July 18, August 21, December 20, 2003; April 11, 2004

How the Formal Equivalence of Grue and Green Defeats What is New in the New Riddle of Induction

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That past patterns may continue in many different ways has long been identified as a problem for accounts of induction. The novelty of Goodman's "new riddle of induction" lies in a meta-argument that purports to show that no account of induction can discriminate between incompatible continuations. That meta-argument depends on the perfect symmetry of the definitions of grue/bleen and green/blue, so that any evidence that favors the ordinary continuation must equally favor the grue-ified continuation. I argue that this very dependence on the perfect symmetry defeats the novelty of the new riddle. The symmetry can be obtained in contrived circumstances, such as when we grue-ify our total science. However, in all such cases, we cannot preclude the possibility that the original and grue-ified descriptions are merely notationally variant descriptions of the same physical facts; or if there are facts that separate them, these facts are ineffable, so that no account of induction should be expected to pick between them. In ordinary circumstances, there are facts that distinguish the regular and grue-ified descriptions. Since accounts of induction can and do call upon these facts, Goodman's meta-argument cannot provide principled grounds for the failure of all accounts of induction. It assures us only of the failure of accounts of induction, such as unaugmented enumerative induction, that cannot exploit these symmetry breaking facts.

[1] I thank Don Howard for first putting the total science objection to me. I am also grateful to Jim Bogen, Jim Brown, Allan Franklin and P. D. Magnus for helpful discussion; and to an anonymous reviewer of this journal for very helpful suggestions for revisions of what is now Sections 4 and 5.

1. Introduction

That patterns can be continued in many ways has long been identified as a problem for induction. That bread has always nourished us, Hume (1748, p.78) urged, does not necessitate that it will continue to do so. Jevons (1874, p.230) recounted examples of numerical patterns, some of which persisted beyond the sample displayed and some that did not. He concluded: “An apparent law never once failing up to a certain point may then suddenly break down, so that inductive reasoning, as it has been described by some writers, can give no sure knowledge of what is to come.” The standard burden of accounts of inductive inference has been to address this problem: to show how the evidence of the known pattern can allow us to choose among the many possible continuations and to show why it warrants the choice.

Goodman (1983, Ch. III) joined this venerable tradition of criticism when he noted that our history of observation of green emeralds does not assure us that emeralds we observe in the future will be green and not, say, blue. His invention of the compound predicate “grue” (so called, if observed to be green before time *t*, but blue otherwise) was a colorful way of stating old news. That was not the novelty of his “new riddle of induction.” The novelty and ingenuity lay in a meta-level argument that threatened to show that efforts by any account of induction to pick among different continuations of existing patterns must fail. That argument depended upon the perfect symmetry of the definitions of grue/bleen and green/blue. Each is defined in terms of the other by the same formulae. Our observation prior to time *t* of green emeralds may be described equally as the observation of green emeralds or of grue emeralds. Thus, using the symmetry, we may assert a meta-claim that apparently applies to all accounts of induction: any account of inductive inference that allows the evidence to support the hypothesis that all emeralds are green must also allow exactly equal support for the incompatible hypothesis that all emeralds are grue.

What I will seek to establish here is that, on further reflection, this essential symmetry requirement also turns out to defeat what is novel in Goodman's new riddle of induction. To do this, I will recall that, in other contexts, perfectly symmetrical accounts are routinely understood to be merely variant descriptions of the same facts. The most familiar example is the symmetric descriptions from different frames of references in physical theories that obey the principle of relativity. One could insist that there are factual differences separating the accounts, but that would be at the cost of accepting that the factual differences are inexpressible in the theory.

Applying this construal of perfectly symmetrical accounts to grue/bleen and green/blue, we arrive at a very different view of them. As long as the descriptions fully respect the symmetry, then we have strong reasons for believing that they are merely notationally variant descriptions of the same physical facts; anyone insisting that the worlds described are factually different must resort to distinguishing facts that are literally inexpressible. That is, the supposed difference is ineffable. I will give two illustrations of how perfectly symmetric, grue-ified descriptions are variant descriptions of the same physical facts. In one, the perfect symmetry of the descriptions can be sustained *very artificially* if we restrict ourselves to a highly contrived, impoverished emerald-world (to be described below), bereft of all but a few physical facts. More plausibly, we might imagine that we preserve the symmetry by somehow grue-ifying our total science. In either case it will turn out that the propositions that all emeralds are green or that they are grue describe the same physical facts; or, if there is a factual difference, it is ineffable. So we should not expect an account of induction to distinguish between them. Indeed we should be suspicious of one that does.

What is new in the new riddle of induction depends essentially on the symmetry of descriptions. The symmetry prevails only in highly contrived illustrations, such as the two to be developed here. In more realistic contexts, the symmetry does not obtain and Goodman's meta-level argument fails. For then accounts of induction are free to exploit the asymmetry between green and grue in order to determine which projection of the past regularity is to be endorsed. This approach is used by many accounts of induction and, I assert elsewhere, with some success. It is not my concern in this paper, however, to examine how they do this or to assess their degree of success, for that belongs to the old problem. Rather my concern is to note that the novel import of Goodman's new riddle lies in showing the failure of accounts of induction that cannot exploit these asymmetries when they are available. The most prominent casualty is a simple form of enumerative induction that places no restriction on the predicates over which induction may proceed.

In the following, in Section 2, Goodman's new riddle of induction is sketched. Section 3 contains a brief excursion into intertranslatable descriptions. I give special attention to symmetric descriptions and urge that they can only fail to represent the same physical systems, if we presume ineffable factual differences. These claims are illustrated by the principle of relativity and by a nocturnal expansion that affects spatial sizes and all related physical quantities. In Section 4 I briefly review how accounts of induction distinguish between ordinary and grue-ified continuations of patterns in the ordinary case in which the symmetry of grue/bleen and green/blue is not respected by the relevant facts of the larger context. In Section 5, I consider the other case in which the context respects the symmetry. That case is illustrated with two examples of how a perfect symmetry can be secured. One is in an impoverished emerald-world. The other is the nocturnal expansion of Section 3, which is essentially similar to grue-ification of our total science. Conclusions are in Section 6.

2. Goodman's "New Riddle of Induction"

Grue and bleen

Goodman (1983, ch.III, pp.74, 79) introduced two new predicates, grue and bleen, defined as

> Grue applies to all thing examined before [some future time]*t*
> just in case they are green
> but to other things just in case they are blue.
> Bleen applies to all things examined before *t*
> just in case they are blue
> but to other things just in case they are green.

The new predicates immediately present fatal problems for any account of induction that licenses the inference:

> Enumerative induction: Some As are B confirms all As are B.

where we may freely substitute any individual term for A and any predicate for B. The reason is that the observation of green emeralds prior to *t* can equally be described as the observation of grue emeralds prior to *t*. Thus, by enumerative induction, the same observations equally confirm that all emeralds are green and that all emeralds are grue. That is, the same observations equally confirm hypotheses one of which entails that emeralds examined after *t* will be green and another that entails that they will be blue. Our past history of observation of green emeralds ends up assuring us that all emeralds we observe in the future will be blue. An account of induction that allows such an assurance is surely defective.

The importance of the formal equivalence of grue and green

So far, there is nothing really novel in Goodman's new riddle. We have a rather clever way of presenting the long known problem for induction that any pattern may be continued in many different ways. Perhaps we are tempted to take a nonstandard continuation—that all emeralds are grue--a little more seriously because it acquired a name that matches a nonstandard description of the evidence. The more natural response is to dismiss the non-standard continuation as somehow bogus because of the explicit mention of time *t* in the definition of grue, a failure not shared by the predicate green. This amounts to a plausible restriction on the scheme of enumerative induction to predicates B that are not grue-ified. The novelty of the new riddle of induction comes from Goodman's ingenious rejoinder, without which grue would surely have disappeared from the literature as a clever but short lived gimmick. There is a formal symmetry between green/blue and grue/bleen, so that if we take grue/bleen as our primitives, green and blue are now the grue-ified predicates:

Green applies to all thing examined before [some future time]*t*

just in case they are grue

but to other things just in case they are bleen.

Blue applies to all things examined before *t*

just in case they are bleen

but to other things just in case they are grue.

What makes Goodman's rejoinder apparently impregnable is the perfect symmetry of the two sets of definitions. They use the same sentences up to a permutation of terms. The original definitions of grue/bleen becomes the definitions of green/blue simply by applying the transformation

green → grue   blue → bleen   grue→ green   bleen → blue

The same transformation (or equivalently, its inverse) converts the definitions of green/blue into the definitions of grue/bleen.

The symmetry allows a general argument that there is no property of grue that allows us to deprecate it in comparison to green. For any formal property of green, there will be a corresponding property of grue; and conversely. For example, our natural intuition is that an observed, green emerald before *t* is the same qualitatively as a green emerald after *t*. But an observed, grue emerald before *t* is not the same qualitatively as a grue emerald after *t*, since the former is really green and the latter really blue. We might like some second order property "same" that can express this difference. However the perfect symmetry of the definitions defeats this. For any predicate "same" that can be defined in a system that takes green and blue as primitives, there will be a grue-ified analog in a system that takes grue and bleen as primitives, as long as the symmetry of description obtains. Any virtue that the former attributes to the green/blue system will be attributed by the latter to the grue/bleen system. The result of this symmetry is that any account of induction must allow that our history of observations of emeralds confirms their greenness and grueness equally.

3. Symmetric Descriptions and Physically Equivalence

The perfect symmetry of green/blue and grue/bleen is reminiscent of equivalences that arise in other areas of philosophy of science. In physics it we find many cases of apparently distinct physical systems whose descriptions are intertranslatable; that is, there is a transformation of the terms in the first description that turns it into the second and conversely. As a result, the two are often judged to be just the same system, described in two different ways. These cases were first made prominent through the principle of relativity. They are now often discussed under the label of gauge freedoms and the mathematical transformation that takes us between the different descriptions of the same system is called a gauge transformation.

One of the simplest cases of intertranslatable descriptions arises in Newtonian gravitation theory. The gravitational field is described by a potential φ, which generates certain motions for planets, comets and falling stones. We get an alternative description of the same field by a gauge transformation that just adds a constant amount K to yield a new field $\varphi'=\varphi+K$. The new field φ' is mathematically distinct from the original field φ since they assign different values to the same point in space. The differences between the two mathematical fields correspond to nothing physical. For example, the fields support identical motions, since the motions are governed solely by differences of potential, and, while φ' and φ disagree in absolute values, they agree in all differences of values. The two fields φ and φ' are physically equivalent.

Intertranslatability and physical equivalence: Four claims

I will state and briefly defend some of the principal claims concerning intertranslatable descriptions, in preparation for their application to grue/green.

As long as two descriptions are intertranslatable but formally or mathematically distinct, we cannot rule out the possibility that the formal or mathematical differences in some way represent something factual so that the descriptions are not physically equivalent. We can readily generate examples of intertranslatable descriptions that pertain to distinct systems by finding cases of distinct systems with similar properties. Both Newtonian gravitational fields and electrostatic fields are governed by an inverse square law, for example. Thus the descriptions of many gravitational fields are intertranslatable with those of electrostatic fields; yet the fields are physically distinct. Hence:

> *A. Intertranslatable descriptions are not assuredly of the very same facts; that is, they are not assuredly physically equivalent.*

When no other considerations enter, whether two intertranslatable descriptions are physically equivalent depends just on our stipulation over what their terms mean. They mean what we choose them to mean. Therefore, because of the intertranslatability, we can choose them to represent the same facts, in which case they are physically equivalent; or we can chose them to represent intertranslatable but distinct facts, in which case they are not physically equivalent.

In practice other considerations almost invariably enter and these realistic cases are the ones I will consider henceforth. In them, the meaning of terms in intertranslatable descriptions is already set by the role these terms play in a larger theory. Thus whether two intertranslatable descriptions represent the same facts is no longer open to stipulation and is already decided by the physical content of the broader theory. That the differences between two intertranslatable descriptions corresponds to no factual differences can be a physical result of some importance in the broader theory. For example, that the two Newtonian gravitational fields φ and $\varphi'=\varphi+K$ represent the very same facts depends on the factual assumption that all that the absolute value of the field corresponds to no physical property of the field; these properties are fully captured in the differences of values of the potentials. Hence:

> *B. That particular intertranslatable descriptions describe the very same facts, that is, are physically equivalent, must be established by argumentation that proceed from factual assumptions.*

This need for factually based argumentation to establish the physical equivalence of intertranslatable descriptions is one of the most important outcomes of the recent discussion of the "hole argument" in philosophy of space and time. (See Earman and Norton, 1987; Norton, 1999.) [2]

The facts that decide whether two intertranslatable descriptions are physically equivalent can enter in two ways, internally or externally. I shall say they enter *internally* if those facts are expressed in the descriptions themselves. For example, the gravitational potential φ_g of a unit mass and the electrostatic potential φ_e of a unit positive charge both vary inversely with distance r from the mass or charge. The two formulae are intertranslatable with $\varphi_g=-1/r$ translating to $\varphi_e=+1/r$ merely by a transformation that multiplies by -1: $\varphi_e=-1\mathrm{x}\varphi_g$. (I set the constants to one by the selection of units.) These intertranslatable descriptions do not describe the same physical systems. The facts that distinguish gravity from electrostatics are at least partly expressed in the formulae themselves. The minus sign of $\varphi_g=-1/r$ expresses the fact that gravitational forces are attractive, whereas the positive sign in $\varphi_e=+1/r$ expresses the fact that electrostatic forces, between like charges, are repulsive.

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[2] In the context of a critique of the underdetermination thesis (Norton, manuscript, Section 4), I have argued that standard examples of observationally equivalent theories in that literature are strong candidates for being mere variant descriptions of the same physical processes. I argue that possibility arises through a special feature of the literature on the underdetermination thesis: to be useful in that literature, the two theories must not merely be observationally equivalent, it must also be easy to demonstrate the observational equivalence in an argument brief enough to fit into a journal article. That assures us of sufficient similarity in their deeper structures to make the theories strong candidates for being variant descriptions of the same physical processes.

I shall say that the distinguishing facts enter *externally* if they are not expressed within the descriptions themselves. A simple example is afforded by a three dimensional Euclidean space. Consider two parallel, flat two dimensional surfaces in the space. Their descriptions are identical since their geometries are the same. So they could represent the same two dimensional surface. That they do not is not expressed as a fact within the description of the geometry of the two dimensional surface. Rather it is expressed by facts outside the description of the geometry of the surfaces; that is, by facts that specify how the surfaces are embedded in the larger three dimensional space.

In this last case, the facts supporting the failure of physical equivalence of the two descriptions had to be external for the simple reason that the geometries of the two surfaces are intrinsically identical, agreeing in all geometrical facts within the surfaces. As a result, their descriptions contain exactly the same sentences and formulae. They are examples of *symmetrical descriptions*, which I define as pairs of descriptions that employ exactly the same sentences and formulae. Since they are exactly the same, there is usually some sort of indexing of terms or quantities so we can keep track of which description is which. For example, symmetrical descriptions of the geometry of a three dimensional Euclidean space are supplied by describing the one geometry in two Cartesian coordinate systems, say (x,y,z) and another (x',y',z'), produced from the first by a rotation. Each description uses the same formulae or sentences with the coordinates entering them in exactly the same way. We keep track of which sentence or formulae belongs to which coordinate system by the absence or presence of the primes. That indexing leads to the common labeling of the descriptions as "notational variants" of one another. In the strongest cases, the very same sentences can be used not just for the descriptions, but also for the transformations that we use to translate between them. Hence:

> *C. For symmetrical descriptions, a failure of physical equivalence must be grounded in external facts, that is, in facts not expressed within the descriptions themselves.*

This claim derives directly from the symmetry of the descriptions. For every sentence or formulae in one, there is a corresponding sentence or formula in the other; and conversely. Thus if we are to be assured that the corresponding sentences or formulae do not represent the same facts, we must call upon further facts that are not expressed within the symmetrical descriptions. As the scope of the symmetric descriptions becomes larger, there are fewer facts that can be called upon. The limiting cases are:

> *Da. Total science: If symmetrical descriptions cover all of science, then facts that establish the failure of physical equivalence must lie outside science.*
>
> *Db. Total knowledge: If symmetrical descriptions cover all of our formally expressed knowledge, then facts that establish the failure of physical equivalence must lie outside our expressed knowledge, that is, in ineffable facts.*

Neither Da nor Db necessitates the physical equivalence of the symmetrical descriptions. Both, however, make the presumption of physical equivalence very strong. If, for example, we have symmetrical descriptions of our total science, we might decide that only one properly describes the total science and that the other somehow fails. But a warrant for that decision must come from facts that lie outside of science; or perhaps from an extrascientific decision to give non-standard meanings to terms in one of the descriptions.

Illustration: the principle of relativity

The best-known examples of symmetrical descriptions are afforded by relativity principles, such as the principle of relativity of inertial motion of special relativity. In that case, the description of the *totality* of all admissible physical processes will be identical no matter which inertial frame of reference is used for description. For example, limiting ourselves to one dimension of space, the totality of possible light signals will be described in one inertial frame (x,t) by the set of world lines $\{x=ct+K, x=-ct+M$: for all real K, M$\}$, where x and t are the space and time coordinates and c the speed light. In another frame, the totality of possible light signals will be described by $\{x'=ct'+K, x'=-ct'+M$: for all real K, M$\}$, an identical description, but where primes have been added to the x and t coordinates to remind us that they relate to a different inertial frame.[3]

One might be inclined nonetheless to insist that one of the frames of reference can be designated as factually preferred—the true rest frame. However all such efforts face formidable obstacles. It is not just that no observable fact distinguishes between the frames. It is worse. No fact expressible in the theory, observable or otherwise, allows any distinction between the frames. For example, we might note that some frame F is distinguished by its admitting a very massive sun at rest with planets orbiting around it. Might this be the distinguishing mark of the preferred frame? No. The symmetry of descriptions derived from the principle of relativity assures us that precisely the same process will be licit for a very massive sun at rest in another

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[3] In this example, the symmetry extends not just to the total physics but to the equations used to transform between the frames. This equivalence is not apparent in the standard way of writing the Lorentz transformation, For velocity v, it is $t' = \gamma(t - v/c^2\, x)$ and $x' = \gamma(x - vt)$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, for the forward transformation; and $t = \gamma(t' + v/c^2\, x')$ and $x = \gamma(x' + vt')$ for the inverse. The two formulae differ in signs. The formulae become the same if we replace (x',t') by (x'', t'') where $x''=-x'$ and $t''=t'$. Then the forward transformation is $t'' = \gamma(t - v/c^2\, x)$ and $x'' = -\gamma(x - vt)$; and the inverse transformation is $t = \gamma(t'' - v/c^2\, x'')$ and $x = -\gamma(x'' - vt'')$.

inertial frame G. The same fate awaits any factual result that we may seek to use to single out frame F over frame G.

In short, when one has a perfect symmetry among the descriptions, one can insist that they describe physically distinct systems; however, as Result C asserts, it is at the cost of insisting that there are physical differences that literally cannot be expressed in the descriptions themselves.[4]

The nocturnal expansion…

It will be convenient to consider one example, already much discussed in the philosophy of space and time literature. (e.g. Grünbaum, 1973, pp. 42-43) What if everything in the universe doubled in size overnight? Would we know? If nothing else changed, we certainly might be able to figure it out. If everything doubled in size overnight, then light would take twice as long to traverse the distance from the sun to here. These and a host of other observable effects would tip us off to the expansion. But what if, at the same time, there were other compensating changes? What if temporal processes slowed to a half so that our clocks take twice as long to complete a cycle and we no longer notice that light is taking twice the time in its journey? It turns out that there is a simple recipe for altering quantities so that all the non-gravitational laws of physics remain unaffected. These laws are built on three basic units: mass M, length L and time T. The rule is simply to double all lengths and times and to halve all masses:

$$L \rightarrow L'=2L \quad T \rightarrow T'=2T \quad M \rightarrow M'=M/2$$

This transformation will leave all the non-gravitational laws unchanged. Most important, the two fundamental constants, the speed of light c and Planck's constant h will remain numerically unchanged.[5] One quickly sees that no non-gravitational law would allow us to discern whether

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[4] Relativity principles are the most familiar example. There are other cases in which this extraordinary circumstance arises. See Norton (1995).

[5] To see this, note that they have units c – LT^{-1} and h – ML^2T^{-1}. Doubling L and T and halving M will leave each numerically unchanged since c $\rightarrow$ $(2L)(2T)^{-1}=LT^{-1}$ and h $\rightarrow$ $(M/2)(2L)^2(2T)^{-1}= ML^2T^{-1}$. We also posit that the transformation leaves electric charge unchanged. As a result electric phenomena will not reveal the expansion. Forces have units MLT^{-2} and, therefore, are quartered by the transformation. Coulomb's law tells us that the force between two charges q and Q separated by distance r is qQ/r^2, which also entails a quartering of force under a nocturnal doubling of sizes. We can now see how inclusion of gravitational phenomena would reveal the doubling, which is why they are excluded. The gravitational constant is $G=Fr^2/mM$, where F is the gravitational force between two masses m and M separated by distance r. Under

this doubling had happened last night or not. For example, quantum theory tells us that the energy E of a photon is related to its frequency ν by the law E = hν. If the expansion happened last night, the photon's frequency ν with units T^{-1} would be halved. We would not notice the halving directly since all our clocks would be correspondingly slowed. And we would not notice it through the law E=hν, which entails that the photon's energy E would be halved. Since energy has units ML^2T^{-2}, all energies would be halved and we would not discern the halved energy of the photon against the background of halving of all energies.[6]

…and its symmetry

The descriptions of the non-gravitational world with and without nocturnal expansion are symmetrical in the sense given above. That is, the totality of admissible physical processes will be described by the same regularities in each case, down to the detail of the numerical constants. We might try to find a way of extending the transformation so that it includes gravitational processes in the symmetrical descriptions, but I am not sure how that could be done. The simplest expedient is this. For the purposes of this discussion and the remainder of the paper, let us set aside gravitational processes, imagining that there is no such thing. That small fiction gives us a simple illustration of what it is like for the expansion symmetry to embrace all physically possible processes.

In this case, one might want to insist that there is a fact of the matter as to whether the type of nocturnal expansion described actually happened last night. But we also know that no factual statement in the expanded world will reveal the expansion; every such statement has an exact, numerically identical counterpart in the unexpanded world. For this reason I regard the two descriptions as merely different descriptions of the same thing. Alternatively, you could insist that there truly is a factual difference between the expanded and unexpanded systems since that is a logical possibility. But you must accept that the difference resides in facts that

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the transformation, it becomes $G' = (F/4)(2r)^2/(m/2)(M/2) = 4G$, so that any experiment that even indirectly allows measurement of G would reveal the nocturnal expansion.

[6] One example does not prove the general proposition. A more general argument is supplied by the standard methods of dimensional analysis. Briefly, we rewrite all our laws in terms of dimensionless numbers. Planck's law becomes the condition of the dimensionless number (E/hν)=1. The transformation leaves all dimensionless numbers unaltered. We might still discern the transformation from a change in value of fundamental constants such as h. However this transformation is chosen to preserve the values of the only two fundamental constants presumed, h and c.

outstrip the expressive power of either system, a most dubious result for anyone who thinks that a total science exhausts physical facts.

There is another way to see that the two descriptions are almost irresistibly of the same physical system. In both unexpanded and expanded systems, corresponding processes will be described by numerically identical laws. One way to account for that is to say that, in the expanded world, all spaces and times have doubled and we measure all processes with correspondingly doubled units. That treats the doubling as a real effect. Another way to account for it is just to say that we have left the world unchanged, but have merely redescribed it in units of space and time that are halved. Using those small units we mistakenly judge the regular units to be doubled and thus we mistake unexpanded processes for expanded processes. That is, the entire apparatus of the nocturnal expansion can be simulated by merely rescaling units without any physical change in the world.[7]

4. When Further Facts Break the Symmetry of Grue/Bleen and Green/Blue

Proper analysis of Goodman's new riddle of induction requires us to separate two cases: when the symmetry of grue/bleen and green/blue *is* broken by other physical facts; and when the symmetry of grue/bleen and green/blue is *not* broken by other physical facts. Each requires very different analysis. The first case will be considered in this section and the second in the following section.

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[7] In greater detail, it might run as follows. In the unexpanded world, a light beam traversing the 149 million km from the sun travels at 300,000 km/sec and requires 496 sec for the trip. In the expanded world, it still travels at 300,000 but the units are now new-km/new-sec, where one new-km equals 2 old-km and one new-sec equals 2 old-sec. That is, the units we use for description have expanded with lengths and temporal processes, where old-km and old-sec are the units of the unexpanded world. Light now requires 496 new-sec = 2x496 old-sec to traverse the 149 million new-km = 2x149 million old-km from the sun. To see the other way of accounting for the nocturnal expansion, imagine that the new-km are really just the original km and we have introduced another system of measure (called old-km above) in which the distance to the sun is 2x149 million km. As a result, we think that everything has doubled in size and doubled in time, but all that has happened is that we have halved the units used to describe spaces and times.

What is the best response to the new riddle of induction when the perfect symmetry of description fails? The literature on grue is enormous. Stalker (1994) includes a 177 page annotated bibliography, so I cannot hope to canvas the many resolutions offered. The best response, in my view, is simply to use an account of induction rich enough to exploit the asymmetries outside the domain in which grue/bleen and green/blue afford symmetrical descriptions. This seems to be the common theme of most responses to the new riddle. Quine (1970), for example, notes that green but not grue is a natural kind term in our broader science, so that we can rescue induction by restricting schemes like enumerative induction to natural kind terms. Alternatively, in a Bayesian analysis, our prior belief against grue-ified predicates could be encoded in our assigning "All emeralds are grue." a much lower (or even zero) prior probability, compared with "All emeralds are green." Conditioning on the observation of a green=grue emerald prior to *t*, would boost the probability of both, but the probability of "All emeralds are grue." would remain small (or zero).

Each of these responses face problems. However they are not problems specifically related to grue. Rather they arise from the awkward fact that we have no unobjectionable account of induction. Each account has its own peculiar weaknesses. If we seek to ground induction in a notion of natural kinds, we must accept the hazardous charge of giving a serviceable account of natural kinds. Or if we seek a Bayesian account, we must establish that a calculus devised to account for games of chance is also adequate to the rational treatment of all forms of uncertainty.

Elsewhere (Norton, 2003; forthcoming), I have defended the view that these problems are ultimately insoluble since, I argue, there are no universally applicable inductive inference schemes. The validity of an inductive inference cannot be established by displaying its conformity to an exceptionless, universal scheme, for there is none. The warrant for an inductive inference resides ultimately in facts that prevail in some larger or smaller domain, but never prevail universally. This is the basic idea of "material theories of induction," in which different inductive inference schemes are only ever warranted locally by material facts. This means that we cannot assess fully the cogency of an inductive inference by looking solely at its form. We must also consider the facts prevailing in the domain in which the induction occurs.

An immediate consequence of this view is that we cannot solve the new riddle of induction by showing that it contradicts some theorem of a universal, formal theory of inductive. We can only treat what the new riddle delivers on a case by case basis. However, when we do, the analysis proceeds much as it does for formal theories. We look to the relevant facts that bear asymmetrically on the original and grue-ified descriptions and hope that they will differentiate between them. That hope need not always be realized. It is realized in the case of green and grue emeralds. The relevant fact is that gems of the same type generally have the same physical properties, including their color. But there is no such constancy among gems for grue-ified physical properties. This fact licenses the inductive inference from the greenness of some emeralds to all; but it does not license the inductive inference from the grue-ness of some emeralds to all. Of course we now ask how we know this fact about gems. That triggers an analysis of further inductions within gemology and chemistry.

Whether these accounts succeed is really the question of whether they are able solve the old problem of discriminating between different continuations of the pattern manifested in the evidence. We may have our doubts since this problem has long proven recalcitrant. My view is that a material theory of induction does succeed and that it explains why different formal accounts of induction also meet with success in limited domains. But that is an issue to be argued elsewhere. Whatever their failings, these accounts of induction are not prone to the novelty of Goodman's new riddle of induction in so far as they can step outside the symmetry of grue/bleen and green/blue.

5. When Further Facts Do Not Break the Symmetry of Grue/Bleen and Green/Blue

The new riddle of induction supplies nothing new as long as the symmetry of grue/bleen and green/blue is not reflected in the broader physical context. If the symmetry is reflected in the broader physical context of all relevant facts, then we face a very different sort of problem. For now, if evidence inductively supports an hypothesis according to some particular theory of induction, then we would expect that evidence to support equally the incompatible grue-ified hypothesis. This must arise with any account of induction whose judgments are based solely on the relevant facts and the formal properties of the descriptions of the evidence, hypothesis and the relevant facts. For every ordinary fact will have a grue-ified counterpart; and the descriptions of the evidence, the hypothesis supported and all background assumptions will be formally identical with the descriptions of the grue-ified case.

My principal contention is that this perfect symmetry is exactly why we should be untroubled by theses cases. For according to Claim C above, the original and grue-ified descriptions can only fail to pertain to the same physical situation if there are relevant facts outside the descriptions that distinguish the two cases. By supposition, there are no such facts. So we have good reason to believe that the two descriptions are merely notational variants of one another. As a result, no account of induction should pick between them. A failure to pick between ordinary and grue-ified hypotheses is just what we should expect of any good account of induction.

In this section, I will illustrate this conclusion by looking at two ways in which the perfect symmetry described here can arise. In one, symmetry is achieved by liberally grue-ifying everything; in the second it is achieved by excising all facts that breach the symmetry. In both cases it will be clear that perfect symmetry can only be achieved by fairly extraordinary contrivances.

5.1 Grue-ifying Our Total Science

The total science objection fails

We saw in Section 4 how the novel problems of the new riddle can be avoided by calling on facts that do not respect the symmetry of grue/bleen and grue/bleen. There is a natural and powerful objection that can be raised to this approach. It was presumed in Section 4 that we can expand our scope into a larger context in which symmetry breaking facts prevail. But what if we provide an alternative, grue-ified description covering that larger scope as well? Then the symmetry breaking facts would now lose their power since there would be corresponding grue-ified facts in the alternative description. That still does not preclude the possibility of further symmetry breaking facts outside that scope. They can be precluded once and for all by extending the scope of the grue-ified descriptions to cover our total science. We would now have two accounts of our total science, the regular and the grue-ified. No symmetry breaking scientific facts would be available and there is no larger context left in which to seek them. Any candidate symmetry breaking fact in one description (e.g., that green is a natural kind term) would be matched by a corresponding fact in the other (e.g., that grue is a natural kind term), so it would fail to distinguish the two cases. The escape is blocked.

Claim Da of the discussion above suggests the answer to the objection. If we grue-ify our total science in this way, then we have simply generated a symmetric description of the very same facts; or at least we have a symmetric description that can only fail to describe the same facts if we presume that there are further distinguishing facts that somehow elude expression in our total science—not a position anyone should be eager to defend. So the total science objection fails exactly because it presumes eradication of any difference between the original and grue-ified descriptions.

Claim Db indicates that a similar failure awaits efforts to protect the novelty of the new riddle of induction by extending the grue-ified descriptions not just to embrace all science but also to cover all formally expressed knowledge of any kind.

An illustration

The principle of this answer is clear. However it would be nice to see some of the details of the grue-ified total science and how it ends up describing the same facts as the original science. I have not been able to see a simple way of implementing universally the original transformation of green and grue. However it turns out that we have already seen an essentially similar transformation in the example above of nocturnal expansion. The effect of the expansion is to double the time of all processes. That means that the expansion will halve the frequency of all light waves. This is not a simple exchange of green and blue at time *t* as in Goodman's example. Instead, at time *t*, all colors are shifted away from the violet towards the red end of the spectrum. Green light of frequency 550 THz would be shifted to infrared light of 275 THz; and ultraviolet light of 1100 THz would be shifted to green light of 550 THz. And so on.

…	1100THz ultraviolet	550 THz green	275 THz infrared
↓	↓	↓	↓
1100 THz ultraviolet	550 THz green	275 THz infrared	…

Let us consider the fate of an emerald. Prior to *t*, it reflects green light of frequency 550 THz. In the ordinary description, after *t*, it still reflects green light of frequency 550 THz. In the description corresponding to the expanded world, however, after *t*, it now reflects light of 275 THz. Emeralds observed before *t* are green; emeralds observed after *t* are infrared. Let us call this compound property "green#", the expanded world's analog of grue.

Prior to *t*, we observe emeralds to be green, or, equivalently, green#. Does this confirm that all emeralds are green; or that all emeralds are green#? The answer is both and that this engenders no problem. To begin, nothing observable depends on whether we decide that all emeralds are green or green#. If the first is true, emeralds observed after *t* will reflect green light of frequency 550 THz. If the second is true, we now shift to what is putatively a new world that expands at *t*. In it, after *t*, emeralds will reflect infrared light of frequency 275 THz. But no experiment in this expanded world will reveal the change. All temporal processes will be slowed correspondingly. Indeed any device capable of measuring the color of light—prisms, diffraction gratings, standard color swatches for comparison—will all be altered so as to make the shift undetectable. Since no fact expressible in the description of the expanded world, whether pertaining to observables or not, can distinguish it from the unexpanded world, I have urged above that we simply regard it as a variant description of the same world.

When we grue-ify or, analogously, nocturnally expand, our total science, the resulting perfect symmetry of the descriptions defeats the new riddle of induction. That symmetry gives us no grounds to presume a factual difference between the assertions that all emeralds are green or grue (or, in the expanded worlds, green or green#). So each can properly be confirmed equally by our history of observation of green emeralds.

Can all emeralds really be both green and grue?

I now want to describe and dissolve a mistaken objection to the presumed physical equivalence of symmetric descriptions in the context of a regular and grue-ified total science. Let us assume, the objection runs, that we are in a world in which all emeralds really are green. Hence it is false that all emeralds not observed before *t* are blue. Hence it is false that all emeralds are grue. That is, the two sentences "all emeralds are green" and "all emeralds are grue" cannot both be true and thus cannot be physically equivalent descriptions.

What the objection overlooks is that the two sentences pertain to different members of a pair of total descriptions.[8] The total descriptions are symmetric and presumed to cover a scope sufficiently broad to preclude the obtaining of symmetry breaking facts outside that scope. In the normal total description, it will be true that "all emeralds are green" and, for the reasons the objection states, false that "all emeralds are grue." Matters will be exactly reversed if we shift to the grue-ified total description. There "all emeralds are grue" will be true and "all emeralds are green" will be false. Thus both "all emeralds are green" and "all emeralds are grue" can *both* be true as long as they are referred to the appropriate total description. Following the earlier analysis, they will turn out to be describing exactly the same physical properties of emeralds.

5.2 Symmetry from Impoverishment

It is unavailing to seek perfect symmetry by grue-ifying our total science. Might we achieve it through another stratagem? Might we seek unusual physical systems that are so

[8] This should not be mistaken for an invocation of Quinean holism. The point is a more modest one. Two assertions may appear superficially to say different things merely because we neglect to note that they are expressed in different languages—a point that can be made by holist and reductionist alike. Consider the assertions that “this object is of size 1” and “this object is of size 2.54”. We may mistakenly think that their factual content is different because we neglect to note that the first derives from a science that uses US units of measure (inches) and second from a science that uses metric units (centimeters). This is essentially the effect in the case of the nocturnal expansion since that expansion amounts to a nocturnal change of units of measure.

impoverished factually that there are no relevant external facts that fail to respect the symmetry of grue/bleen and green/blue? While such systems are logically possible, they must be very contrived, as the example of the emerald world below indicates. In any case, such cases do not provide a place in which the novelty of the new riddle can create problems for induction. In these cases, there are no external facts to distinguish the two descriptions. So, as Claim C suggests, we cannot preclude that they are merely variant descriptions of the same physical situation.

The emerald world

Let us imagine a fictitious world in which emeralds come in three types: a left-handed type, a right-handed type and a perfectly symmetrical type. The left-handed emeralds are perfect mirror images of the right-handed emeralds; and (therefore) conversely. This is not so unrealistic an assumption since many crystalline substances exist in left and right-handed crystals. For present purposes, it will be quite adequate to imagine that each emerald is an elaborately carved left hand; or an elaborately carved right hand that is the perfect mirror image of the left hand; or a perfect sphere. We begin observing emeralds, noticing that all we observe prior to time *t* are right-handed emeralds. We infer inductively that all emeralds are right-handed.

We now introduce grue-ified predicates "reft" and "light":

Reft-handed applies to all thing examined before *t*
just in case they are right-handed
but to other things just in case they are left-handed.

Light-handed applies to all things examined before *t*
just in case they are left-handed
but to other things just in case they are right-handed.

By familiar argumentation, our evidence is equally well described as the observation of nothing but reft-handed emeralds prior to *t*. If we now infer to reft-handness of all emeralds, our evidence equally supports two incompatible hypotheses of the type familiar in grue-ified problems.

This incompatibility arises because we do not yet have systems of descriptions that are fully symmetric in right/left and reft/light. There are symmetry breaking facts in the world. For example, take an emerald observed to be right-handed prior to *t* and compare it with another properly called right-handed but unobserved prior to *t*. We readily affirm that they have the same handedness. We can do this by transporting one through space to the other and aligning them; or we may have a right-handed glove that we transport between them, confirming that the glove fits both. These tests will fail for the analogous case of reft-handed emeralds. If a glove fits one—say the emerald observed reft-handed prior to *t*—then it will not fit the other—a reft-handed emerald unobserved prior to *t*. These facts break the symmetry.

We achieve perfect symmetry by some fairly fanciful alterations to our emerald world that excise the physical facts that make comparison of handedness of different emeralds physically meaningful. We imagine that each emerald lives in its own Euclidean space, spatially disconnected from the spaces of every other emerald. The emerald world is the totality of these spaces. We might imagine each space as a parallel mini-universe subsisting independently of all the others; or we might imagine them on opposite sides of successive big bang/big crunch spacetime singularities in which all spatiotemporal structures are obliterated. Within each space, it makes perfect sense to say that the emerald is spherical or handed. If it is handed, an arbitrary stipulation will fix whether we call the emerald right-handed or left-handed; and that stipulation will fix the right or left-handedness of gloves and other handed objects in that one space. But there will be no fact of the matter as to whether a handed emerald designated as right-handed in one space has the same handedness as a handed emerald designated as right-handed in another space. Each space will require its own independent stipulation as to whether its handed emerald is to be called right or left-handed. There is, for example, no larger or higher dimensioned embedding space that might allow us to transport an emerald or a glove from one space to another to check sameness of handedness. The issue is not merely verificationist: that there might be a fact of the matter but we have no means to verify it. The supposition is that there is no overarching fact in the physical laws that gives sense to sameness of handedness across the spaces. The laws governing all physical processes in each space are unchanged under mirror reflections; that precludes, for example, quantum mechanical weak interactions, since they distinguish left from right and would give us a way to check if two emeralds in two spaces had the same handedness.

An observer in each space observes its emerald and communicates the result back to us. Exactly how this communication occurs is not easy to say given the disconnectedness of the spaces. We need merely presume that it happens in a way that does not compromise the factual lack of a unified sense of handedness across all the spaces. We also presume that the reports arrive at different times and that these arrival times will be the times relevant for the definitions of reft and light. Finally we presume that all reports prior to *t* are of right-handed emeralds; or, equivalently, of reft-handed emeralds. As before, we take this history of reporting to confirm equally well that:

All emeralds are right-handed.

All emeralds are reft-handed.

It might seem that we have reverted to the original problem and are confirming incompatible extensions of the evidence. Since we have been scrupulous in retaining the full symmetry of reft/light and right/left in our doctored world, that is not so. The two universal generalizations are merely notational variants of the same fact; that is, their factual content is equivalent to

All emeralds are not spherical.

For where the two generalizations appear to differ is over the emeralds not observed prior to time *t*. The first generalization declared them to be right-handed; the second, left-handed. But that difference is no factual difference. Whether the handed emerald of one space is called right-handed is merely the result of a stipulation fully independent of the corresponding stipulations in other spaces. The factual content, as far as trans-space comparison is concerned, of the assertion that the emerald is right-handed is merely that the emerald is handed; and that is the same factual content as the assertion that the emerald is left-handed.

This example of the emerald world is extraordinarily contrived. The problem lies with grue. While the definitions of grue/bleen and green/blue are symmetric, that symmetry does not extended to the context in which these predicates normally appear. Extraordinary contrivances are needed to eradicate its symmetry breaking facts.

6. Conclusion

We are either in a situation in which the symmetry of grue/bleen and green/blue obtains in the broader context or it does not. In either case, what is new in Goodman's new riddle of induction fails to establish a new, principled problem for all accounts of induction.

To arrive at the first case, we imagine ourselves in a world, impoverished or otherwise contrived, so that grue/bleen and green/blue provide fully symmetric descriptions These descriptions are strong candidates for descriptions of the same physical world; any physical difference between them must be inexpressible in the descriptive apparatus presumed. No account of induction ought to be expected to distinguish between such descriptions. Their failure to do so is their success at not distinguishing what should not be distinguished.

The second case arises in ordinary circumstances. There the symmetry of grue/bleen and green/blue fails. Our accounts of induction are now free to exploit the asymmetry so that evidence may bear unequally on different projections of past observations of emeralds. How they are to do this and whether they succeed is a matter to be addressed outside this paper, although I have indicated above in Section 4 my belief that we have had some success in this problem. What matters here is just this: the novelty of Goodman's new riddle of induction gives us no principled reason for the failure of all such accounts. It does, however, establish the failure of any account of induction that is not able to exploit the asymmetry, such as an unqualified enumerative induction. That failure is the full extent of the new difficulties brought by Goodman's new riddle of induction.

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