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# INFINITE IDEALIZATIONS

1. Approximations of arbitrarily large but finite systems are often mistaken for infinite idealizations in statistical and thermal physics. The problem is illustrated by thermodynamically reversible processes. They are approximations of processes requiring arbitrarily long, but finite times to complete, not processes requiring an actual infinity of time.
2. The present debate over whether phase transitions comprise a failure of reduction is confounded by a confusion of two senses of "level": the molecular versus the thermodynamic level and the few component versus the many component level.

## 1. INTRODUCTION

In my talk at the celebration of the 20th Anniversary of the Vienna Circle Institute, I sketched results of recent work on approximation and idealization (Norton, 2012). A goal of that work was to clarify the widespread use of infinite limits in statistical physics to introduce what are informally described as idealizations. This literature examines the behavior of systems composed of very many – but always finitely many – components. Certain properties of these systems settle down to stable values if the number of components is very large, as is the case with ordinary thermal systems. These stable values can be approximated quite closely by taking a limit of the property. Each of these properties is a function of the number of components and one merely takes the limit of these functions. The key point is that the limit of these functions can be taken without considering a system that consists of an actual infinity of components.

Infinite limits can be treacherous, and especially so, if we take a limit to a system consisting of an actual infinity of components. When things go badly, as they often do, this limit may yield a limit system of infinitely many components with quite unintended properties. Worse, if the limit is sufficiently badly behaved, there may be no limit system at all. What results is confusing talk about limit systems that have properties different from those announced; or, worse, the attribution of properties to systems that cannot exist at all.

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To help clarify these issues, I called for a more disciplined use of the related terms "idealization" and "approximation." It then followed that many of what are called idealizations in the present statistical physics literature are really approximations. Below in Section 3, I will give another example of a misleading infinite limit, the notion of a reversible process in thermodynamics. My goal is to show how the terms "approximation" and "idealization" can be used to separate two quite distinct forms of analysis. The example has been chosen because it is technically much simpler than the infinite component number limits employed in statistical physics. The infinite time limit of reversible processes will display the confusion of approximation and idealization with fewer distractions from the technical difficulty of the infinite component number limit of statistical physics.

Infinite idealizations in statistical physics are often associated with claims of a failure of reduction; and there is presently some debate in the literature over when and whether the reductions succeed. My concern is that much of this literature conflates two distinct senses of reduction in statistical physics. They are a reduction between macro-thermodynamic and micro-molecular systems; and a reduction between system of many components and few components. While I do not think that separating these two senses will resolve all disputes, I do think that attending directly to the two senses may dissolve some of the incomprehension in the disputes. In particular, recent work on phase transitions displays a failure of reduction between many component and few component system; but it does not display a failure of reduction between thermodynamic and molecular systems. These two senses of reduction will be developed in Section 4.

## 2. A WELL-BEHAVED IDEALIZATION

My goal is to demonstrate how easy it is to mistake an idealization for something less. To prepare for these troublesome cases, I will review an unproblematic case in this section. An ideal gas provides an example of an idealization in which everything works as we would expect. Describing it will also enable me to introduce the definitions that will be employed for approximation and idealization.

Dilute gases under normal conditions of temperature and pressure conform quite closely to the ideal gas law

$$PV = nRT \tag{1}$$

Here P is the pressure, V the volume, n the number of moles, T the temperature and R is the ideal gas constant. The law has surprisingly wide scope. It applies also to the osmotic pressure exerted by dilute solutions. While often good, the fit of the law to the behavior of real systems is never perfect. Errors become larger if, for example, we consider regimes of high pressure and high density.

The law expressed in (1) is a proposition that describes the target system inexactly. These I shall approximations:

> An *approximation* is an inexact description of a target system. It is propositional.

So far, the ideal gas law has been introduced as an empirical generalization. There is a second way to introduce it. We consider another fictitious system of very many non-interacting, spatially localized components. The system is fictitious since there are no real systems like that. All molecules occupy some volume of space and interact at least weakly with other molecules. The same is true of solute molecules in solution. While these real, target systems conform only inexactly to the ideal gas law, it turns out that the fictitious system obeys it exactly.[2]

This fictitious system, I will say, is an idealization of the target systems of real dilute gases and dilute solutions. Loosely speaking, it is an idealization because it looks like those systems in the particular aspects that interest us; that is, in the relationship between their pressures, temperatures and other magnitudes in the ideal gas law (1). More carefully, this fictitious system is an idealization of those real systems in that an *exact* description of the fictitious system for the aspects of interest turns out to provide an *inexact* description of the target system. Put more tersely:

> An *idealization* is a real or fictitious system, distinct from the target system, some of whose properties provide an inexact description of some aspects of the target system.

This characterization describes the essential relationship between idealizations and approximation. We take the description that is exactly true of the idealization. It will be propositional, such as the ideal gas law (1). This same proposition then turns out to be an inexact description of the target system.

It follows that we can always demote an idealization into an approximation by discarding the idealizing system and preserving only the relevant propositions that describe it. A corresponding promotion from approximation to idealization will not always be possible. Cases in which the promotion fails are just those that will interest us here.

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[2] It is not generally recognized how few assumptions are needed to generate the ideal gas law. One does not need the familiar assumptions of many point-like particles propagating in straight lines in vacuo and bouncing elastically off the walls confining the gas. One merely needs to assume that there are many, spatially localized components that do not interact with each other. They may interact with other components present in the bulk of the system without compromising the derivation. For a short demonstration employing very few assumptions, see Norton (2006, §2.1).

## 3. REVERSIBLE PROCESSES IN THERMODYNAMICS

An idealization is a system, fictitious or real, whose properties, propositionally described, supply inexact descriptions of the target system. The essential starting point of the notion of idealization is that we have a consistently describable system, even if it is fictitious. Without that, there is no idealization. Reversible processes in thermodynamics, I shall now argue, fail this minimal condition. Whatever they may be, they are not idealizations. Recognizing that fact helps us dispel the fog of paradox that surrounds them.

### *3.1 The Informal Idea of Reversible Process*[3]

All real thermodynamic processes are dissipative in the sense that they increase the total thermodynamic entropy of the universe. For example, when a quantity of heat Q passes from a hotter body at temperature $T_{hot}$ to a colder one at $T_{cold}$, the entropy created is[4]

$$\Delta S = Q/T_{cold} - Q/T_{hot} = Q(T_{hot} - T_{cold})/ (T_{hot}.T_{cold}) = Q.\ \Delta T/(T_{hot}.T_{cold}) \qquad (2)$$

where $\Delta T = T_{hot} - T_{cold}$. This creation of entropy amounts to a loss of the ability of the heat to perform work, for a smaller portion of the degraded heat at the lower temperature can be converted to useful work. It is an inefficiency that the science of thermodynamics was founded to minimize.

The inaugural insight of thermodynamic theory, in Sadi Carnot's original work of 1824, is that the least dissipation and greatest efficiency is approached in the unachieveable limit of reversible processes. No real thermodynamic process is reversible. Heat will only pass from a hotter body to a colder one if there is a temperature difference between them; and, as along as that temperature difference is maintained, the process cannot be reversed. Heat cannot spontaneously pass from the colder to the hotter body.

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[3] For a detailed examination of the notion of reversibility in thermodynamics, see Uffink (2001).

[4] It is assumed that the heat passes by conduction and without appreciable change in the temperature of either body. For that Q may be very small in relation to their internal energies; or their internal energies may be kept constant by work performed reversibly on each system.

We can convert this process into a thermodynamically reversible process by making the temperature difference ΔT between the hot and cold body "infinitesimally small," so that the temperature difference that drives the heat transfer is negligible. To assure that the desired quantity of heat, Q, still passes, we must wait an arbitrarily long time, since we make the temperature difference arbitrarily small. However taking the limit of infinite time returns a process with remarkable properties. Throughout, the two bodies are at the same temperature, that is, they are at thermal equilibrium. As a result, the process now no longer has a favored direction for the passage of heat. It is the same process as would be created had we started with a temperature difference that passed heat in the reverse direction. In this sense, the process produced by this ideal, infinite time limit is reversible; it proceeds equally well in either direction. Finally, since its temperature difference ΔT is zero, equation (2) tells us that no entropy is created by the passage of heat Q between the two bodies. We have achieved minimum dissipation.

### *3.2 Why it is Nonsense*

The account of the last section is familiar from thermodynamics treatises. But is it also nonsense in the precise sense that there is no possible referent for the term "reversible process." For the reversible process just described cannot exist if we want to preserve some of the simplest elements of thermal physics. The process described must satisfy two incompatible conditions:

(a) A non-zero quantity of heat Q passes from one body to the other.

(b) The two bodies are at the same temperature.

Once the bodies are at the same temperature as (b) requires, no heat passes between them and (a) fails. This follows directly from the fact that heat only passes from one body to a second if the first body is hotter than the second. Condition (b) allows no driving temperature difference. Even an "infinitesimal difference" is too much since it will favor passing heat in one direction and not the other, thereby contradicting reversibility.

We may hope that the magic of taking the infinite time limit will somehow preserve the passing of heat. For, in every finite time process considered in approaching the limit, we do succeed in passing Q of heat. When the limit delivers a process consisting of the two bodies at the same temperature in contact over an infinity of time, we imagine that somehow the passage of heat is preserved. It is not.[5]

Our vain hope is that a zero rate of heat transfer over an infinity of time balances out to the passing of the requisite non-zero quantity Q of heat. In effect we are pinning our hopes on the faux equation:

$$\text{“}Q = 0 \times \infty\text{”}$$

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[5] The infinity of time also makes it a process that never ends, so we will never see its results. Moreover, we will not be able to append a subsequent process. For there is no finite time at which the subsequent process can start.

Alas, we know that this equation is nonsense. Once we take the limit of zero temperature difference and infinite time, we have a static system. It consists of two bodies at the same temperature that pass no heat between them in any finite time. Waiting longer will not change the fact that no heat passes. Even waiting an infinity of time will make no difference. That infinity of time consists of infinitely many periods of finite time in each of which no heat passes. The sum of these infinitely many zeroes of heat passed is still zero.

In short, we have used a limit to infinite time incautiously. We take a process that is well defined for all finite times. We let the time go to infinity and then speak uncritically about the process that results, attributing properties (a) and (b) to it. There is no such process on pain of violating the basic notion that heat passes between bodies only when driven by a temperature difference.

### *3.3 Reversible Processes as Approximations*

The prospects look bleak for reversible processes in thermodynamics. How it is possible that they can figure so centrally in thermodynamic theory? The answer is that they figure as approximations and not as idealizations. The infinite time limit procedure does not generate a new system that is our idealization. Rather, the limit procedure takes the exact propositional descriptions of the finite time processes and converts them into a simpler, inexact description of the finite time process. That is, the limit process merely generates approximations.

To see how this works, recall the rules that govern heat transfer. The total heat transferred between two bodies with temperature difference $\Delta T$ over some time is given as:

$$\text{heat} = \text{constant x } \Delta T \text{ x time} \tag{3}$$

and the entropy created in the process is given by (2). Let us tabulate these quantities as the limit of infinite time is approached. In order to maintain passing of a fixed, non-zero quantity of heat, $\Delta T$ and time are adjusted in concert so that ($\Delta T$ x time) retains the same value. To keep the numbers simple, we can set the constant in (3), so that heat=100 when $\Delta T$ = time = 1:

| | $T_{hot}$ | $T_{cold}$ | $\Delta T$ | time | heat passed Q | entropy created $\Delta S$ |
|---|---|---|---|---|---|---|
| | 10 | 9 | 1 | 1 | 100 | 1.11 |
| | 10 | 9.5 | 0.5 | 2 | 100 | 0.526 |
| | 10 | 9.9 | 0.1 | 10 | 100 | 0.101 |
| | 10 | 9.99 | 0.01 | 100 | 100 | 0.010 |
| | 10 | 9.999 | 0.001 | 1000 | 100 | 0.001 |
| | 10 | 9.9999 | 0.0001 | 10000 | 100 | 0.0001 |
| | ... | ... | ... | ... | ... | ... |
| Limit of columns at time → ∞ | 10 | 10 | 0 | ∞ | 100 | 0 |
| Properties of limit process | 10 | 10 | 0 | ∞ | 0 | 0 |

*Table 1. The infinite time limit of a heat transfer*

The table shows how the various quantities stabilize in their approach to limiting values:

$$\Delta T = 0 \quad Q = 100 \quad \Delta S = 0 \tag{4}$$

What are to make of these limiting values? They are the properties that we had uncritically attributed to a single process that we called "the reversible heat transfer." However, as we have just seen, they cannot be the properties of a single process. Once we set $\Delta T = 0$, then we must have $Q = 0$ as well, no matter what the time. To underscore the point, the last row of the table displays the corresponding values for the static system that arises when we take the limit. That limit process consists of two bodies at the same temperature passing no heat between them.

The importance of the propositions comprising (4) is that they are inexact descriptions of each of the heat transfer processes that arise when we have a very small $\Delta T$ and a correspondingly large time. For, by virtue of their construction, the limiting values reported in (4) must be very close to the actual values possessed by heat transfer operations in the table that require very large, but finite times. That is, the limiting value of $\Delta S = 0$ is a very good approximation of the correct values of $\Delta S = 0.0001$ for the process with time = 10000 and $\Delta S = 0.001$ for the process with time = 1000.

To revert to the terminology introduced above, the function of the infinite time limit is to create propositions (4) that are *inexact descriptions* of finite, long time heat transfers; that is, the function of the infinite time limit is to generate an *approximation* of finite, long time heat transfers. Confusion arises only if we try to present the propositions of (4) as describing just one idealized process exactly, for then we are asked to imagine a fictitious process with impossible properties.

This analysis tells us how we should reform talk of reversible processes in thermodynamics. If there is a need to be precise, we would not talk of a reversible heat transfer in which heat $Q = 100$ is passed at $\Delta T = 0$ while creating no entropy, $\Delta S = 0$. Rather we would say that real heat transfer processes can be slowed so that they come arbitrarily close to the parameters set in (4).[6] If we manage to slow the heat transfer sufficiently, the differences between the parameters of our real system will become so small that we will not be bothered by the difference. However no amount of slowing will ever allow us to realize a process that is described exactly by the propositions of (4).

In sum, the infinite time limit generates approximations of long, finite time heat transfers in the form of the propositions of (4). The limit does not create an idealized limit process, that is, an idealization, of which these propositions are an exact description.

### *3.4 Other Examples in Statistical Physics*

The example of reversible processes in thermodynamics provides a simple illustration of a problem that arises repeated in statistical physics. In that science, we examine statistically the behavior of large systems of components – molecules, spins, radiation modes – in order to recover their thermal properties. Since the systems have very many components, the taking of an infinite component limit is common. It is also common to treat these infinite limits incautiously as if they were creating idealizations. Yet they less commonly do so. Their role will vary with the particular way the infinite limit is taken and almost every possibility is realized somewhere.

The simplest case arises with the "continuum limit," described by Compagner (1989). In it, the number of components passes to infinity in such a way that both system volume and the fraction occupied by matter remain constant. One finds (Norton, 2012, §4.2.1) that there is no limit system; some of limits that would return its properties are not convergent.

The Boltzmann-Grad limit (Lanford, 1975, 70-89; 1981) also passes the number of components to infinity while keeping the system volume constant. However it does it such that the fraction of space occupied by matter becomes zero and the matter itself becomes an infinity of dimensionless points. This limit system has unexpected properties. In it, the familiar rules of mechanics are no longer able to resolve collisions uniquely, so that, if there are collisions in the system, they are indeterministic. (See Norton, 2012, §4.2.2).

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[6] This arbitrarily close approach is possible only as long as we operate within phenomenological thermodynamics. Once we allow for the atomic constitution of matter, then we operate within statistical mechanics and fluctuations intervene. When the temperature difference becomes small, some creation of entropy is necessary to overwhelm the fluctuations and allow the process to proceed to completion. This last observation, I argue in Norton (2011, manuscript), comprises a fatal no go result for the theory known as the thermodynamics of computation.

The most commonly used limit is the "thermodynamic limit," in which both the number of components and the system volume pass to infinity in such a way that their ratio, the component density, remains fixed. One can find two versions of the limit in the literature. In what I call the "weak" form, such as in Le Bellac et al. (2004, pp. 112), one merely tracks the behavior of the system's properties as the number of components becomes arbitrarily large. In what I call the "strong" form, as in Ruelle (2004, p.2), the limit is used to take us to an actually infinite system whose properties are then examined as a surrogate for large systems of finitely many components. The weak form corresponds to using the limit merely to create approximations of large systems with finitely many components. The strong form corresponds to using the limit to create an idealization that consists of a new system with an actual infinity of components.

These last idealizations are inherently fragile since most infinite systems of components behave differently from their finite counterparts. Most notably, they are generically indeterministic. An infinite system of masses connected by Hooke's law springs, for example, is a natural model of an infinite crystal. It can remain quiescent for an arbitrary time. Then it can spontaneously set itself into motion by excitations propagating in from infinitely far away in the crystal.

If the strong version is to be used, one must contrive additional constraints to block this unwanted behavior. This is the practice of the more mathematically oriented theorists such as Lanford (1975). However it leaves the reader wondering what *other* unwanted behavior unique to infinite systems remains unblocked.

Most other invocations of the thermodynamic limit leave vague just how the limit is used. While they may talk of infinite systems and their properties, it is more often the case that the analysis proves to be not directly of an actually infinite systems. There are, for example, no additional conditions applied to block the unwanted generic indeterminism of infinite systems of components. Rather, the analysis seems to arrive at its results by tracking the limiting behavior of the propositions describing large systems with finitely many components. That is, their results are the analogs of the propositions (4) and provide approximations, not idealizations. I have argued in Norton (2012, §4.3) that this is case with the analysis of critical phenomena by renormalization group methods. They employ approximations and not infinite idealizations.

## 4. REDUCTION AND EMERGENCE IN STATISTICAL PHYSICS: AN OVERLOOKED DISTINCTION

The presence of infinite component limits in the statistical physics literature has ignited a debate in the philosophy of science literature. It is over the question of whether phase transitions are emergent properties, in the sense that they indicate a failure of reduction between thermodynamics and statistical mechanics.[7] This is a frustrating debate, rife with mutual incomprehension. My conjecture is that at least a part of the incomprehension comes from a failure to distinguish two senses of "level." The two senses should be kept separate, since one favors reduction relations and the other does not. Participants in the debate can press toward conclusions favoring reduction or emergence according to which notion of level appears more in their analysis.

### *4.1 A Familiar Distinction: Ontological and Theoretical Reduction*

First a preliminary: the distinction is *not* between ontological and theoretical (or explanatory) reduction, which will be sketched in this section. In this distinction, a thesis of ontological reduction asserts that thermal systems just are systems of many molecules, spins, radiation modes, and so on. A thesis of theoretical reduction asserts that the theories of these microscopic components are able to generate the macroscopic thermodynamic theories.

In the case of ontological reduction, the debate has been settled in favor of the reductionists.[8] While the ontological thesis is quite ambitious, the evidence in its favor is so massive that, now, no one who doubts it is or should be taken seriously. The situation with theoretical reduction is less clear. In some cases it clearly works: the recovery of the ideal gas law from microphysics is a good and popular example. However the case is yet to be made that it will always succeed.

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[7] A few of the contributions to this debate are Batterman (2005), Belot (2005), Butterfield (2011, 2011a), Callender (2001), Jones (2006).

[8] Lest one think that this is a trivial outcome, it is well to recall that atomic and similar corpuscular theories of the constitution of matter were little more than imaginative speculation for most of the millennia of their existence. Their correctness was only established within the last century or so. See Chalmers (2009).

The difficulty is that a theoretical reduction requires manipulation within a theory and theories employ human created methods and human selected postulates. That creation and selection must be done sufficiently well to enable computations at the molecular level that will return the requisite results at a thermodynamic level. The reach of derivations within theories can be less than we expect. Consider the simple Euclidean theory of figures built from circles and straight lines. The novice in geometry expects that all constructions are achievable with Euclidean straight edge and compass. Yet there are simple figures in the theory's ontology that outstrip the constructive powers of a straight edge and compass. Merely trisecting an angle is beyond their reach. The more extreme but familiar example is Gödel's celebrated discovery that some truths of arithmetic will escape the theorems of any particular, finite axiomatization. What happens when we pass to more complicated thermal systems? Will their properties always be within reach of the computations with molecules and spins of some nominated theory?

### *4.2 The Neglected Distinction*

The present debate is whether these last limits of theoretical reduction have already been reached with phase transitions. It is often couched in terms of levels. There is a lower level in which we attend to the properties of individual components molecules, spins or radiation modes. There is a higher level in which we attend to the thermodynamic properties of large collections of these components. We ask if a theoretical reduction relation obtains between the two levels.[9] There are two ways of identifying the higher and lower level:

> *I. Molecular-Thermodynamic:*
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> The lower molecular level is the theory of the components, characterized by the canonical positions and momenta of a phase space. The basic quantities of the theory are functions of the phase space: the Hamiltonian characteristic of the system and a probability distribution over the phase space. Other quantities, including mean energy, free energy and entropy, are derived from them.
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> The higher thermodynamic level is the theory of thermal states, characterized by the state variables: pressure, volume, temperature, density, and so forth. Internal energy, free energy and entropy are functions of these state variables.

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[9] I use this symmetrical language to retain compatibility with the different usages of the term "reduce" and "reduction" by philosophers and physicists. Following Nagel (1961, Ch. 11), philosophers ask if the higher level theory can be derived from the lower level theory. If so, the higher level theory is said to reduce to the lower level theory. Physicists ask if the lower level theory transforms into the higher level theory when an appropriate limiting operation is applied, such as the taking of a limit of infinitely many components. If that transformation succeeds, then the lower level theory is said to reduce to the higher level theory. This is a reverse of the language of the philosophers in terms of what reduces to what. Both are expressing the same idea, however. Loosely speaking, the higher level theory is some kind of special case of the lower level theory.

This is the familiar distinction that appears explicitly in most of the discussions in the philosophy of science literature. There is a second distinction that delineates the levels differently:

> *II. Few Components-Many Components.*
>
> Both of these levels reside within the molecular level of the first distinction. The lower "few" level consists of the analysis of systems of just a few components. The higher "many" level consists of the analysis of systems of very many components, where the systems of many components are still understood to have molecular constitution.

The ideal gas law provides a success of reduction between these levels. A single molecule can be treated as a gas, albeit one with wildly fluctuating properties.[10] Nonetheless one can determine the mean values of these fluctuating quantities, such as the gas' mean density and the mean pressure it exerts. These mean quantities can be scaled up from the few component level to the level of the very many components of a macroscopically sized samples of a gas of non-interacting molecules. What results is the ideal gas law.

This second delineation of levels is routinely suggested. It is already alluded to in Anderson's (1972) half century old and much celebrated slogan "more is different." However it is rarely distinguished from the first delineation. Rather we pass from a few component, lower level account to a many component and thermodynamic, higher level account without separating the two senses of lower and higher level just described. That obscures the source of any breakdown in the reduction relations. Did the breakdown arise in the passage from few to many components? Or did it arise in the passage from the micro-molecular to the macro-thermodynamic?

For each of these two delineations of levels, we can ask if a theoretical reduction relation obtains between the lower and higher level. In my view, we recover a different answer in each case.

In the molecular-thermodynamic case, the renormalization group method analysis of phase transitions supplies a successful theoretical reduction. Because of the technical brilliance and difficulty of the method and the presence of oddly behaved limits, it is possible to miss this simple fact. The key observation is that the renormalization group analysis is carried out in the lower, molecular level. It is, in the end, a molecular analysis. The basic units of the analysis are large systems of components, understood at the molecular level, with their associated

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[10] The idea that a single component – a molecule or microscopically visible corpuscle – exerts a pressure by virtue of its thermal motions was used to good effect by Einstein in his 1905 analysis of Brownian motion that established the reality of molecules. See Norton (2006, §3). However the same idea in Szilard's 1929 single-molecule engine has engendered another tradition in the thermodynamics of computation that is less successful for its neglect of fluctuations. See Norton (2011).

Hamiltonians. The notable successes of the methods, such as the recovery of critical coefficients, arise within computation in this statistical-molecular context.

In the few-many component case, matters are otherwise. We cannot scale up the properties of a single component molecule or spin and recover phase transition behavior. Something more is needed. Mean field theory was an attempt to provide that something more. It considered the behavior of a few components within the mean field generated by the many more components of the system. Since the analysis focused on just a few components, using the mean field as a surrogate for the rest, it belongs in the few component, lower level. The failure of mean field theory to return correct results quantitatively concerning phase transitions is a failure of reduction. The advance of renormalization group methods was to realize that analyzing the fluctuation behavior of systems of very large numbers of components is essential to representing phase transitions, quantitatively. In this sense, the analysis of phase transitions requires the higher, many component level and is not reducible to the lower, few component level.

The net outcome is that there is a failure of reduction as one passes from the lower level analysis of just a few components to the higher level of thermodynamic analysis. The passage is in two steps: first we pass from the few to the many component level within the molecular analysis; then we pass from the molecular to the thermodynamic level. The failure of reduction arises in the first step and not in the second.

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