August 21, November 12, 28, 2017

Correction to John D. Norton "How to Build an Infinite Lottery Machine"

John D. Norton[1]
Department of History and Philosophy of Science
University of Pittsburgh
jdnorton@pitt.edu http://www.pitt.edu/~jdnorton
and
Alexander R. Pruss
Department of Philosophy
Baylor University
alexander_pruss@baylor.edu http://alexanderpruss.com

This note corrects an error in John D. Norton, "How to Build an Infinite Lottery Machine."

Norton (2018) examines physical devices intended to realize an infinite lottery machine that can select from a countable infinity of outcomes without favoring any. Readers will be helped by consulting it before proceeding. Successful proposals seem to require exotic processes, such as supertasks or probability zero processes. The most promising of the proposals considered in Norton (2018) was one that employs an infinite array of coin tosses. Section 11.3 of Norton (2018) argues that its successful operation is a nonmeasurable event in the background probability measure of the coin flips of the array. The argument is fallacious and successful operation is provably a probability zero outcome.

This zero probability of success makes the machine of Section 11 less interesting and comparable in significance to the pointer on a dial machine of Section 2.3, since both now only succeed with probability zero. More significantly its failure means that the investigation has not found a design for an infinite lottery machine that employs only finite randomizers, like binary

[1] JDN: I thank Alex Pruss for drawing my attention to the problems in Section 11.3.

coin flips, that choose among a finite number of outcomes. This is so even when we employ infinitely many finite randomizers and accelerate our processes with supertasks. It makes more plausible the conjecture that no such design can succeed, except with probability zero. In light of this change, the most promising design among those of Norton (2018) is now the exotic quantum mechanical lottery machine of Section 10. Its primitive process is, by supposition, already a randomizer that selects among an infinity of outcomes.

The details of the fallacy in Section 11.3 are not especially illuminating, but are included here for completeness. The argument by rows arrives at the correct conclusion of zero probability in classical, countably additive probability theory. Moreover, the argument shows that the event of successful operation is measurable. Its measure is constructed from the measurable outcomes of individual coin tosses with complements, countable intersections and countable unions. There is no supposition of measurability that could figure in a reductio argument against measurability. Contrary to the suggestion of Section 11.3, no constructive argument within the probability calculus can yield a different probability for the outcome, unless the calculus is inconsistent. The arguments by column and by rectangles of *M* rows and *N* columns fail. Each is based on a limiting process that should produce a set of all arrays that do not contain the row HTTTTT… The limiting process in each produces only a proper subset of this set. A counterexample for the "by columns" argument is the array:

HTHHHHH…
HTTHHHH…
HTTTHHH…
HTTTTHH…
…

This array does not contain the row HTTTTT… but it is not in the limit set. Analogous counterexamples can be found for the argument by *M*x*N* rectangles.

If the number of rows in the array is uncountable, then the event of the array having at least one row that encodes a counting number is nonmeasurable.[2] However identifying the first row encoding a counting number is problematic. The notion of such a first row requires a

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[2] This follows from the fact proved by Norton in Section 11.3 that the probability that a particular fixed row contains the encoding of a counting number is zero together with the Proposition in Pruss (2017).

nonconstructive well-ordering of the uncountable set of rows, and additionally one would need a process that that traverses that uncountable set in that order.

References

Norton, John D. (2018) "How to Build an Infinite Lottery Machine," *European Journal for Philosophy of Science*, this issue.

Pruss, Alexander R. (2017) "Uncountable Independent Trials," blog post, http://alexanderpruss.blogspot.com/2017/08/uncountable-independent-trials.html.