JOHN D. NORTON

THE N-STEIN FAMILY*

1. THE STORY OF NEWSTEIN

The work of Newstein is now so familiar to us, thanks to Professor Stachel's efforts, that it bears only the briefest recapitulation. Sometime after 1880 but before the advent of general relativity, Newstein brooded on the equality of inertial and gravitational mass. Through an ingenious thought experiment—the Newstein elevator—he hit upon the idea of an essential unity of gravitation and inertia. This was expressed in the indistinguishability of the effects of acceleration in a uniformly accelerated frame of reference from a homogeneous gravitational field in an inertial frame of reference. Now having to consider the behavior of the gravitational force as it is transformed from unaccelerated to accelerated frames of reference, Newstein found it no longer behaved like the familiar vector. Puzzled, he turned to his mathematician friend Weylmann, another neglected figure in history of mathematics. His extraordinary achievement, as revealed by Professor Stachel, was to formulate the notion of affine connection around 1880, decades before the much better known formulation of Levi-Civita of 1917. Weylmann recognized that the puzzling transformation behavior of gravitational force was simply that of the components of a four-dimensional affine connection.

This provided the insight needed to write the now famous Newstein-Weylmann paper. It developed a formulation of Newton's theory of gravitation akin to Cartan and Friedrich's later proposals of the 1920s. In it, the chronogeometrical structure of spacetime remained absolute, but inertia and gravitation are combined in an affine structure. The Poisson equation for the gravitational potential is absorbed into an equation relating the Ricci tensor of the connection with the gravitational field's sources. With the association of gravitation with a curved, four-dimensional affine structure, the scene was now set for an Einstein to merge this viewpoint with the chronogeometry of special relativity, as captured in the spacetime metric of Minkowski, to yield the general theory of relativity.

A. Ashtekar et al. (eds.), Revisiting the Foundations of Relativistic Physics, 55–67.

2. THE FATAL OBJECTION?

Sadly, however, the Newstein-Weylmann proposal was neglected. As Professor Stachel tells it:

> Their work was regarded by contemporaries, in so far as they took any notice of it at all, as an ingenious mathematical tour-de-force; but since it had no new physical consequences, it did not much impress Newstein's positivistically-inclined physics colleagues.

There is no doubt that this diagnosis reveals part of reasons for the hesitation over the Newstein-Weylmann proposal. But there is more to say. We are inclined now to draw an analogy between special relativity and the Newstein-Weylmann proposal. Special relativity proceeds from the recognition that classical theories proposed the existence of an aether state of rest. What was objectionable in that proposal was that the aether state of rest was itself unobservable. That in turn resulted from its indeterminate nature. Any inertial state of motion proved to be an equally viable candidate for the aether state of rest. Both observation and theory were powerless to decide between them. Here we accord fully with the positivist sentiments of Newstein's physics colleagues in so far as they regarded the unverifiable aether state of rest as something to be purged from our physical theories.

The Newstein-Weylmann proposal seems very similar. The classical theory portrays free fall motions as the resultant of inertial motion and a gravitational deflection. But which of all possible motions are we to choose as the true inertial motion? All we observe are the resultant free fall motions. It would seem that the background inertial structure that fixes these inertial motions is as indeterminate as an aether state of rest. We eschew this aether state of rest in special relativity and build our theory of inertial motions alone. Should we do the same in gravitation theory: eschew the background inertial motions and build our theory on what is observed, the free fall motions, to which Newstein-Weylmann directly adapt their affine structure?

Compelling as this consideration may seem to us now, Newstein's colleagues were unconvinced. There was a telling disanalogy between the two cases. While the true inertial motions were not directly observable, they could be picked out uniquely by very natural conditions in the standard examples used in gravitation theory. Take the case of the gravitational field of the sun. We make the standard and natural presumptions of classical theory: the background inertial structure can be represented by a flat affine structure and the gravitational field of the sun is spherically symmetric in the space about the sun. This now provides a unique decomposition of the free fall motions around the sun into a background inertial structure and a gravitational deflection. The background inertial structure is perfectly determinate. Not even a strong dose of positivistic skepticism can undo that and repeal the sense that this determinate split into inertial motion and gravitational deflection reflects reality.

While this objection seems fatal, there was an answer. Natural conditions may pick out a unique inertial structure in some cases, but there are others in which demonstrably no such conditions can succeed. The realm of possibility is large and we may well wonder whether someone hit upon these examples and their import in the history of physics.

3. THE N-STEIN FAMILY: EINUNDZWANZIGSTEIN

My primary purpose in this paper is to announce the discovery not just of a single unnoticed 'stein in the history of science, but of a family of such figures:[1] Einstein, Newstein, Zweistein, ... The first two of these family members now enjoy the celebrity that their work warrants, thanks to the efforts of Professor Stachel. The mathematically inclined reader will immediately see that they form not just a family but an n-parameter family, where n takes suitable values: Ein, New,... For our purposes what is important is that one family member did hit upon the response that defeats the objection sketched above to the Newstein-Weylmann proposal. The work of this hitherto unrecognized figure, Albert Einundzwanzigstein, was revealed using techniques of historical research pioneered by Professor Stachel.[2] The content of the 1905 volume number 17 of *Annalen der Physik* is widely known; it contains the five papers of "Einstein's Miraculous Year."[3] What has remained unrecognized until now is the existence of a supplementary volume (see fig. 1) in which Einundzwanzigstein's "On the Cosmology of Free Falling Bodies" was published (see fig. 2). There Einundzwanzigstein showed that there is one case in Newtonian gravitation theory in which no natural conditions on the inertial structure and gravitational field can enforce a unique split of free fall motions into a background inertial motion and a gravitational deflection.

Einundzwanzigstein's result was expressed as the recognition that Newtonian cosmology is covariant under transformations between inertial frames and accelerated frames and that this covariance reflects the equivalence of observation for inertial and accelerated observers. It follows immediately that there are no unique background inertial motions identifiable, for these inertial motions cannot be invariant under a transformation to an accelerated frame. Einundzwanzigstein's argument is closely analogous to that of Einstein's 1905 "On the Electrodynamics of Moving Bodies." In Einstein's theory, an absolute state of rest is purged from the laws of physics by the principle of relativity since that state fails to remain invariant under a transformation between inertial frames of reference. We shall see that this similarity of strategy is reflected by closer analogies in the two papers.

ANNALEN
DER
PHYSIK.

BEGRÜNDET UND FORTGEFÜHRT DURCH
F. A. C. GREN, L. W. GILBERT, J. C. POGGENDORFF, G. UND E. WIEDEMANN.

VIERTE FOLGE.

BAND 17.

DER GANZEN REIHE 322. BAND.

BEILAGE
JOHANNES STACHEL
ZU SEINEM SIEBZIGSTEN GEBURTSTAG

KURATORIUM:
F. KOHLRAUSCH, M. PLANCK, G. QUINCKE,
W. C. RÖNTGEN, E. WARBURG.

UNTER MITWIRKUNG
DER DEUTSCHEN PHYSIKALISCHEN GESELLSCHAFT
UND INSBESONDERE VON
M. PLANCK

HERAUSGEGEBEN VON
PAUL DRUDE.

MIT FÜNF FIGURENTAFELN.

LEIPZIG, 1905.
VERLAG VON JOHANN AMBROSIUS BARTH.

Figure 1.

1823

27. *Zur Kosmologie frei fallender Körper; von A. Einundzwanzigstein.*

Daß die Kosmologie Newtons — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z.B. an die freie Fallbewegung von Körpern im homogenen Weltraum. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung der Körper, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der beschleunigte sei, streng voneinander zu trennen sind.

Ferner ist es wohlbekannt, daß die Newtonsche Grenzbedingung des konstanten Limes für das Potential räumlich Unendlichen zu der Auffassung hinführt, daß die Dichte der Materie im Unendlichen zu null wird. Wir denken uns nämlich, es lasse sich ein Ort im Weltraum finden, um den herum das Gravitationsfeld der Materie, im großen betrachtet, Kugelsymmetrie besitzt (Mittelpunkt). Dann folgt aus der Poissonschen Gleichung, daß die mittlere Dichte rascher als $1/r^2$ mit wachsender Entfernung r vom Mittelpunkt zu null herabsinken muß, damit das Potential im Unendlichen einem Limes zustrebe. Die mittlere Dichte der Materie ist die Dichte, gebildet für einen Raum, der groß ist gegenüber der Distanz benachbarter Fixsterne, aber klein gegenüber den Abmessungen des ganzen Sternsystem. In diesem Sinne ist also die Welt nach Newton endlich, wenn sie auch unendlich große Gesammtmasse besitzen kann.

Figure 2.

4. NEWTONIAN COSMOLOGY

Einundzwanzigstein's paper addressed a natural formulation of the cosmology of a homogeneous universe as afforded by Newton's theory of gravitation. Space is assumed to be infinite and Euclidean and filled with a uniform matter distribution of density $\rho(t)$, which will vary as a function of time. The gravitational potential φ is governed by the Poisson equation

$$\nabla^2\varphi = 4\pi G\rho \tag{1}$$

where G is the constant of universal gravitation. These assumptions combined provide the framework of Newtonian cosmology. One might expect that, these assumptions are sufficient to fix the gravitational potential uniquely. But that is not so. Any of the class of solutions

$$\varphi(\boldsymbol{r}) = \left(\frac{2}{3}\right)\pi G\rho(t)(\boldsymbol{r}-\boldsymbol{r}_0)^2 \tag{2}$$

satisfies the condition, where the vector position $\boldsymbol{r} = (x, y, z)$, for Cartesian spatial coordinates x, y and z and $\boldsymbol{r}_0$ is any arbitrarily chosen position in space.[4] It follows directly from (2) that the force on a unit test mass is

$$\boldsymbol{f} = -\left(\frac{4}{3}\right)\pi G\rho(\boldsymbol{r}-\boldsymbol{r}_0). \tag{3}$$

This in turn enables a very simple expression for the gravitational tidal force. The differential force $\Delta\boldsymbol{f}$ on two unit masses separated by a distance[5] $\Delta\boldsymbol{r}$ is given by

$$\Delta\boldsymbol{f} = -\left(\frac{4}{3}\right)\pi G\rho\Delta\boldsymbol{r}. \tag{4}$$

Since no other forces are presumed to prevail on the bodies forming the matter distribution ρ, these cosmic masses are in free fall with accelerations and relative accelerations given by (3) and (4) respectively.

5. THE ASYMMETRY OF NEWTONIAN COSMOLOGY

In addressing this simple system, Einundzwanzigstein commenced his "On the Cosmology of Free Falling Bodies" by noticing the existence of an asymmetry between theory and observation in the system that was strongly reminiscent of the asymmetry Einstein used to launch his "On the Electrodynamics of Moving Bodies" Einundzwanzigstein wrote:

> It is known that Newton's cosmology—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the motion of bodies in free fall in a homogeneous space. The observable phenomenon here depends only on the relative motion of the bodies, whereas the customary view draws a sharp distinction between the two cases in which one or the other of the bodies is accelerated.

Einundzwanzigstein's point is recoverable immediately from equation (4).

The observables sustain a perfect equivalence of all bodies in the cosmology. What is observable is the relative motion of the bodies. That observable is the same for any of the cosmic bodies. Each is in free fall and, according to (4), each sees neighboring masses accelerating towards it with an acceleration proportional to distance. As far as the observables are concerned, every body is fully equivalent to every other. If we find ourselves on one of them, no observation of motions can decide which that is. Inertial forces can supply no guide; since every body is in free fall, none of them experience inertial forces.

Newtonian gravitation theory, however, is unable to sustain this equivalence. According to it, at most *one* of all the cosmic masses of the distribution ρ can be unaccelerated, that is, in inertial motion. All the rest are truly accelerated. That body has the role of a unique center of the universe. All the other bodies accelerate towards it. It is designated by the position vector r_0. While that position vector appears in the expression for the many different fields φ of (2) and f of (3), it does not appear in the equation (4) that governs the observable of motion, the tidal force.

Einundzwanzigstein's response was analogous to Einstein's response to the corresponding problem in the electrodynamics of moving bodies.[6] The aether state of rest Einstein observed in 1905, was superfluous for the treatment of electrodynamics. All inertial motions are equivalent. The designation of any reference system as "at rest" is purely a matter of convenience. Electrodynamics embodies a relativity of inertial motion. Correspondingly Einundzwanzigstein declared the notion of a preferred class of inertial motions as superfluous to the cosmology. All inertial and uniformly accelerated motions are equivalent. The designation of any reference system as "inertial" is purely a matter of convenience. Newtonian cosmology embodies a relativity of uniform acceleration.

6. COVARIANCE OF NEWTONIAN COSMOLOGY UNDER ACCELERATION TRANSFORMATIONS

In 1905, Einstein gave formal expression to this relativity of inertial motion by demonstrating that electrodynamics is covariant under the transformations that connect inertial systems of reference, the Lorentz transformation. Correspondingly, Einundzwanzigstein demonstrated that Newtonian cosmology is covariant under an acceleration transformation. To display this covariance, he chose a reference system (x, y, z, t) as "inertial." In it, there is just one cosmic body whose motion is inertial (i.e. its position coordinates are linear functions of the time coordinate). The origin of the reference system is so selected that this body remains at position $x = y = z = 0$. The gravitational potential and the acceleration of cosmic bodies are given as

$$\varphi = \left(\frac{2}{3}\right)\pi G r^2 \quad \frac{d^2 r(t)}{dt^2} = -\left(\frac{4}{3}\right)\pi G r \tag{5}$$

where $\boldsymbol{r} = (x, y, z)$ and $r^2 = |\boldsymbol{r}|^2$. Einundzwanzigstein now selected arbitrarily another comic body at position $\boldsymbol{R}(t)$. Its trajectory over time is governed by

$$\frac{d^2 \boldsymbol{R}(t)}{dt^2} = -\left(\frac{4}{3}\right)\pi G \rho \boldsymbol{R}(t). \tag{6}$$

This arbitrarily chosen body in turn can be used to define an acceleration transformation[7] from the original reference system to the new system (x', y', z', t')

$$\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{R}(t) \qquad t' = t. \tag{7}$$

If we write $\boldsymbol{R}(t) = (X(t), Y(t), Z(t))$, this transformation can also be written as

$$x' = x - X(t) \quad y' = y - Y(t) \quad z' = z - Z(t) \quad t' = t.$$

Under this transformation, the gravitational potential and the acceleration of cosmic bodies is now given as

$$\varphi' = \left(\frac{2}{3}\right)\pi G r'^2 \quad \frac{d^2 \boldsymbol{r}'(t)}{dt'^2} = -\left(\frac{4}{3}\right)\pi G \boldsymbol{r}' \tag{5'}$$

expressions identical in form to (5). The Lorentz covariance of Maxwell's theory expresses the relativity of inertial motion; the elimination of the aether state of rest lies just in the failure of that state to be invariant under Lorentz transformation. The covariance of Newtonian cosmology under transformation (7) expresses a relativity of acceleration. The selection of one class of motions as inertial corresponds to a choice of one subclass of the reference systems of the theory. That choice is not invariant under the transformation (7); motions that are inertial in (x, y, z, t) are accelerated in (x', y', z', t') and *vice versa*. Further, the distinction between the different potential and force fields of (2) and (3) loses physical significance. That is, the designation of which body occupies the preferred position $\boldsymbol{r}_0$ of the unique inertial moving body is not invariant under the transformation (7). By suitable choice of $\boldsymbol{R}(t)$, any body can be brought to the origin of coordinates and thus to this preferred position.

The transformation of (5) to (5') requires that the gravitational potential φ *not* transform as a scalar. Rather it must transform as[8]

$$\varphi' = \varphi + \boldsymbol{r} \cdot \left(\frac{d^2 \boldsymbol{R}}{dt^2} \right) + \varphi(\boldsymbol{R}). \tag{8}$$

That φ does not transform as a scalar has no effect on observables. There are two additional terms in the transformation law (8). The second, $\varphi(\boldsymbol{R})$, is just the adding of a constant to the potential; such a constant does not affect the observables, since it has no effect on the motions. The first term added, $\boldsymbol{r} \cdot (d^2\boldsymbol{R}/dt^2)$, corresponds to the addition of a homogeneous field to the force field associated with $\dot{\varphi}$. That force field is given by the negative gradient of φ and is $-\nabla\varphi' = -\nabla\varphi - (d^2\boldsymbol{R}/dt^2)$. It is augmented by a vector $d^2\boldsymbol{R}/dt^2$, which is a constant over space at any instant. Such a homogeneous field does affect accelerations, but it does not affect the observable, relative accelerations, since it accelerates all bodies alike.

This new transformation law for the gravitational potential corresponds to the Lorentz transformation law for electric and magnetic fields in special relativity.

7. THE GEOMETRIC FORMULATION

Einundzwanzigstein's point is that there was a relativity of acceleration built into Newtonian gravitation theory that is closely analogous to the relativity of inertial motion of special relativity. That relativity of acceleration is hard to see in the context of the usual examples. In the case of the gravitational field of the sun, for example, the *observable* inhomogeneity of the field picked out a preferred trajectory in space (that of the sun) and this in turn defined a preferred inertial motion. The case of Newtonian cosmology allowed no such selection. In terms of observables, the motion of all bodies were fully equivalent, even though they were in relative acceleration.

The methods and formalism of Einundzwanzigstein's paper was that of Einstein's 1905 paper on special relativity. Just as the ideas of Einstein's paper were soon translated by Minkowski into a geometrical language, the same translation was possible for Einundzwanzigstein's paper. It could be expressed in the language of the Newstein-Weylmann proposal, in which the free falls of Newtonian cosmology are just the geodesics of the affine spacetime structure. Now the Lorentz covariance of special relativity embodies a relativity of inertial motion because the Lorentz transformation is a symmetry of the Minkowski metric. Correspondingly covariance of Newtonian cosmology under (7) is expressed geometrically as the symmetry of the geometric structures of the spacetime, including the affine structure, under the transformation (7), now read as an active point transformation.[9] In each case, the relativity of a motion is expressed as a symmetry of the geometric structure.

In this context, Einundzwanzigstein's point can be given it sharpest expression. The attempt to preserve some absoluteness of inertial motion corresponds to the attempt to find some way to split the affine connection into a connection defining true inertial motions and a gravitational deflection. *No invariant condition can effect this split in a way that privileges the motion of any one cosmic body over any other.* For it follows immediately from the symmetry of the geometry that any property of one

such motion must be shared equally by any other.[10] It is not even sufficient to require that the inertial affine structure be flat—this condition is met by each of the different, natural splits that render one or other body's motion inertial.

The transition to the geometric formulation can be made very quickly on the basis of the equations (1), (3), and (4). If we introduce an index notation so that $r = (x, y, z) = (x^1, x^2, x^3)$ and the time coordinate $t = x^0$, then, according to (3), the trajectories of masses in free fall are governed by

$$\frac{d^2x^i}{dt^2} + \left(\frac{4}{3}\right)\pi G\rho x^i = 0 \qquad (3')$$

where $i = 1, 2, 3$. These motions are just the geodesics of the affine connection with symbols Γ^i_{km}, so that this condition (3') can be rewritten as

$$\frac{d^2x^i}{dt^2} + \Gamma^i_{00} = 0 \qquad (3'')$$

where t is an affine parameter and the only non-zero symbols are

$$\Gamma^i_{00} = \left(\frac{4}{3}\right)\pi G\rho x^i \qquad (9)$$

which fixes the affine structure. Further, since Γ^i_{00} represents the gravitational force on a unit mass in the reference systems used by Einundzwanzigstein, we see that this force must transform like the coefficients of the connection.
From (4) we read off an expression for the relative acceleration of neighboring bodies in free fall

$$\frac{d^2\Delta x^i}{dt^2} + \left(\frac{4}{3}\right)\pi G\rho\Delta x^i = 0. \qquad (4')$$

This corresponds to the equation of geodesic deviation

$$\frac{d^2\Delta x^\alpha}{dt^2} + R_{\gamma\delta\beta}{}^{\alpha}\Delta x^\beta\left(\frac{dx^\gamma}{dt}\right)\left(\frac{dx^\delta}{dt}\right) = 0 \qquad (4'')$$

where $\alpha, \beta, \gamma, \delta = 0, 1, 2, 3, 4$.

The comparison of (4') and (4'') is very fruitful. To begin we can see that the coefficients of the affine curvature tensor represented in (4'') must be constant. This suggests, but does not prove, the uniformity of the affine structure expressed in its symmetry under transformation (7). We can read sufficient of the coefficients of the curvature tensor to allow recovery of the Ricci tensor

$$R_{00k^i} = \left(\frac{4}{3}\right)\pi G\rho\delta^i_k \qquad (10)$$

where $i, k = 1, 2, 3$. Contraction over the indices i and k allows us to recover[11] the R_{00} component of the Ricci tensor as

$$R_{00} = 4\pi G\rho. \qquad (1')$$

This is the analog of the Poisson equation (1) in the geometric formulation.

8. CONCLUSIONS, REFLECTIONS AND ADMISSIONS

Lest any readers be in doubt, Newstein, Weylmann, and Einundzwanzigstein are all fictitious and the history reported a fable—inspired by Professor Stachel's own creative endeavors. I have tried to ensure however that all footnoted material in the above fable is historically correct. The fable is intended to convey a serious moral and one that I have laid out in (Norton 1995), in response to David Malament's demonstration (Malament 1995) that the paradoxes of Newtonian cosmology are eradicated by the geometric approach. The usual decision to represent gravitational free falls by a curved affine structure in Newtonian theory is akin to extending the relativity of motion to acceleration, but there are significant disanalogies between it and Einstein's original introduction of the relativity of inertial motion in special relativity. Einstein introduced the relativity of inertial motion to express the indistinguishability of inertial motions that was itself revealed in the failure of experiments that would have picked out the aether state of rest. In general, the representation of gravitational free falls by a curved affine structure does not express a corresponding indistinguishability and the case for it is correspondingly weaker. Newtonian cosmology supplies a clear instance in which it does express such an indistinguishability and is hard to resist. But once it has been admitted in this case, the attempt to avoid it elsewhere becomes all the more contrived.

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NOTES

* With great pleasure, I join the contributors to this volume in honoring Professor Stachel and celebrating his many achievements. My debt to him is great. I learned the real craft of history of science at his elbow when he generously allowed me to visit the Einstein Papers Project in 1982 and 1983 in Princeton and my career owes a great deal to his generosity and kindness. We have all learned so much from Professor Stachel's researches. However, when he revealed the hitherto unknown figure in history of physics, Newstein, in his (Stachel forthcoming) we may have learned somewhat more from him than even he intended, as this paper will demonstrate.

1. I am grateful to Don Howard for pointing out another 'stein that truly belongs to the family: Howard Stein for his (1967). We might also adopt Wolfgang Pauli as an honorary family member on the strength of his nickname, recalled for me by Professor Stachel: "Zweistein."

2. The long standing debate over whether Einstein knew of the Michelson-Morley experiment prior to his work on special relativity of 1905 was settled by the discovery of a letter from Einstein to Mileva Maric of September 1899) in which he recalls reading a paper by Wien (1898) that includes a report on the experiment. That paper was located in an 1898 supplement to the volume of *Annalen der Physik und Chemie*. See (Stachel 1987, 233-34, 407).
3. So named in (Stachel 1998).
4. That the presumptions of this cosmology did not force a unique solution for φ produced great confusion at this time that is not reflected in the above exposition. It was widely expected that any potential φ in the cosmology ought to respect the homogeneity and isotropy of the spatial geometry and matter distribution so that a constant φ was sought. The indeterminacy of φ, as expressed by the admissibility of any member of (2), surfaced in the result that the integral expressions for the gravitational potential, gravitational force and tidal force were not uniformly convergent; they could be integrated to give many conflicting results. A common response was the conclusion that the result was fatal to Newton's law of gravitation, which must be supplemented by other terms to eradicate this indeterminacy. Einstein (1917) used a related argument to motivate the cosmological constant in general relativity, for example. He noticed that the solutions (2) require the density of lines of force to grow without limit at r increases. For a detailed survey of the problem up to 1930, see (Norton 1999).
5. Δr need not be infinitesimally small because of the linearity of f in r according to (3).
6. Correspondingly, Einstein in 1905 argued that the observable phenomena of electrodynamics depend only the relative motions of bodies, whereas Maxwell's electrodynamics distinguished the cases according to which body was at rest in the aether. His example was the electric current induced by the relative motion of a magnet and conductor. The observable, the current, depended only on the relative motion of the magnet and conductor, but Maxwell's electrodynamics gave a very different account of the process according to which of the conductor or magnet was deemed at rest in the aether. If the conductor was at rest, the motion of the magnet led to the induction of a new entity, an electric field, which was not present in the case in which the magnet was at rest in the aether. This example, Einstein suggested, was typical.
7. This transformation (7) corresponds to a uniform acceleration in this sense. Let the trajectory of some body be $\boldsymbol{S}(t)$. At some instant t, its acceleration will be $d^2\boldsymbol{S}(t)/dt^2$. Under transformation (7), that acceleration becomes $d^2\boldsymbol{S}'(t)/dt^2 = d^2\boldsymbol{S}(t)/dt^2 - d^2\boldsymbol{R}(t)/dt^2$. The acceleration has been reduced by the term $d^2\boldsymbol{R}(t)/dt^2$, which is a constant over all space at time t, but will vary with t. That is, at a fixed instant, all accelerations in space are altered by the same amount, but that amount will vary from time to time.
8. Then we have $\varphi + \boldsymbol{r}\cdot\left(\frac{d^2\boldsymbol{R}}{dt^2}\right) + \varphi(\boldsymbol{R}) = \left(\frac{2}{3}\right)\pi G\rho(r^2 - 2\boldsymbol{r}\cdot\boldsymbol{R} + \boldsymbol{R}^2) = \left(\frac{2}{3}\right)\pi G\rho(\boldsymbol{r} - \boldsymbol{R})^2 = \varphi'$.
9. This symmetry is set up and proved in (Malament 1995).
10. This result is the analog of the result in special relativity that no invariant condition can pick out a preferred state of rest from the inertial motions.
11. Recall that $R_{000}{}^0$ vanishes identically.

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