the same calculation probably from late September 1915 in which Einstein finds the error.

lack of rotational covariance would be fatal to Einstein's hopes of generalizing the principle of relativity to acceleration. So if he now realized that his "Entwurf" theory could not supply it, he might well return to the last candidate gravitation tensors considered in the context of the rotation transformation (18). That would be T_{il}^{x} and the related proposals around page 22 of the notebook. Now wiser and desperate and suspicious of all his methods and presumptions, Einstein might just finally be able to see past his objection to the coordinate condition (11) to the recognition that there was something improper in the core of his objection, his interpretation of what I have called the hole construction. That realization would have simultaneously allowed him to see that the hole argument does not succeed in showing the inadmissibility of generally covariant gravitational field equations. For it also depends on the same interpretation of the hole construction. Because of the close connection between the two errors, some such scenario among many obvious variants is credible.

As we saw in section 3.2, Einstein gave several accounts of the error of the hole argument. None mentioned above contain autobiographical remarks on how Einstein found the error. There is one exception, a recollection in a letter of January 23, 1917, to de Sitter (CPAE 8, Doc. 290) concerning the errors of Einstein (Einstein 1914c)

> ...there were the following two errors of reasoning [in (Einstein 1914c)]:
>
> 1) The consideration of §12 [the hole argument] is incorrect, since occurrences can be uniquely determined without the same being true for the functions used for their description.
>
> 2) In §14 at the top of page 1073 is a defective consideration.
>
> I noticed my mistakes from that time when I calculated directly that my field equations of that time were *not* satisfied in a rotating system in a Galilean space. Hilbert also found the second error.

Here Einstein assures us that he found the errors of his 1914 review article, with the hole argument listed as the first of the two errors, because he discovered the lack of rotational covariance of his "Entwurf" field equations.[72] Without the conjecture of this part, it is hard to see why Einstein would proceed without great detours from that lack of rotational covariance to the rejection of the hole argument.

What is More Plausible?

In the absence of decisive evidence, we once again ask after the plausibility of the conjecture. To my mind, the one factor that speaks against the conjecture is this very lack of evidence. Things might have transpired as conjectured without more decisive evidence surviving. Einstein was not obligated to annotate his private calculations or later recount every misstep, so as to save the labor of future historians. The resulting

72 By a "Galilean space," Einstein refers to a Minkowski spacetime in the coordinates of (5). The second error is presumably the one Einstein discusses with Hilbert in a letter of March 30, 1916, to Hilbert (CPAE 8, Doc. 207) and concerns the failure of a variation operator to commute with coordinate differentiation. For discussion, see (Norton 1984, end of §6).

paucity of evidence, however, is also compatible with a simpler explanation: things just did not go as conjectured. One factor makes this case a little different from the earlier deliberations on Einstein's supposed unawareness of the use of coordinate conditions: the conjecture ties Einstein's misturnings to the error of the hole argument. In that case we have no doubt of Einstein's reticence to leave later traces of his error and that reticence would carry over to the related rejection of the tensor T^x_{il}. But now we tread on dangerous ground. We offer an account that also predicts that it will be difficult to find evidence for that account. Such accounts can be correct. They can also be signal that a defective account has been protected illegitimately from refutation. There are earnest accounts of how our small planet is routinely visited by aliens intent on abductions. They face a sustained lack of concrete evidence. So we are assured that no irrefutable evidence of the visits survives because of a massive government conspiracy or the ingenuity and thoroughness of the aliens in eradicating all such traces!

These serious hesitations should be weighed against the need for some account of Einstein's twisted path. Again we risk a pitfall. If we are willing to multiply the errors Einstein is supposed to have committed, there is scarcely any pathway that we could not explain. What is appealing about the conjecture is that it requires us to posit no new errors. Aside from outright blunders of calculation and self deception, as documented in (Janssen 1999), Einstein was led astray for nearly three years by two groups of misconceptions. The first surrounded his presumptions on the form of the static metric and the weak field equations. The second pertained to the hole argument and the independent reality of the coordinate systems.

To arrive at the second, we need only ask that Einstein was consistent and thorough in his support of the misconception the hole argument. Then just one error leads Einstein to reject the use of coordinate conditions, to acquiesce to the gravely restricted covariance of the "Entwurf" theory and not to pursue its generally covariant generalization. The recognition of that same error both frees Einstein from the hole argument late in 1915 and allows him to propose T^x_{il} as his gravitation tensor.[73] If I must choose an account, I find this one plausible.

CONCLUSION

Why did Einstein reject the candidate gravitation tensor T^x_{il} in the notebook? His own answer emphasized his "fateful prejudice," the $\{\}$ prejudice. He did not see that the Christoffel symbols are the natural expression for the components of the gravitational field. As a result he could not properly relate the gravitation tensor to the requirement of energy conservation. Instead he was tempted to multiply out the

73 For comparison, consider the alternative account in which Einstein is just unaware of the use of coordinate conditions. This awareness must come if T^x_{il} is to be admissible as a gravitation tensor. So the preparation for the new proposals of November 1915 must include recognition of two independent errors, that of the hole argument and the neglect of coordinate conditions.

Christoffel symbols to recover expressions explicitly in the metric tensor that would prove unwieldy.

That may well have been all that it took to convince Einstein to abandon the proposal. We must then discount as unrelated his anomalous concern with questions of covariance on the pages surrounding page 22R on which the gravitation tensor is analyzed. While Einstein had clearly mastered the mathematical manipulations needed to apply a coordinate condition to expressions of general or near general covariance, his treatment of them suggests that his interpretation of the conditions was idiosyncratic. His concern for their covariance properties cannot be reconciled with his later attitude to them. So we have presumed that his treatment and interpretation of these coordinate condition supplied a further fateful prejudice that precluded admission of the candidate gravitation tensor T_{il}^{x} by somehow obstructing his extraction of the Newtonian limit. The supposition of this additional fateful prejudice makes Einstein appear far less capricious. In finding the gravitation tensor T_{il}^{x} he had circumvented the tangled cluster of problems he had imagined facing the Ricci tensor as gravitation tensor. We suppose that he abandoned the new proposal not just because the calculation looked complicated but because deeper matters of principle also seemed to speak against it.

Just how did Einstein's treatment of coordinate conditions defeat him? There is clear evidence in the notebook that Einstein used the requirements as what we call "coordinate restrictions": they are not just applied in the case of the Newtonian limit but universally. That alone does not explain why Einstein would think his candidate gravitation tensor unable to yield the Newtonian limit in a satisfactory manner. We have found two additional hypotheses that would supply the explanation. The first supposes an obtuse Einstein, overlooking a natural option. It supposes he just persistently failed to see that coordinate conditions could be invoked selectively as part of the restriction on covariance imposed in recovery of the Newtonian limit. The second, which I favor, portrays an excessively acute Einstein, zealously consistent even in his errors. He would soon improperly accord an independent reality to coordinate systems in the hole argument and the conjecture is that he did the same thing earlier in applying coordinate conditions. Both hypotheses have the same outcome. Einstein would come to an impossible demand: the requirement that reduces the candidate gravitation tensor to a Newtonian form must have sufficient covariance to support a generalization of the principle of relativity to acceleration. The first is a dim Einstein, felled by overlooking a standard device in general relativity that he later used without apology. The second in an Einstein of Byzantine sophistication, pursuing his errors, even when only dimly aware of them, to their farthest catastrophe. Perhaps another Einstein, the real Einstein, neither dim nor Byzantine, still waits to be discovered.

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