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Observationally Indistinguishable Spacetimes:

A Challenge for Any Inductivist

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1. Introduction[1][2]

For several years, through the "material theory of induction," I have urged that inductive inferences are not licensed by universal schemas, but by material facts that hold only locally (Norton, 2003, 2005). My goal has been to defend inductive inference against inductive skeptics by demonstrating how inductive inferences are properly made. Since I have always admired Peter Achinstein as a staunch defender of induction, it was a surprise when Peter identified me as one of the skeptical enemies in "The War on Induction," in his (Achinstein, forthcoming). Peter reproached me for "taking on" his inductive heroes, Newton and Mill, and their celebrated rules of inductive inference.

That my project could lead me to become a foe of induction was unimaginable. Or it was, until I began analysis of a problem in philosophy of physics, whose elaboration is the purpose of

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[1] It is a pleasure to present this paper in honor of Peter Achinstein, with gratitude for his scholarship and dedication to philosophy of science.

[2] I thank Peter Achinstein for his thoughtful discussion in Achinstein (forthcoming) and also for correspondence, in which he clarified his views and made suggestions for editing this note. I also thank Claus Beisbart and John Manchak for helpful discussion on an earlier draft.

this note. I wanted to endorse certain inductive inferences whose cogency seems unassailable: we have never seen certain pathologies in spacetime; so inductive inference should assure us that we never will. However I was unable to display the facts that license these inferences. After some reflection, I believe this problem shows that induction has less reach than we thought. There are problems we expect inductive inference to solve but that it cannot solve.

This admission is no failure of the material theory of induction. No other approach to inductive inference fares any better with them. I will argue below that attempts to ground the inductive inferences in universal inductive principles founder on both the vagueness of the principles and the tendency of the applicable principles to contradict each other. While Peter has identified me as a foe of induction, I doubt that his analysis will help these principled approaches. Contrary to my expectations, Peter's (forthcoming, Section 3: Enter John Norton) does not mount a full-blown defense of Newton's and Mill's rules as a priori, even though he finds the rules to codify "a type of reasoning that is crucial to empirical science." Rather, he agrees with me that empirical considerations do determine whether an inductive inference is good. An inference that merely has the familiar inductive form "That A is B, so all A's are B" may fail to be good if the underlying facts are inhospitable.

Indeed, the analysis of the problem presented here is a success for the material theory of induction, provided one is prepared to limit the reach of inductive inference in this and similar cases. For the material theory enables us to delineate which inductive problems are tractable and which are not. That decision reduces to whether we can identify appropriate warranting facts. Theories of inductive inference based on universal principles are unable to make the corresponding discrimination, for their universal principles must hold everywhere. A failure of inductive inference is for them inexplicable.

The problem at issue concerns observationally indistinguishable spacetimes, described in the following section. In them, deductive inference cannot determine which is our spacetime, no matter how extensive a portion of the spacetime is observed. In Section 3, I will argue that these results do not illustrate an underdetermination of theory by evidence, since they make no decision between competing theories and they make little contact with the inductive considerations that must ground such a decision. Rather, in Section 4, I will describe how they exemplify a different sort of failure manifested by physical theories, a form of generic indeterminism in general relativity. In it, a specification of the observable past always fails to fix the remainder of a spacetime. While we may have no means to distinguish deductively among different cosmic futures in the cases considered in this literature, I will urge in Section 5 that we can pick among them with quite familiar sorts of inductive arguments whose cogency seems unassailable. Nonetheless, in Section 6, I will urge that these inductions are troubling in that they are what I shall call "opaque." That is, we cannot see through the inductive inferences to an unproblematic warrant, whether it be in matters of principle or fact.

2. Observationally Indistinguishable Spacetimes

The existence of observationally indistinguishable spacetimes in general relativity was brought to the attention of philosophy of science by Clark Glymour (1977) and David Malament (1977). An observer at any event in a spacetime is depicted as having full knowledge of everything in the temporal past of that event. The temporal past is that set of events from which ordinary processes, propagating at less than the speed of light, may reach the observer's event. It may happen that exactly the same observable past arises somewhere in a second spacetime. The first spacetime of the observer is observationally indistinguishable from the second, if this finding is assured no matter where the observer may be in the first spacetime.

Manchak (2009) proved that any well-behaved[3] spacetime always has many geometrically distinct, nemesis spacetimes from which it is observationally indistinguishable. Moreover the nemesis spacetimes will be "locally" the same as the observer's spacetime. In the first spacetime, one might have a condition that holds at each event, such as the Einstein gravitational field equations; or, more simply, a different condition that just asserts everywhere vanishing geometric curvature. The locality clause of the theorem assures us that the nemesis spacetimes will satisfy these same conditions.

The theorem and its proof involve some sophisticated spacetime geometry. But the basic idea is simple. A loose analogy, shown in Figure 1, illustrates it. Imagine an ant on an infinite, flat (Euclidean) sheet of paper who can survey only the surrounding 10,000 square foot patch. No matter where the ant may be, it cannot distinguish its sheet from a nemesis sheet, which

[3] The theorem excludes spacetimes in which the entire spacetime is observable from one event. They are "bizarre," because they include closed timelike curves, which permit time travel.

consists of a copy of the original sheet of paper rolled into a cylinder with a circumference of one mile.

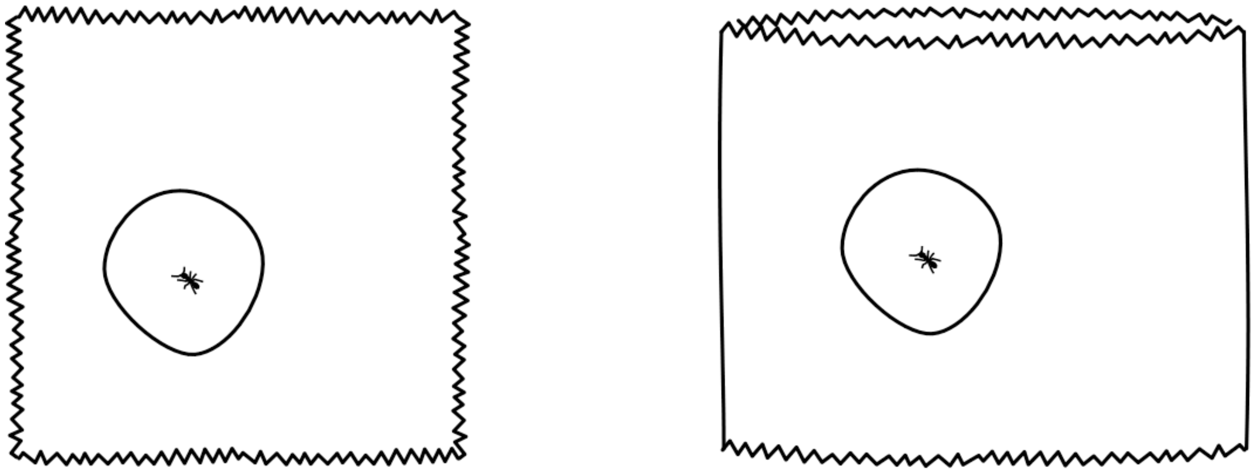

Figure 1. Ant on a sheet of paper

3. What the Significance is Not: Underdetermination of Theory

The thesis of the underdetermination of theory by evidence, in its strong and interesting form, asserts that all evidence necessarily fails inductively to fix theory.[4] The similarity in the terms "observationally equivalent theories" and "observationally indistinguishable spacetimes" requires some thought.[5] Are the observationally indistinguishable spacetimes illustrations of the thesis of the underdetermination of theory by evidence? I will argue that they are not. I agree with the opening remarks of Manchak (2009). He notes the distinctness of his result from the skeptical thesis that acceptance of some particular scientific claim can be resisted in the face of all evidence by revision to background theories.

The assurance of observationally indistinguishable spacetimes in general relativity fails to bear on the thesis on the underdetermination of theory by evidence in two ways.

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[4] In a recent paper (2008), I urge that the thesis is groundless. For further analysis of the relation of these examples to the underdetermination thesis and the possibility that the observationally indistinguishable spacetimes may just be variant presentations of the same facts, see Magnus (2005).

[5] It requires more thought than I gave it in writing footnote 13 of Norton (2008).

First, the indistinguishability does not pertain to theory. We are not presented, for example, with general relativity and some competitor theory, indistinguishable from it. Rather, what we cannot distinguish is whether *this* spacetime is the one of our observations or whether it is *that* one.[6]

Second, the indistinguishability asserts a failure of deductive inference, whereas the thesis of the underdetermination of theory by evidence asserts a failure of inductive inference. Many spacetimes are logically compatible with the fragment of spacetime that we observe. So deductive inference does not allow us to fix which of them is our spacetime. This failure is no failure of inductive inference, which routinely fares better with such problems. Deductive inference cannot assure us that our spacetime will continue to manifest the conservation of electric charge, even though it has been observed to do so exceptionlessly. The simplest inductive inferences can give us that assurance.

Manchak's theorem, however, is stronger. It does preclude particular sorts of inductive inferences from distinguishing the spacetimes. Our observable spacetime is four-dimensional and has a Lorentz signature metrical structure. We are allowed the inductive inference that this will persist in the unobserved part. More generally, we are allowed to infer inductively to the persistence of any local condition, such as the obtaining of the Einstein gravitational field equations, in both the observer's and the nemesis spacetimes. These inductive inferences, the theorem shows, will still not enable us to separate the spacetimes, for both will agree on them.

What is not shown, however, is whether other inductive inferences would enable us to separate the two spacetimes. It is essential to the theorem that the observer's spacetime and its nemesis are factually distinct. What needs to be shown is that these factual differences cannot be exploited by an inductive inference that can separate the two spacetimes. I will suggest below in Section 5 that apparently routine inductive inferences are capable of exploiting these factual differences to discriminate a spacetime from its nemesis. In Section 6, however, I will urge that this routine appearance is deceptive in that the warrants of these inductive inferences are unclear.

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[6] There is an ambiguity in the use of the term "theory." One might conceive an individual spacetime as a theory in its own right. The geometry of Minkowski spacetime is the special theory of relativity. However this use is unnatural in general relativity, in which the particular spacetimes are models of the general theory.

4. What the Significance Is: a Form of Indeterminism

The results on observationally indistinguishable spacetimes amount to this: we fix some part of the spacetime and, within the context of some physical theory like general relativity, the rest of the spacetime remains undetermined. This result is a form of indeterminism.

Indeterminism in physical theories arises whenever the full specification of the present fails to fix the future. Indeterminism is routine in standard, collapse versions of quantum theory. The full specification of the present state of a radioactive atom does not determine when it will decay in the future.

Determinism arises commonly in the spacetimes of general relativity. In Robertson-Walker spacetimes of relativistic cosmology. if we fix the spacetime geometry and matter fields of the universe at one moment of cosmic time, a "Cauchy surface" they are fixed by the theory for all times.

The failure of determinism in quantum theory in the 1920s was shocking since it implied a profound limit on what we could know about the future. It told us that no matter how much we knew about the present state of some suitably chosen quantum system, we could not know assuredly what it would do in the future.

This kind of principled epistemic limit on what we can know is not captured well by seeking to implement determinism in terms of the Cauchy surface "nows" of cosmic time in relativistic spacetimes. For no observer can observe the entirety of one of these surfaces. Rather, what an observer can know at one moment is delimited better by the observer's temporal past, even though it represents an excessively optimistic assessment of our observational abilities. Then the results on observationally indistinguishable spacetimes place powerful constraints on just what can be inferred directly from our spacetime theories about the remaining unobserved parts of our spacetime. They tell us that, even with the assistance of local spacetime theories, we can never assuredly fix a unique extension to the portion we have observed. In this regard these results are the appropriate analog of the indeterminism of quantum theory.[7]

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[7] Claus Beisbart has pointed out to me that, aside from Manchak's result, there is a familiar expectation of this sort of indeterminism. Fixing one's temporal past leaves open the possibility

However, there is a strong disanalogy to the indeterminism of quantum theory. Both forms of indeterminism express an impossibility of the past *deductively* determining the future. They differ markedly when we consider the possibility of *inductive* determination of the future. While inductive discrimination is possible in both cases, as we shall see below, they employ different sorts of inductive inferences.

5. Some Cosmic Inductions

Inductive inferences can discriminate a spacetime from an observationally indistinguishable nemesis arising in the results on observationally indistinguishable spacetimes. A simple example illustrates the types of induction required. Consider a Minkowski spacetime. It is observationally indistinguishable from a "half Minkowski spacetime"; that is a Minkowski spacetime in which half has simply been excised. This excised half is the "t=0" hypersurface, in a standard coordinate system, and all events to its future. The observational indistinguishability depends on the fact that every observer's past in either spacetime is identical to every other observer's past in either spacetime; they are all geometric clones of one another, as illustrated in Figure 2.

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of influences propagating to one's future from the region of spacetime outside one's past light cone.

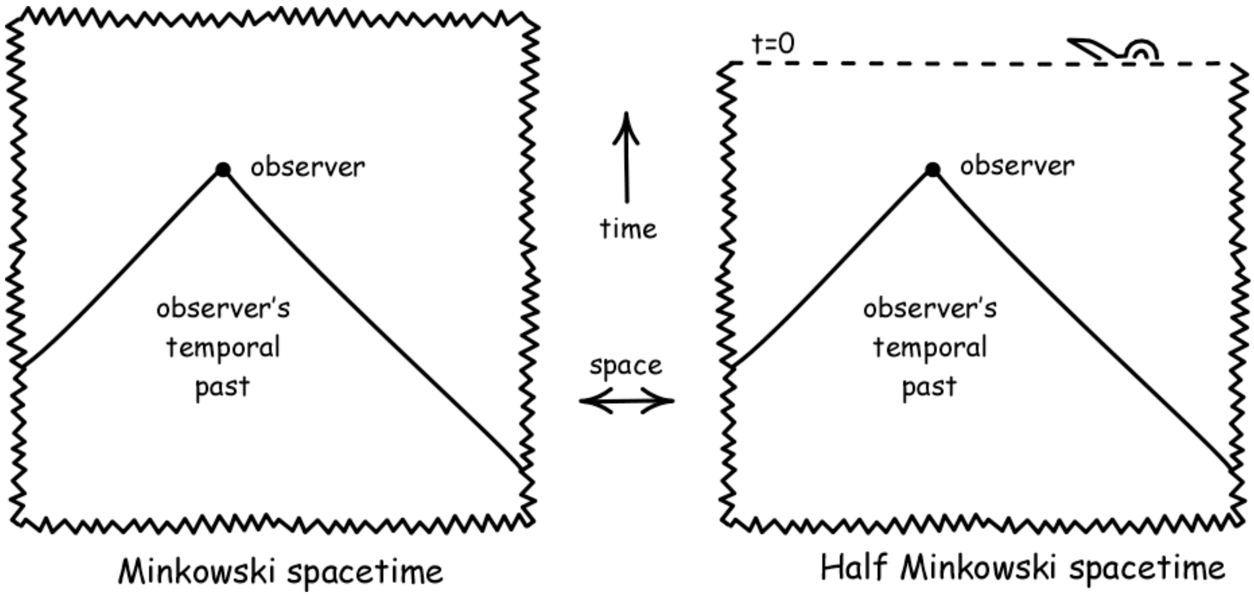

Figure 2. Minkowski and half Minkowski spacetimes

Consider the timelike curves of any inertial observer in either spacetime. No such observer would ever detect a failure of the observer's world line to extend by a millisecond of proper time.[8] Every observer would have repeatedly done the experiment of waiting a millisecond and found always that their worldline was extended by a millisecond, as shown in Figure 3. The natural inductive inference is that all future terminated inertial worldlines can be extended by one millisecond of proper time. But that condition can only be met in the full Minkowski spacetime. Hence, even though the two spacetimes are observationally indistinguishable as far as deductive discriminations are concerned, this induction indicates in favor of the full Minkowski spacetime.

[8] In the half Minkowski spacetime, some worldlines will not extend by a millisecond, when the observer's worldline runs into the non-existent t=0 excision. That observer ceases to exist and there is no detection or record of the failure.

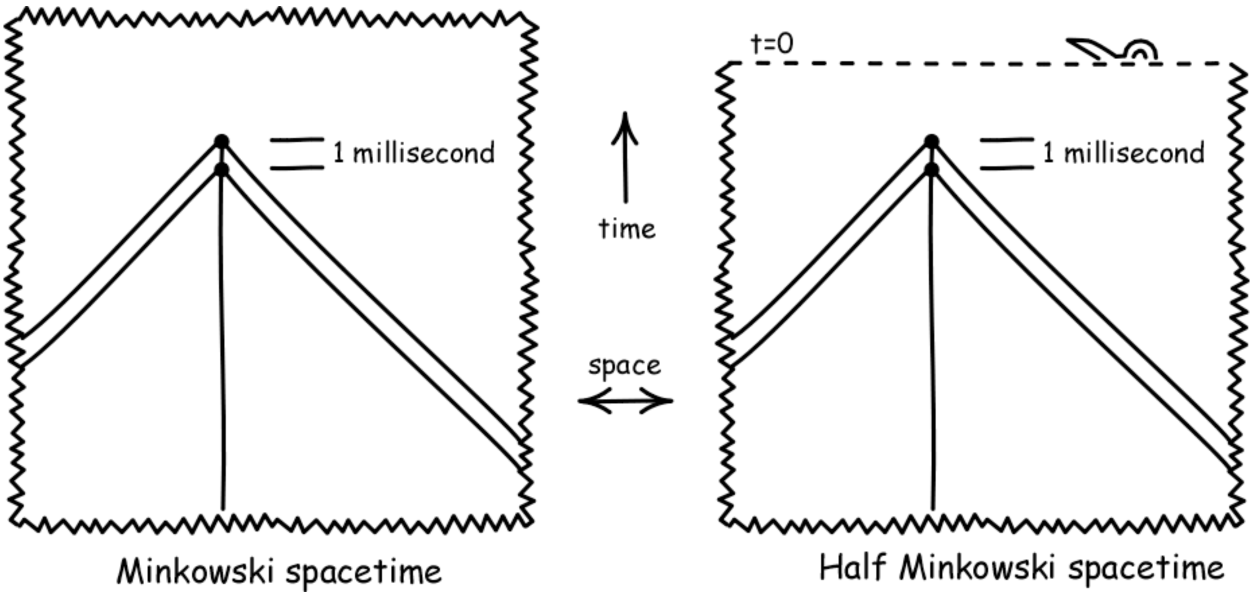

Figure 3. Millisecond extensions of inertial observers' worldlines

This last example uses a peculiar spacetime, an extendable half Minkowski spacetime These extendable spacetimes only a little more peculiar than the constructions used to generate indistinguishable spacetimes. Where the nemesis of a Minkowski spacetime was created by subtracting spacetime structure, more common examples in the literature create the nemeses by adding. The ingenious chain construction of Manchak's proof requires us to build an infinity of duplicate spacetimes and then stitch them together in an infinite chain by what amounts to wormholes. In the case of a full Minkowski spacetime, observers would never detect the wormholes in the portions of spacetime they observe. Thus must remain forever unsure of whether such a wormhole link to the duplicated Minkowski spacetimes will eventually enter the growing portion that they can observe. Deduction cannot rule out the possibility. Induction can; these odd structures have never been seen, so one expects never to see them.

There are more examples in which spacetime structure is added. A familiar case is a two-dimensional de Sitter spacetime and versions of it spatially unrolled into larger spacetimes of twice, thrice, etc. the spatial size of the original spacetime.[9] This de Sitter spacetime can be

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[9] This spacetime has only one spatial dimension. The spatial duplications described are harder to implement with three dimensional spaces. The simplest case arises with a topologically toroidal Euclidean space. It is created by taking a cube of Euclideian space and identifying opposite faces. The space can be unrolled by connecting its faces to duplicate cubes of Euclidean space.

pictured as a two-dimensional hyperboloid in a three dimensional Minkowski spacetime. Its spatial slices are circles and the unrolling just consists of replacing them by larger circles of twice, thrice, etc., the circumference. The original and unrolled versions are depicted in Figure 4.[10]

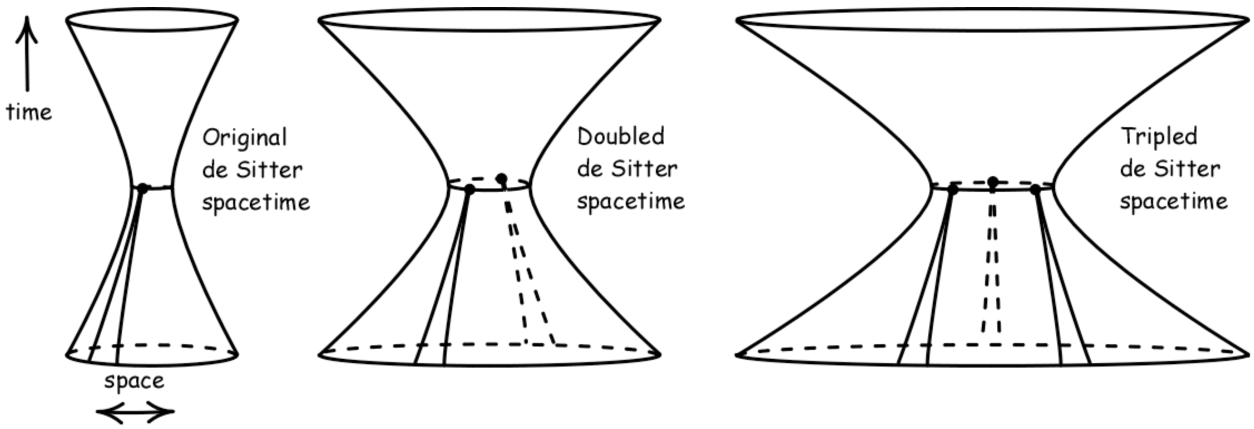

Figure 4. Two dimensional de Sitter spacetimes

The unrolled versions have the curious property of harboring spatial regions that are duplicated twice, thrice, etc. according to the extent of the unrolling. This property is illustrated in the figure by the presence of a single observer's temporal past in the original de Sitter spacetime; and then two copies of it in the doubled de Sitter spacetime; and three copies in the tripled de Sitter spacetime. A spacetime with no duplications and a spacetime with 27 duplications of the observer's past will be observationally indistinguishable by deductive means. However Occam's razor motivates an inductive inference to the first spacetime.

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[10] The figures are misleading in so far as it appears that the doubling is achieved by a uniform expansion of the spacetime. That would alter the spacetime curvature at every point of the spacetime, so that the temporal pasts in the two spacetimes would no longer be isometric. Rather the doubling is effected by a cutting and pasting that leaves local spacetime structure unaffected. Take a de Sitter spacetime "1" and copy of it, de Sitter spacetime "2". Cut each spacetime along a timelike geodesic that then exposes edges "1L" and "1R" in spacetime 1 and "2L" and "2R" in a spacetime 2. Glue 1L to 2R and 1R to 2L to form the doubled de Sitter spacetime.

6. The Opacity of Cosmic Inductions

While we can discriminate inductively among possible futures in both cases, the indeterminism arising through observationally indistinguishable spacetimes is more troubling than the indeterminism of quantum theory. In the case of quantum theory, the warrant for the inductive inferences is quite clear and unproblematic. In the spacetime case, it is hard to see through the inductions to the warrant that lies behind them. In so far as warrants can be found, they are suspect. I will call these latter inductions “opaque.”

In the case of quantum theory, the theory supplies physical chances to help us pick among the possible futures. Take the radioactive decay of an atom. We are equally sure that the atom will or will not decay over a single half-life; both outcomes have the same physical chance of 1/2. We can be very sure, inductively, that decay will have happened if we wait ten half-lives; the physical chance of decay is then $1-(1/2)^{10} = 0.999$. These inferences from the known past to the unknown future are grounded by the physical chances of the quantum theory itself. We can see through these inductions to the physical theory that grounds them; in this sense, they are “transparent.”

The inductions arising in observationally indistinguishable spacetimes are of a different kind.[11] Relativity theory provides no physical chances to weight the different spacetime extensions that it allows for our temporal past. The theory itself merely declares that the various alternatives are possible and nothing more. It leaves to us the task of determining if one or other of them is somehow preferred. We must look outside the physical principles of cosmology to decide this question.

This is a natural project for inductive inference. However the examples of Section 5 above reveal no single, principled inductive approach that can be applied across the many cases of indeterminism. Rather we must consider each on a case-by-case basis and hope that we can find principled grounds in each. Take the extrapolation of the extendability of observed

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[11] The problem has been long discussed in the context of justifying the cosmological principle. Its justification requires an inductive inference from the large scale, spatial homogeneity and isotropy of the observed part of spacetime to all spacetime. For a recent discussion, see Beisbart and Jung (2006).

spacetime curves to all spacetime curves. Can it be grounded in an inductive principle that asserts that what has always been *so*, will always be *so*? Such a universal principle is untenable. It can only apply to some things that have been *so*, otherwise we rule out the possibility of any novel changes in the future. It must be modified so some future novelty is possible. But what could these modifications be, if we are to secure a universal inductive principle applicable beyond the one case? The danger is that we reduce the principle to a blatant circularity, in which we solemnly declare that it applies except when it does not. Worse, we must also be able to overrule the principle if further facts demand it. In a cosmology with a future "big crunch" singularity, we will have the same records of assured millisecond extensions, yet our inertial trajectories will not be indefinitely extendable. That failure can be deduced from present facts through the singularity theorems of general relativity.

We face similar problems when we try to rule out the funhouse mirror duplications of the unrolled de Sitter spacetimes or the extravagantly duplicated spacetimes, connected by wormholes. We would like to ground the inductive inference in a principle like Occam's razor. However, the idea behind it, that simplicity is often accompanied by truth, is more a convenient and fallible rule of thumb than a guarantee. These problems are deepened by an absence of any clear rules as to just what counts as simple.[12]

I have long harbored dissatisfaction with the evident failure of any universal inductive principle such as the ones just listed. My solution has been to propose that we abandon the search for universal, formal approaches to inductive inference. In a material theory of induction, I urge (2003, 2005) that inductive inferences are not warranted by general principles, but by facts. A fitting application of this material approach is to the inductive inferences just seen on radioactive decay. The laws of quantum theory are the facts that warrant them.

What is troublesome from the material perspective is the absence of warranting facts for the inductions in the spacetime case. It seems natural to infer inductively to the fully extended Minkowski spacetime rather than the extendable half Minkowski spacetime; or to avoid

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[12] I set aside Bayesian analyses. All they will do is take something like these principles and use them to determine prior probabilities and likelihoods. The resulting analysis will be no more secure than the principles used to set these quantities, although this will be harder to see because of the complications of the computational machinery.

admitting holed spacetimes that are created from other spacetimes by excising even quite small parts. However it is very hard to specify just what facts ground the inference. That we have never seen holes in spacetime does not settle the matter. By their construction, there cannot be an observable trace of holes. That remains true even if our world tubes pass directly through the hole. We would cease to be for the portion of our world tubes coinciding with the excision. However the portion of our world tubes in the future of the hole would be reconstituted with all the memories and other traces of the excised spacetime. If observed facts do not ground the inductive inference, what of physical laws? We could cite the common postulate in relativity texts that spacetimes are inextendable. However that postulate is merely the supposition of precisely what is at issue and is distinctive as being dispensable from a physical perspective. It is present as much for mathematical convenience.

In his (manuscript), Manchak reports the justifications usually given for assuming inextendability. They amount to invoking Leibniz's principle of plenitude. Manchak quotes from the writings of the mathematical physicists Robert Geroch as a typical justification: "Why, after all, would Nature stop building our universe … when she could just as well have carried on?"

One cannot help but be struck by how tenuous the grounding has become. We are now to secure our inductions in abstract metaphysics. The principle of plenitude itself is sufficiently implausible that we need to prop it up with anthropomorphic metaphors. We are to image a personified Nature in the act of creating spacetime, much as I might be painting my fence on the weekend. Just as I might not want to stop when there is one board remaining unpainted, so Nature is supposedly loath to halt with a cubic mile-year of spacetime still uncreated.

If the complete arbitrariness of the principle of plenitude is not already clear, we might pause to apply it elsewhere. We are supposed to prefer spacetimes without duplications by virtue of a metaphysics of simplicity. Yet surely the metaphysics of plenitude would direct the opposite result. Why would Nature, guided by the slogan "make all you can make," eschew yet another duplication of the spacetime if it is possible?

All these inductive inferences are opaque in that we cannot see through them to their warrants. If we seek to justify them by means of general inductive principles, we resort to principles that are clearly false, or, if not, so hedged as to be inapplicable. If we seek to justify them materially in facts, we arrive almost immediately in the dubious, abstract metaphysics of plenitude and simplicity. This circumstance is to be contrasted with the transparent inductive inferences in the quantum context. Their grounding is found directly in the laws of quantum theory; and we can in turn satisfy ourselves of those laws by tracing back further warrants in the experimental and theoretical foundations of quantum theory.

In sum, we have what appears to me to be an intractable problem.[13] On the one hand, it seems completely unjustified to presume that wormholes we have never seen in our past spacetime will appear in the future. It seems completely unjustified to presume that processes we observe here are duplicated many times over in an unrolled spacetime, when those duplications are by construction, necessarily invisible to us. It seems completely unjustified to assume that there are holes in spacetime, when the spacetime would, by construction, look identical to us if there were no holes. Indeed, even if our world tubes had no past, we would have memories of a past that never was. The inductive inference from those memories to the reality of the past seems irresistible, as do the inductive inferences that reject spatial duplications and future wormholes to new universes. To deny these inductive inferences would, in other contexts, be denounced as delusional. We routinely dismiss as desperate zealots those who tell us our universe was created last Wednesday complete with all records of an ancient past.

Yet, when we try to display the proper warrant of those inductive inferences we favor, whether the warrant is in general principles or material facts, the ground crumbles around our feet.

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[13] Peter Achinstein has urged me to explain how this problem differs from another notorious intractability in induction, "grue." In Norton (2006) I argue that "grue" only provides a novel inductive challenge if we grue-ify our total science. However then the standard and grue-ified alternatives are isomorphic, so we cannot rule out the possibility that they are merely notational variants of the same facts. Hence we should not expect an inductive logic to separate them. A variation on this approach may assist us in the case of spacetimes with excisions. Since no experience will ever enable us to learn whether ours is the fully extended or mutilated spacetime, strict empiricist leanings may incline us to say that the two do not really differ factually. However this sort of thinking cannot help us if we are choosing among spacetimes that may have wormholes in our future. These wormholes will eventually manifest in our observations.

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