Out of the Labyrinth? Einstein, Hertz, and the Göttingen Answer to the Hole Argument

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In his lifetime, Einstein became a living oracle. We are told time and time again of lesser-known scientists grappling with overwhelming problems who made the pilgrimage to consult Einstein, perhaps just for encouragement or endorsement, or perhaps in the hope that he might hand them the thread that would lead them out of their labyrinth. Our paper tells the story of a scientist who had become hopelessly lost in a labyrinth of his own making as he struggled with the most important discovery of his life. A correspondent gives him the thread that could be followed out of the labyrinth, but the scientist impatiently dismisses this gift as a confused distraction, only to discover a similar way out a few months later. What makes our story special is that the scientist was not just anyone—it was Einstein himself—and the discovery was general relativity.

The time was 1915. Einstein's correspondent was Paul Hertz, then a physicist working in Göttingen and taking regular part in the activities of the group centered around David Hilbert. The problem was the so-called hole argument, through which Einstein had convinced himself that no physically acceptable version of his still-incomplete general theory of relativity could be generally covariant. We will conjecture that Hertz provided Einstein with a serviceable and sophisticated escape from this ill-fated conclusion, and that Einstein misunderstood and dismissed it, only to arrive at a similar escape a few months later in the form of his point-coincidence argument. Finally, on the basis of an intriguing similarity in wording and timing, we will suggest that Einstein may have drawn immediate inspiration for the final formulation of his point-coincidence argument from another hitherto unrecognized source.

Our argument for our main conclusion will be somewhat unusual, resting, as it does, upon our conjectural reconstruction of letters from Hertz to Einstein on the basis of Einstein's surviving replies to Hertz. Such an approach raises obvious methodological and historiographical questions about the use of evidence that is as much conjectured as discovered. However, in the absence of more direct evidence, our only alternative is to say nothing at all; but this is an issue too interesting and important to pass over in silence.

1. Background: General Covariance Lost and Regained

In the summer of 1915, when our story is set, Einstein's long struggle toward his general theory of relativity was drawing to a close. Roughly two years earlier, he and Marcel Grossmann had published the first outline of the theory, complete in all essential details excepting the gravitational fields equations offered, which were not generally covariant (Einstein and Grossmann 1913). To make matters worse, Einstein soon suppressed his concern over this lack of general covariance by convincing himself that any generally covariant field equations that one might propose must be physically uninteresting. His principal argument for this surprising conclusion was the "hole argument," published in its final and most complete form in Einstein 1914b, pp. 1066–1067 (see Norton 1987; Stachel 1989).

In the hole argument, Einstein considered a "hole," a region of spacetime devoid of "material processes" (the stress-energy tensor $T_{ik} = 0$), and a solution g_{ik}, in a coordinate system x^m, of supposedly generally covariant field equations for the metric tensor g_{ik}, given a matter distribution that is nonvanishing only outside the hole. He then showed that the general covariance of the field equations allowed him to construct a second solution, with components g'_{ik}, in the *same* coordinate system x^m, that agreed with the first solution g_{ik} outside the hole but came smoothly to differ from it within the hole. Einstein found the existence of two such solutions in the same coordinate system unacceptable, for he took it to violate the "principle of causality," which seemed here to amount to the requirement that the field and matter distribution outside the hole should determine uniquely the processes or events within the hole. His presumption, apparently, was that there is a unique, real state of affairs within the hole (and elsewhere) that is supposed to be described, uniquely, by a theory of gravitation (see Howard 1992).

In brief, Einstein constructed these two solutions by means of a transformation from the original coordinate system x^m to a new coordinate system $x^{m'}$ that agreed with the original outside the hole but came smoothly to differ from it within the hole. Under this transformation the first solution g_{ik}, in x^m, becomes g'_{ik}, in $x^{m'}$, which general covariance guarantees is also a solution of the field equations. To recover the second solution mentioned above, Einstein looked upon the components g'_{ik} as ten functions of the arguments $x^{m'}$ and imagined that these arguments were replaced by numerically identical values of the original x^m without changing the functional form of g'_{ik}. The result is two differing solutions of the field equations in the same coordinate system x^m. (See Figure 1.)

It will be important for later discussion to pause here and note that these two solutions have the following characteristic property, although Einstein did not stress this fact: There exist two coordinate systems x^m and $x^{m'}$ that agree outside the hole but come smoothly to differ within the hole, such that the components of the second solution, in the coordinate system x^m, are precisely the same functions of the coordinates as are the components of the first solution, in the second coordinate system $x^{m'}$.[1]

For example, in the case of the two-dimensional space-time of Figure 2, if the matrix of values of the second solution is $\left[\begin{smallmatrix}1 & 0\\ 0 & -2\end{smallmatrix}\right]$ at (1, 1) in the first coordinate system, then the matrix of values of the first solution is also $\left[\begin{smallmatrix}1 & 0\\ 0 & -2\end{smallmatrix}\right]$ at (1, 1) in the second coordinate system. Notice, however, that if (1, 1) are the coordinates of a point p inside the hole, then, by construction, (1, 1) in the second coordinate system will be the coordinates of a *different* point, p', in the hole.

The hole argument forced Einstein to limit the range of coordinate systems used in his theory in such a way that, for any arbitrarily selected region of space-time, he could not use two coordinate systems that agreed outside but came smoothly to disagree within the region. To see how close the covariance of his 1913 theory came to this limit, Einstein defined the notion of the "adapted coordinate system," analyzed most completely in Einstein 1914b. The coordinate system adapted to a given field was defined by a variation principle so contrived that it selected a single coordinate system from all those that came smoothly to agree on the boundary of any given region of space-time. This entails a result that will become important below: *For any region of space-time, it is impossible for there to be two different adapted coordinate systems that come smoothly to agree at the boundary.* Einstein could also show that his 1913 field equations were covariant under transformations between these adapted coordinate systems, so that while these field equations were not generally covariant, they had at least the maximum covariance permitted by the hole argument.[2]

Einstein's failure to offer generally covariant field equations was a great worry and embarrassment to him. His frequent protestations of the unacceptability of generally covariant field equations, however, such as Einstein 1914a, and his publication in October 1914 of a lengthy review article (Einstein 1914b) of the theory suggested that he felt the theory had achieved some stability in its then non-generally covariant formulation.

In late June and early July of 1915, Einstein visited Göttingen and gave six lectures on his theory to a group including David Hilbert, Felix Klein and, more likely than not, Emmy Noether and Paul Hertz. Einstein described this visit to several correspondents. Thus, on August 16, he wrote to Berta and Wander Johannes de Haas: "To my great delight, I succeeded in convincing Hilbert and Klein completely" (EA 70-420).[3] And one month earlier, on July 15, Einstein had reported enthusiastically to Sommerfeld:

> In Göttingen I had the great pleasure of seeing everything understood, down to the details. I am quite enthusiastic about Hilbert. A man of consequence. (EA 21-381; reprinted in Hermann 1968, p. 30)[4]

That report to Sommerfeld, however, also showed that Einstein was not yet entirely reconciled to his new theory. He wrote Sommerfeld that he would prefer not to include one or two papers on his new theory (Einstein 1911b, 1914b) in the collection *Das Relativitätsprinzip*, since none of the current presentations were "complete."

As it turned out, Einstein had been understood in Göttingen even better than he realized. Hilbert was particularly excited, writing to Karl Schwarzschild on July 17, 1915: "During the summer we had here as guests the following: Sommerfeld, Born, Einstein. Especially the lectures of the last on gravitational theory were an event" (quoted in Pyenson 1979a, p. 193, n. 83). The excitement in Göttingen was tempered, however, by a widely shared belief that Einstein's mathematical abilities might not be up to the task of perfecting the new theory of gravitation. Typical of this attitude are a couple of remarks found in Felix Klein's lecture notes on general relativity from the summer of 1916. Thus, on the first day of the lectures, July 15, 1916, Klein remarked to his audience that, in the popular mind, relativity theory was surrounded by a "fog of mystery" [*Nebel der Mystik*], adding:

> Einstein's own way of thinking is partly to blame for this mystery, for it starts out again from the most general philosophical speculations and is guided, above all, more by strong physical instinct than by clear mathematical insight.[5]

More to the point, however, is a remark later in that same lecture, in the middle of a section entitled "On the Choice of Coordinates Encountered in Einstein." In Einstein's new theory, Klein tells his students, we enter upon the terrain of arbitrary coordinates, "familiar" to us from the work of Lagrange, Gauss, and Riemann, where the $g_{\mu\nu}$ and the ds^2 must be treated according to the rules of Ricci's absolute differential calculus, or "more

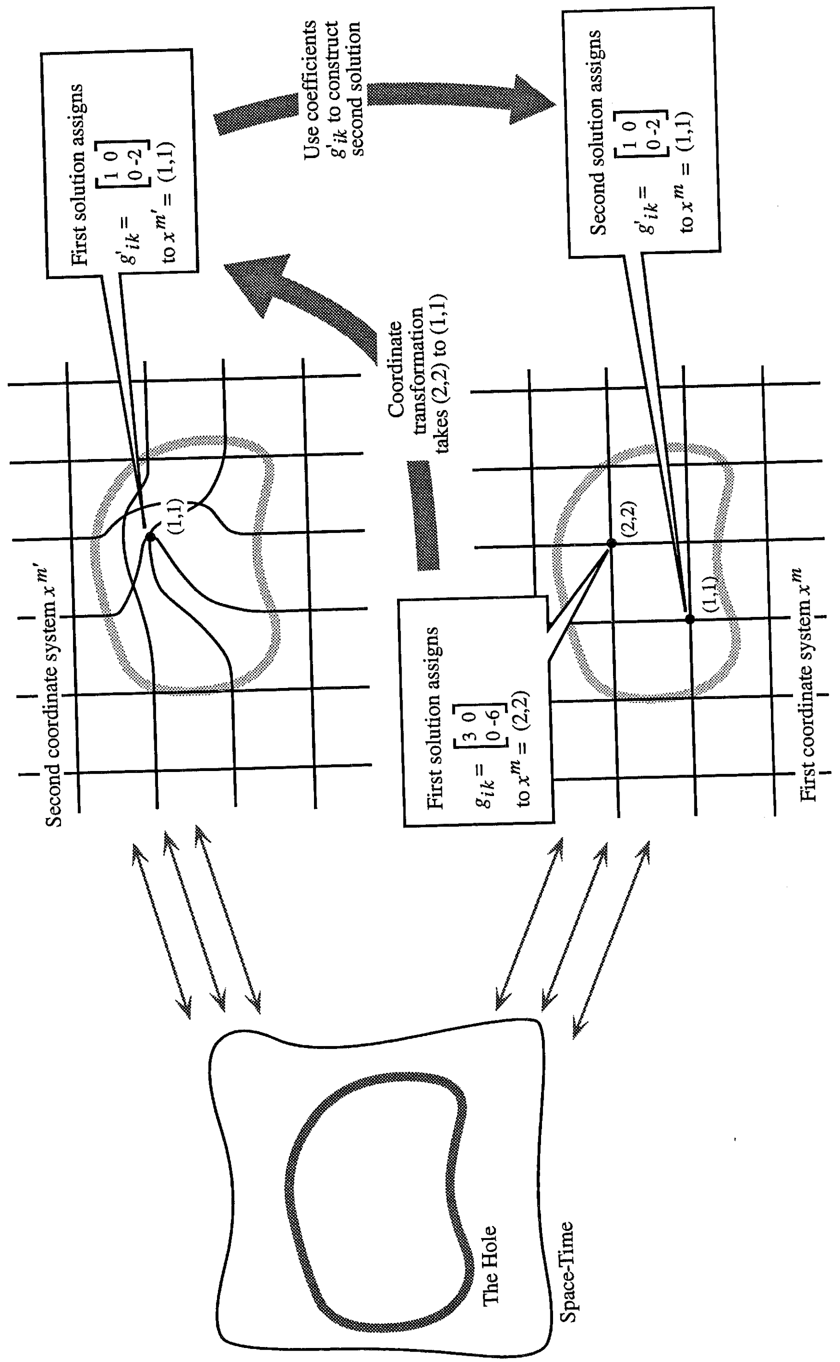

Figure 1. Construction of the two solutions of the hole argument.

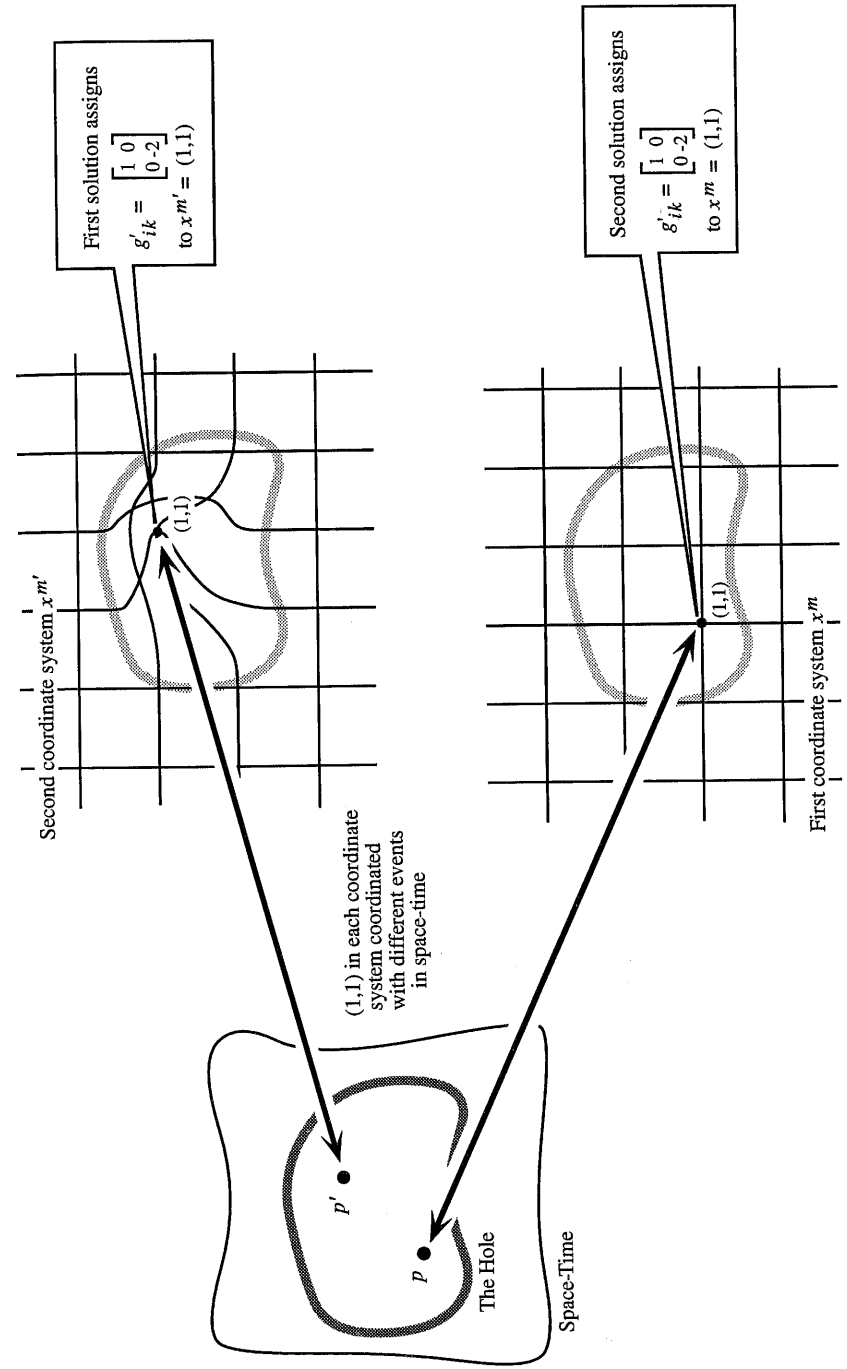

Figure 2. Properties of the two solutions of the hole argument.

objectively expressed," according to the rules of the theory of invariants of the group of arbitrary point transformations applied to the differential invariant ds^2. Everything we learned about Lagrange, Gauss, and Riemann may be clear in itself, says Klein. Still,

> It is nevertheless a good idea to explain it further, because there are here, in Einstein's work, imperfections [*Unvollkommenheiten*], which do not impair the great ideas in his new theory, but hide them from view.
>
> This is connected with the repeatedly mentioned circumstance that Einstein is not innately [*von Hause aus*] a mathematician, but works rather under the influence of obscure [*dunkelen*], physical-philosophical impulses. Through his interaction with Grossmann and on the basis of the Zurich tradition he has, to be sure, gradually become acquainted with Gauss and Riemann, but he knows nothing of Lagrange and overestimates (parenthetically) Christoffel, under the influence of the local Zurich tradition.[6]

One senses in Klein's words a hint of jealousy, but they still help us understand how members of the Göttingen group may have regarded Einstein's mathematical failings with more than a little condescension.

Undeterred by the hole argument, and determined, perhaps, to demonstrate how the vaunted Göttingen expertise at the mathematics of mathematical physics might yield dividends of a kind not yet achieved by the "obscure physical-philosophical impulses" of Einstein, Hilbert himself turned to the task of finding generally covariant field equations for his version of Einstein's theory, a fusion of Einstein's gravitation theory and Mie's matter theory. He communicated the modern gravitational field equations of general relativity to the Göttingen Gesellschaft der Wissenschaften on November 20, 1915 (Hilbert 1915). Meanwhile, Einstein had lost confidence in the lack of general covariance of his theory and returned to the quest for generally covariant field equations. He arrived at the same gravitational field equations as Hilbert, and they were communicated to the Prussian Academy on November 25, 1915, five days after Hilbert had communicated the same equations in Göttingen.[7]

Einstein soon turned to the task of informing his correspondents of how he reconciled his hole argument with his return to general covariance by means of a consideration now known as the "point-coincidence argument."[8] The latter was first published in Einstein's comprehensive 1916 review article, "Die Grundlage der allgemeinen Relativitätstheorie" (Einstein 1916, pp. 117–118). Whereas previously he had argued that generally covariant equations typically can be made to yield different solutions for one and the same coordinatization of the physical space-time, Einstein now argued that while the two solutions g_{ik} and g'_{ik} may be mathematically distinct, they

are not physically distinct, for both solutions catalogue the identical set of space-time coincidences, which exhaust the reality captured by the theory. Thus, Einstein wrote to Paul Ehrenfest on December 26, 1915:

> The physically real in the world of events (in contrast to that which is dependent upon the choice of a reference system) consists in *spatiotemporal coincidences*.* Real are, e.g., the intersections of two different world lines, or the statement that they *do not* intersect. Those statements that refer to the physically real therefore do not founder on any univocal [*eindeutige*] coordinate transformation. If two systems of the $g_{\mu\nu}$ (or in general the variables employed in the description of the world) are so created that one can obtain the second from the first through mere space-time transformation, then they are completely equivalent [*gleichbedeutend*]. For they have all spatiotemporal point coincidences in common, i.e., everything that is observable.
>
> *)and in nothing else! (EA 9-363)

An example of these space-time coincidences would be the collision of two point-masses.

We illustrate Einstein's point-coincidence argument in a way that will be suggestive below. Let two point-masses originate at a point-event q outside the hole, separate, and then collide at some point-event within the hole. See Figure 3. According to the second solution, g'_{ik}, the particles will collide at the point[-event] with coordinates (1, 1) in the first coordinate system, x^m. According to the first solution, g_{ik}, the particles will collide at the point with coordinates (1, 1) in the second coordinate system, $x^{m'}$. As illustrated in Figure 2, Einstein had earlier assumed that the two sets of coordinates would represent *different* point[-event]s, p and p', in the physical space-time. He now understands that, on the contrary, they must represent the same point[-event], because the two sets of trajectories agree in all physically significant quantities and thus cannot pick out physically different point[-event]s. For example, measurements of physical time elapsed along the trajectory qap as determined by the first solution g_{ik} would be identical to that along qap' as determined by the second solution g'_{ik}.[9]

2. Letters from Paul Hertz

Einstein later recalled the intense emotions that simmered and boiled within himself through the years of his struggle with general covariance when he wrote of the episode: "But the years of anxious searching in the dark, with their intense longing, their alternations of confidence and exhaustion and final emergence into the light—only those who have experienced it can understand that" (Einstein 1934, pp. 289–290). Into this emotional

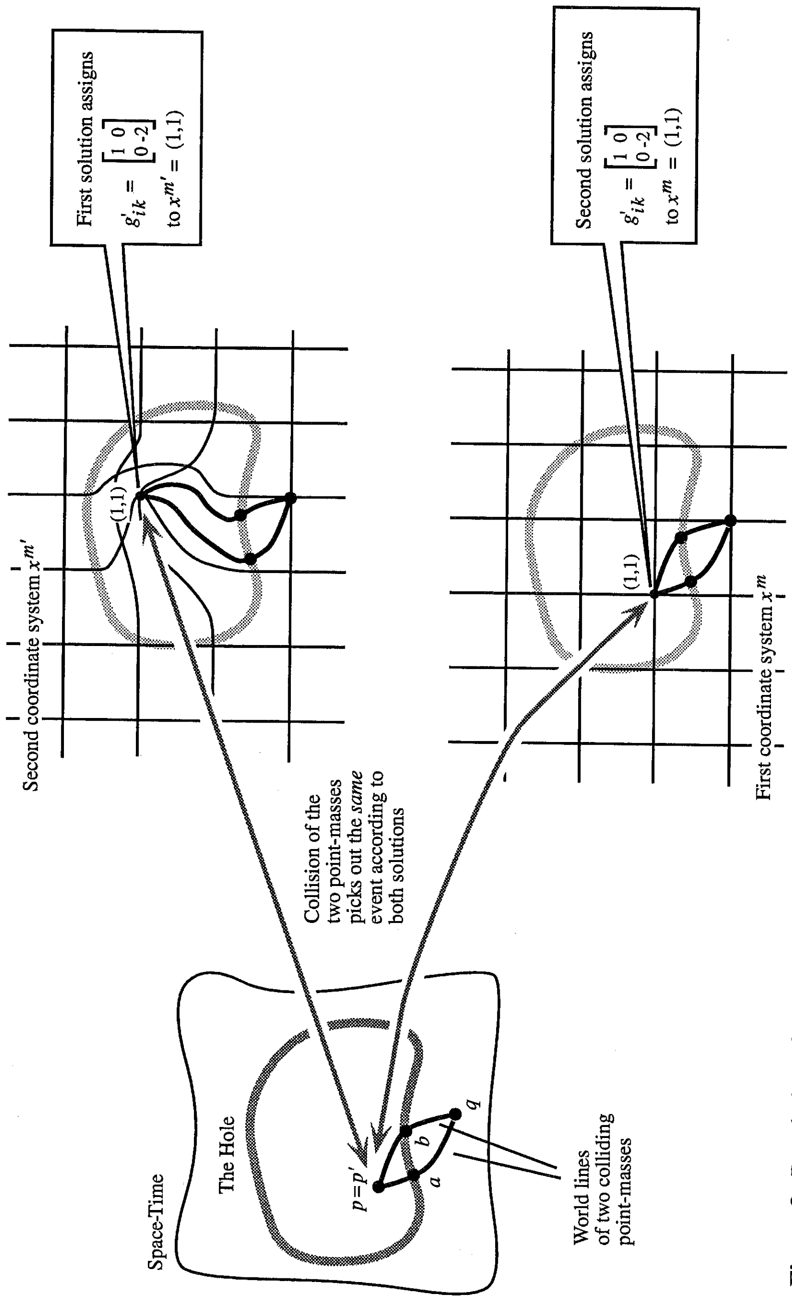

Figure 3. Resolution of the hole argument.

and intellectual cauldron around August 1915 was added an exchange in correspondence with Paul Hertz, just a few months before the struggle drew to its dramatic close that November.

Hertz was born in 1881 in Hamburg. In 1915 he was a *Privatdozent* at Göttingen and a member of the group clustered around Hilbert and Klein. He had taken a degree at Göttingen in 1904 under Max Abraham, with a dissertation on discontinuous movements of an electron (Hertz 1904).[10] After publishing a few additional studies on electron theory, he turned his attention to the foundations of statistical mechanics, an interest that culminated in his seminal 1916 monograph in the *Repertorium für Physik* (Hertz 1916), and also led to his acquaintance with Einstein. This acquaintance was a direct result of Hertz's critical remarks (Hertz 1910) on Einstein's early papers on the subject (Einstein 1902, 1903, 1904), remarks to which Einstein replied in a short note in the *Annalen* in 1911 (Einstein 1911a). They had begun corresponding by August 1910 and had become personally acquainted no later than early September 1910, at a meeting of the Schweizerische Naturforschende Gesellschaft in Basel.[11] Hertz was by this time acquainted with several of Einstein's closer friends and colleagues, most importantly Paul Ehrenfest, who had been a student in Göttingen at the same time as Hertz,[12] and Jakob Laub, another fellow student from Göttingen, who was a colleague of Hertz's in Heidelberg from 1909 to 1911.[13] In 1921, Hertz finally received an appointment as Ausserordentlicher Professor in Göttingen, the same year that he and Moritz Schlick published their influential edition of Helmholtz's epistemological writings (Helmholtz 1921). And in later years, Hertz turned his attention to various topics in the philosophy of science, including pioneering studies, very much in the Göttingen tradition, of the formal axiomatics of scientific theories.[14] Einstein provided a letter of recommendation for Hertz after his emigration to the United States (EA 12-221). He died in Philadelphia in 1940.

We do not know for certain that Hertz was present when Einstein lectured in Göttingen in late June and early July of 1915. Given the nature of the previous relationship between Hertz and Einstein, given Hertz's role in the group around Hilbert, and given the character of Hertz's correspondence with Einstein later that summer, it is more than likely, however, that he was present.

We know of the letters that Hertz wrote to Einstein only because Einstein's replies still exist (EA 12-201 and EA 12-203). Einstein's letter EA 12-203 is dated "22. VIII" (August 22). The content is compatible only with the years 1913–1915. The year must be 1915 because of the mention in a postscript of a coming visit to Zurich ("Aug. 26 to about September 15"), the address of his friend Heinrich Zangger being given for correspondence. Einstein made a visit to Zurich fitting this description in 1915.[15]

Einstein's letter is written in a friendly and encouraging tone. It reflects on the great problems Einstein had faced in finding a way to restrict the coordinate systems of his theory and sketches the difficulties still facing the theory in this area. The letter begins:

> One who has himself poked about so much in the chaos of possibilities can understand very well your fate. You haven't the faintest idea what I, as a mathematical ignoramus, had to go through until I entered this harbor.

And about his specific restriction to "adapted" coordinates, he comments:

> How can one pick out a coordinate system or a group of such? It appears not to be possible in any way simpler than that which I have chosen. I have groped about and tried everything possible.... The coordinate restriction that was finally introduced deserves particular confidence because it can be brought into connection with the postulate of the complete determination of events.

This last remark alludes to the fact that adapted coordinate systems were first introduced by Einstein in order to block the conclusion of the hole argument.

The letter's primary purpose, however, is to respond encouragingly to an idea of Hertz's alluded to in the first paragraph, which presumably concerns the restriction of the coordinate systems. Hertz's idea is presumably also the one that Einstein refers to in both the opening sentence—"A surface-theoretical interpretation of preferred systems would be of very great value"—and the closing sentence of paragraph five—"Perhaps one could get an overview on the question if one succeeded in finding the geometrical interpretation for which you seek"—for such an interpretation is not given or even mentioned by Einstein anywhere else in the letter. And Einstein's other letter, EA 12-201, contains a response to a proposal by Hertz that is cast in the older language of the theory of two-dimensional Gaussian surfaces.[16]

Einstein's EA 12-201 is dated "Berlin, Saturday" but, because of the close similarity of content, it was quite plausibly written at about the same time as EA 12-203. The earliest possible date is August 14, since Hertz's son, Hans, who is mentioned at the end of the letter, was born on Sunday, August 8.[17] The letter was probably written no later than about Saturday, October 9, since it betrays no doubts on Einstein's part about the restricted covariance of the Einstein–Grossmann (1913) theory, whereas by October 12 Einstein is writing to Lorentz that he now realizes that something is amiss with the theory.

The letter responds to another proposal by Hertz, but, as we shall see, it is written in a very different tone. The letter is at times impatient, discouraging and almost hostile—Einstein did not like Hertz's proposal! On the basis of Einstein's reply in EA 12-201, we reconstruct Hertz's proposal to amount to an escape from the hole argument, coupled with a proposal for setting up generally covariant gravitational field equations. The reconstruction that follows is the only one we have found that is compatible with the entirety of Einstein's response.

At this point, some readers might like to scan ahead and read the letter EA 12-201, which is quoted in full in Section 4, in order to see the raw material upon which our reconstruction is based. Readers who like puzzles might even want to try to build their own reconstruction before reviewing the one we offer below in Section 3.

3. Our Reconstruction of Hertz's Proposed Escape from the Hole Argument

Hertz tried to show Einstein that he should not be troubled by the differences between the two solutions considered in the hole argument. He considered the hole argument for the case of a two-dimensional Gaussian surface. We would now write the line element of such a surface in the quadratic differential form $\mathrm{d}s^2 = g_{11}(\mathrm{d}x^1)^2 + 2g_{12}\,\mathrm{d}x^1\,\mathrm{d}x^2 + g_{22}(\mathrm{d}x^2)^2$, where Hertz used the older notation introduced by Gauss, wherein one writes $\mathrm{d}s^2 = E\,\mathrm{d}u^2 + 2F\,\mathrm{d}u\,\mathrm{d}v + G\,\mathrm{d}v^2$. In the case of variable curvature, this geometry seems to allow the defining of a special coordinate system (u, v), whose curves are the curves of constant curvature and of maximum curvature gradient, and are thus adapted to the geometry. We shall call such systems "Hertz-adapted" to avoid confusing them with Einstein's "adapted" coordinate systems. Presumably such coordinates were proposed because they would be defined in terms of invariant features of the surface and because they might be proved to exist for spaces of both positive and negative curvature, unlike isometric coordinates.

Hertz examined the two solutions of the hole argument in the way outlined in Section 1 above. He considered one solution with coefficients E, F, and G in his original coordinate system (u, v) and the other with coefficients E^x, F^x, and G^x in the second coordinate system (u^x, v^x) so that the E, F, and G are the same functions of the variables u and v as the functions E^x, F^x, and G^x are of the variables u^x and v^x.[18] Moreover, Hertz ensured that the coordinate system (u, v) is Hertz-adapted to the geometry represented by E, F, and G, which entails that the coordinate system (u^x, v^x) is also Hertz-adapted to the geometry represented by E^x, F^x, and G^x.

He then asked after the nature of the underdetermination of the geometry revealed by the admissibility under general covariance of the two solutions constructed in the hole. To do so, he asked after the geometry within the hole according to the two solutions at two points that correspond in the sense that the coordinates of the first point in the first coordinate system (u, v) are numerically equal to the coordinates of the second point in the second coordinate system (u^x, v^x). To find the points, one must follow the two coordinate curves corresponding to the coordinate values selected and pursue them until they meet in the hole. Since the two coordinate systems are Hertz-adapted to superficially different geometries, the coordinate curves must diverge upon entering the hole, according to whether the system was adapted to the first or second solution of the field equations. For the coordinate system adapted to the first solution, the curves would meet at the point $P(u, v)$. For the coordinate system adapted to the second solution, the curves would meet at the point $P^x(u^x, v^x)$. See Figure 4, which is our rendering of the diagram Einstein gives in his letter (which is reproduced as Figure 5).

But what are the differences between the two solutions revealed by the construction? Hertz could point to no geometrically significant differences. Spelling out the argument in a way that employs the equations Einstein writes in his letter EA 12-201, the points selected by the construction would have the same coordinate values in each of the geometrically significant Hertz-adapted coordinate systems so that

$$u^x = u \quad \text{and} \quad v^x = v.$$

Moreover, the geometries at each point in the corresponding solutions are the same. For if E, F, and G are the coefficients assigned by the first solution to P, and if E^x, F^x, and G^x are the coefficients assigned by the second solution to P^x, then the geometries at the two points are the same in so far as $E^x = E$, $F^x = F$, and $G^x = G$.[19]

Perhaps Hertz might now have said that the two solutions are geometrically the same in every respect, for these identities would hold for corresponding points covering every point of both solutions. We can think of each solution as representing a different geometric surface. The construction shows how one of them can be mapped into the other by the map that takes point P to point P^x while preserving all geometric properties. In modern language, the two are isomorphic.

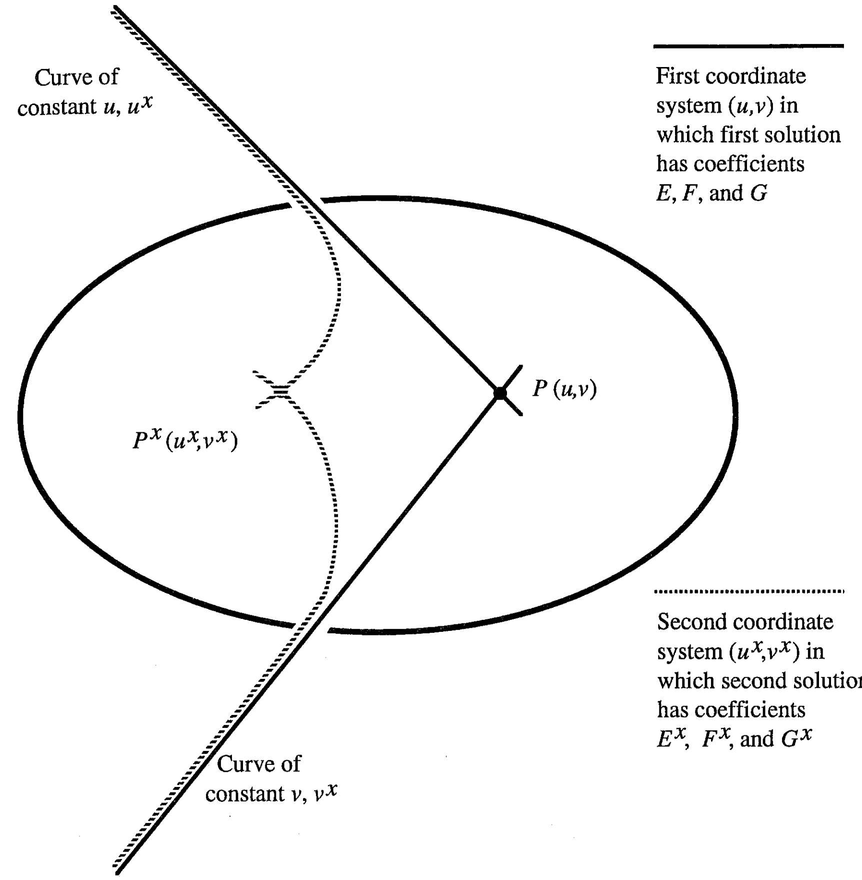

Figure 4. Interpretation of figure in Einstein's letter (cf. Figure 5).

We might rephrase this last point using the only direct quotation Einstein gives of Hertz: Since the two solutions amount to the same surface geometrically, we merely recall that, by the construction, this surface "is developable [i.e., isomorphically mappable] into itself," a clumsy but intelligible way of making the point. This usage of the term "developable" as meaning isomorphically mappable was standard at the time and was even applied to precisely the case Hertz treats using exactly the same set of equations.

Consider, for example, the discussion of two two-dimensional Gaussian surfaces embedded in a three-dimensional space that is found in Johannes Knoblauch's *Grundlagen der Differentialgeometrie* (Knoblauch 1913, pp. 121–124), then regarded as a standard text in Göttingen.[20] If the two surfaces could be laid upon one another without deformation, they are said to be "developable onto one another." The two surfaces have this property if they both admit two-dimensional coordinate systems (u, v) such that at corresponding points on the two surfaces, where the coordinate values are the same, the coefficients E, F, and G of one surface have the same values as the coefficients E_1, F_1, and G_1 of the second surface. Knoblauch wrote this requirement in the now-familiar equations:

$$E_1 = E, \quad F_1 = F, \quad G_1 = G.$$

4. Einstein's Immediate Response

The escape from the hole argument sketched above is obviously very close in strategy to the escape Einstein himself would offer shortly as the point-coincidence argument, but Einstein's immediate response to Hertz's proposal was just a list of protests and complaints. Einstein took Hertz-adapted coordinates to be the same as the adapted coordinates Einstein himself had defined (see Section 1 above). The letter from Einstein began with the protest that Hertz had misrepresented Einstein's adapted coordinate systems, since he had failed to retain the crucial property stressed in Section 1 above, namely that two different (Einstein-)adapted coordinate systems could not come smoothly to agree on the boundary of some region of spacetime. And in any case—whether or not the two coordinate systems were adapted—they were supposed to have properties that, in general, could not obtain. Einstein wrote:

Berlin, Saturday

Dear Herr Hertz,

If I have understood your letter correctly, then you make a completely erroneous representation of that which I call "adapted coordinate systems." How do you come to require that a pair of coordinate systems [Figure 5 = figure from Einstein's letter] should exist, such that for

$$u^x = u$$
$$v^x = v$$

one has also

$$E^x = E$$
$$F^x = F$$
$$(G^x = G)\ \phi^x = \phi$$

and over and above this they agree on the boundary of the region?

I am rather convinced that (excepting perh.[aps] quite special fields) *this is never allowed to be possible.* I have never posited the existence of systems equivalent in this sense.[21]

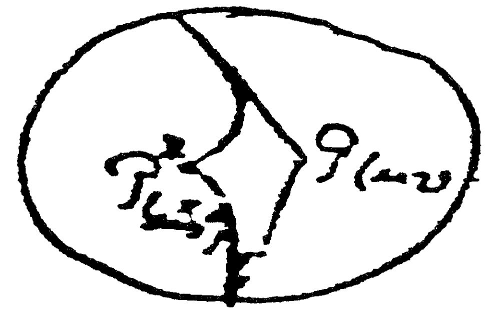

Figure 5. Diagram in Einstein to Hertz, "Berlin, Samstag" [1915] (EA 12-201).

We can only conjecture about how Einstein came to see the adapted coordinates of Hertz's proposal as being the same as the adapted coordinates he himself had defined for his 1913 theory. Both would use the term "adapted" naturally as an appropriate term for coordinate systems that they define in a way that responds to the geometry of the metric field, but it is hard to see that the use of the term alone would be sufficient to lead to this misunderstanding. Recall that in EA 12-203 Einstein had encouraged Hertz in his attempts to find a "surface-theoretic interpretation" of the preferred systems of coordinates of Einstein's theory. If EA 12-203 was written before EA 12-201, we could well imagine Einstein anticipating such a proposal from Hertz when he received EA 12-201. Or, even if EA 12-201 did predate EA 12-203, Hertz himself might have thought his adapted coordinates would serve as the surface-theoretic interpretation of Einstein's adapted coordinates and offered them as such. Finally, a minor factor that might well be crucial in such circumstances: Einstein complains later in the letter that he cannot read Hertz's handwriting on page five of his letter. We might well wonder, then, how clearly written the other pages were.

Einstein's more general complaint about the inadmissibility of the two coordinate systems (u^x, v^x) and (u, v) is readily explicable. All he need assume is that both coordinate systems with their components (E^x, F^x, G^x) and (E, F, G) are coordinate systems and components of the *same* field, not of two different fields as is crucial to both the hole argument and the proposal of Section 3 above. (Perhaps this is already assumed in Einstein's objection that the two systems cannot both be adapted coordinate systems.) As Einstein points out, only quite special fields can be transformed in the way indicated. A coordinate transformation in general produces a quite different set of components for the field that will fail to match in the indicated way.

Einstein continued in what seems to be an attempt further to worry Hertz's proposal. He pointed out that the defined special coordinate system would become degenerate in the case of a space of constant curvature and then mentioned the problem of extending the definition of these coordinate systems to the four-dimensional case in a way that suggested some doubt about its feasibility. If Einstein did intend doubt here, he was shortly proven wrong about the general program of finding four-dimensional coordinate systems that fit the natural structure of a region of space-time, for less than two years later Kretschmann showed how a four-dimensional coordinate system could be constructed in general relativity from curvature invariants (Kretschmann 1917, pp. 592–599).[22] The search for coordinates somehow "adapted" to the intrinsic geometry of the space was, in any case, characteristic of the Göttingen approach to general relativity, as reflected in Hilbert's employment of what he termed "Gaussian coordinates" (Hilbert 1916, pp. 58–59), which are now commonly designated geodesic normal coordinates.[23] The passage quoted above continues thus:

> Independently of this, I understand how you establish a special coordinate system on a two-dimensional manifold by curves of constant curvature and those of maximal curvature gradient. What is problematic [*verdächtig*] about this, however, is that, in regions of constant curvature, the ⟨surfaces⟩ curves (or surfaces) of constant curvature are shifted infinitely far away from one another. The difference, in principle, of the two coordinates that have been introduced is also problematic. You could, nevertheless, attempt to see whether such a thing can be done in a four-dimensional manifold.

Hertz had apparently also coupled his analysis with a proposal for a generally covariant field equation. Einstein replied sharply, asking whether or not Hertz agreed with the need to restrict the covariance of his theory, which again suggests that Hertz had been less than clear in explaining that the proposal, as outlined in Section 3, was intended as an escape from the hole argument. Einstein wrote:

> I have not understood the proposal for the setting-up of a gravitation law, because I cannot read your writing on page 5. After all, I have said in my work that a usable gravitation law is not allowed to be generally covariant. Are you not in agreement with this consideration?

Einstein then returned to his earlier objection about the two coordinate systems that Hertz had introduced and closed with these words:

> So once again: I would not think of requiring that the world should be "developable onto itself," and I do not understand how you require such a dreadful thing of me. In my sense, there is certainly a huge manifold of adapted systems that do *not*, however, agree on the boundary.

> With best regards to you, your wife, and your gentleman son, who is already surprisingly affable and fond of writing, I remain, riveted upon your further communications, yours
>
> A. Einstein

Einstein had understood, in effect, that Hertz required the transformation relating the two coordinate systems to be an isometry of the surface, so that he could say that the surface could be developed onto itself by the transformation. As Einstein had pointed out, surfaces admitting such isometries are exceptional and, in any case, the transformation could not be between Einstein's adapted coordinate systems, since such systems would never agree on the boundary of the region in the way Hertz required.

5. Einstein's Eventual Assimilation of the Lessons Hertz Tried to Teach Him

Even though Einstein's immediate response to Hertz was so prickly and defensive, he eventually came to appreciate and advocate Hertz's central point: If a system is developable onto another, the two represent the same reality. This advocacy is nowhere more in evidence than in Einstein's correspondence with Ehrenfest in late December and early January 1916. Ehrenfest was reluctant to accept the generally covariant form of the theory of gravitation announced by Einstein in November 1915, and he pressed his reservations by reminding Einstein, as had other correspondents, of the earlier hole argument. More specifically, in a letter that no longer exists

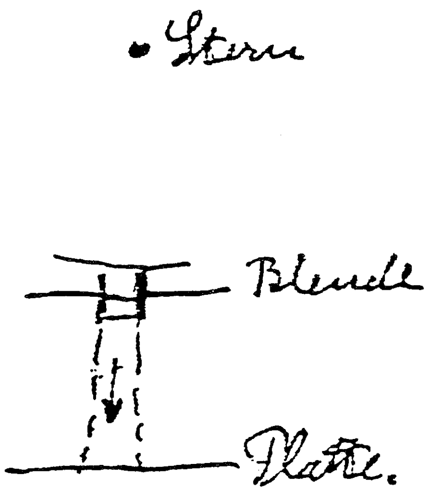

Figure 6. First diagram in Einstein to Ehrenfest, January 5, 1916 (EA 9-372).

from late December 1915, Ehrenfest evidently asked Einstein to consider a situation in which light from a distant star passes through one of Einstein's notorious holes and then strikes a screen with a pinhole in it that directs the light onto a photographic plate.[24] Given that generally covariant equations allow for two different solutions, $g^A_{\mu\nu}$ and $g^B_{\mu\nu}$, inside the hole, Ehrenfest asks how we can be sure that light from the distant star following different paths through the hole determined by the two different solutions can be guaranteed to strike the same place on the plate.[25]

We quote the relevant section of Einstein's detailed answer in its entirety:

> In the following way you obtain all of the solutions that general covariance brings in its train in the above special case. Trace the little figure above [see Figure 6] on completely deformable tracing paper. Then deform the tracing paper arbitrarily in the paper-plane. Then again make a copy on stationery. You obtain then, e.g., the figure [Figure 7]. If you now refer the figure again to orthogonal stationery-coordinates, then the solution is mathematically a different one from before, naturally also with respect to the $g_{\mu\nu}$. But physically it is exactly the same, because even the stationery-coordinate system is only something imaginary [*eingebildet*]. The same points of the plate always receive light....
>
> What is essential is this: As long as the drawing paper, i.e., "space," has no reality, the two figures do not differ at all. It is only a matter of "coincidences," e.g., whether or not the point on the plate is struck by light. Thus, the difference between your solutions A and B becomes a mere difference of representation, with physical agreement. (EA 9-372)

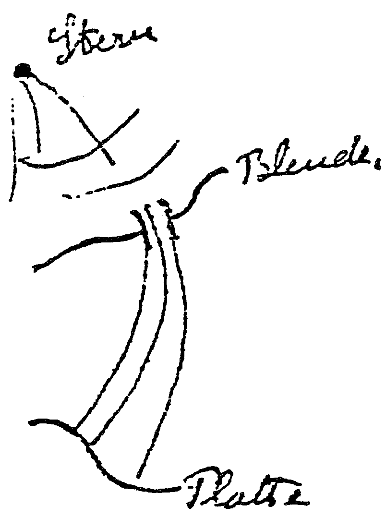

Figure 7. Second diagram in Einstein to Ehrenfest, January 6, 1916 (EA 9-372).

Aside from the talk of "coincidences," Einstein's point here is exactly Hertz's, namely, that one can have two solutions that are mathematically different, while being physically or geometrically (they come to same thing in this context) indistinguishable.

6. Hilbert's Escape from the Hole Argument

The reconstruction of what Hertz wrote to Einstein as conjectured in Section 3 above was based on an analysis of Einstein's letters. We then sought some independent evidence for our conjecture, but the existing documentation provided none. There is additional correspondence between Einstein and Hertz from early October 1915, concerning whether or not Hertz should resign his membership in some society seemingly concerned with political matters. And something that Einstein wrote in this connection so irritated Hertz that he threatened to break off the correspondence, an eventuality that Einstein earnestly sought to avoid.[26] Further communication was no doubt made even more difficult by the fact that Hertz soon found himself in the military, posted to a flight school in Posen.[27]

If we could not confirm independently that Hertz suggested such an escape from the hole argument, then, we asked ourselves, could we at least determine whether or not such an escape was common knowledge in Göttingen at the time so that Hertz was either initiating or reflecting a standard response? To our surprise and pleasure we found—after we had completed the construction of the conjecture of Section 3—that Hilbert had offered almost exactly the escape in the second of his famous papers on general relativity and the foundations of physics (Hilbert 1916).

The relevant remarks are found in Hilbert's somewhat labored discussion of the "causality problem" in general relativity, the designation Einstein often used for the hole argument (Hilbert 1916, pp. 59–63).[28] Hilbert points out that the Cauchy problem is not well posed for his own generally covariant version of general relativity (Hilbert 1915). That theory has fourteen independent variables—the ten gravitational potentials, $g_{\mu\nu}$, and the four electromagnetic field potentials, q_s—but the gravitational field equations and Maxwell's equations provide only ten independent field equations. Hilbert illustrates this underdetermination with a pair of solutions, the first of which represents an electron at rest throughout all time, with the gravitational and electromagnetic fields everywhere time-independent. In a manipulation reminiscent of the hole argument, the second solution is obtained by a coordinate transformation that is the identity for the time coordinate $x_4 \leq 0$, but comes to differ for $x_4 > 0$. In the second solution, the electron adopts a nonvanishing velocity and the fields become time-dependent after $x_4' = 0$. While the possibility of such different solutions at first seems to threaten the principle of causality, however, Hilbert proposes to rescue it by offering a definition of what it means for an object, a law, or an expression to be "physically meaningful." According to Hilbert, something should be regarded as physically meaningful only if it is invariant with respect to arbitrary transformations of the coordinate system. And in this sense, the causality principle is satisfied, since, he asserts, all physically meaningful expressions, which is to say, all invariant expressions, are unambiguously determined by the generally covariant equations.[29]

It is at this point in Hilbert's exposition that his argument converges upon what we believe Hertz proposed to Einstein. Hertz, we believe, exploited a geometrically adapted coordinate system to display the essential agreement between the two solutions E, F, G and E^x, F^x, G^x. Hilbert summarized his basic claim and then promised to prove the claim by exploiting the geometrically adapted Gaussian coordinate system:

> *The causality principle holds in this sense*:
> From a knowledge of the 14 physical potentials, $g_{\mu\nu}$, q_s, follow all assertions about them for the future necessarily and uniquely, *insofar as they have physical significance*.
> In order to prove this claim, we employ the Gaussian space-time coordinate system. (Hilbert 1916, p. 61; Hilbert's emphasis)

Hilbert begins by noting that the selection of Gaussian coordinates provides the four extra constraints needed to ensure that the fourteen potentials are determined uniquely by fourteen equations. The Gaussian coordinate system is uniquely defined, and, most importantly, the unique assertions then made about the potentials in the Gaussian coordinate system are of invariant character. Thus, the present can uniquely determine the invariant and therefore physically meaningful content of the future and no contradiction with the causality principle remains.

Hilbert proceeded to indicate three ways in which invariant assertions can be given mathematical expression. Reminiscent of our reconstruction of Hertz's proposal, the first two of Hilbert's ways resorted to specially adapted coordinate systems.[30] The first recapitulated the use of invariant coordinate systems, such as what he termed Gaussian (geodetic normal) coordinates, and elaborated on its application to the example of the electron at rest. The second allowed invariant character for an assertion that *there exists* a coordinate system in which some nominated relation holds. As an illustration, he resorted again to the case of the electron and claimed invariant character for the assertion that there exists a coordinate system according to whose x_4 time coordinate the electron is at rest.

That Hertz, as we reconstruct him, and Hilbert, both working in Göttingen, should rely so heavily on specially adapted coordinate systems to reveal the physically significant elements of a theory provides strong evidence for our reconstruction. It also raises the further question of the origin of these ideas. Were they Hertz's own? Or was he acting, in effect, as a spokesperson for Hilbert and the Göttingen group?

7. Other Influences on Einstein's Resolution of the Hole Argument

Hertz's proposal to Einstein—as reconstructed by us—would have provided a serviceable escape from the hole argument. The escape route actually followed by Einstein, however, his point-coincidence argument, differed in crucial ways from that of Hertz and the Göttingen group. The latter was the mathematician's escape, relying principally on the mathematical notion of invariance; the former was the physicist's escape, relying principally on general dicta about physical reality. Was the point-coincidence argument another unprimed outpouring of Einstein's genius? Or can we identify who primed the pump? We believe that there are at least two plausible candidates.

The first of these, chronologically, is Joseph Petzoldt, a Privatdozent at the Technische Hochschule Berlin–Charlottenburg, founder in 1912 of the Gesellschaft für positivistische Philosophie (of which Einstein was a founding member), and author of numerous books and articles promoting a point of view that Petzoldt labeled "relativistic positivism," a mélange of ideas from Mach and Einstein, the chief aim of which was a critique of the traditional metaphysical notion of substance. Petzoldt's most important contribution for the purposes of our discussion was his introduction in 1895 of what he termed "Das Gesetz der Eindeutigkeit" ("The Law of Uniqueness" or "Univocalness") (Petzoldt 1895), according to which, in one of its forms, a theory would be acceptable only if it determined a unique model of the reality it aimed to describe. Petzoldt's "law of uniqueness" and the major discussion stimulated by it form an essential part of the background to Einstein's hole and point-coincidence arguments, since it is this very methodological principle that lies at the root of both.[31]

By 1915, Einstein and Petzoldt were in personal contact with one another. There is evidence that Petzoldt was attending Einstein's lectures on relativity in Berlin in either the winter semester of 1914–1915 or the summer semester of 1915. A postcard from Einstein to Petzoldt in late 1914 or early 1915 makes it clear that Einstein had been reading Petzoldt's work and approved of its general tendency: "Today I have read with great interest your book in its entirety, and I happily infer from it that I have for a long time been your companion in your way of thinking" (EA 19-067); the book was most likely Petzoldt's *Das Weltproblem vom Standpunkte des relativistischen Positivismus aus, historisch-kritisch dargestellt* (Petzoldt 1912b).[32]

Against this background, one may wonder whether Einstein had absorbed the point of view exemplified by a remark in Petzoldt's "Die Relativitätstheorie im erkenntnistheoretischer Zusammenhang des relativistischen Positivismus" (Petzoldt 1912a), which would have appeared early in 1913 in the proceedings of the Deutsche Physikalische Gesellschaft. The relevant remark concerns the way Petzoldt's epistemological perspectivalism is allegedly embodied in special relativity. Petzoldt writes,

> The task of physics becomes, thereby, the unique [*eindeutige*] general representation of events from different standpoints moving relative to one another with constant velocities, and the unique setting-into-relationship of these representations. Every such representation of whatever totality of events must be uniquely mappable onto every other one of these representations of the same[1)] events. The theory of relativity is one such mapping theory. What is essential is that unique connection. Physical concepts must be bent to fit for its sake. We have theoretical and technical command only of that which is represented uniquely by means of concepts.
>
> [1)] Better: representations of events in arbitrarily many of those systems of reference that are uniquely mappable onto one another are representations of "*the same*" event. Identity must be *defined*, since it is not given from the outset. (Petzoldt 1912a, p. 1059)

It is the footnote that grabs one's attention, for it expresses a fundamental presupposition of Einstein's point-coincidence argument. What is interesting about Petzoldt's remark is that this way of talking about identity under a mapping, especially of what are clearly, from context, Minkowskian point-events, was not commonplace in the pre-1915 literature on relativity.

To appreciate the role of the second figure possibly influencing Einstein's formulation of the point-coincidence argument, recall that Einstein's struggle to find generally covariant field equations came to a close with his November 25, 1915 communication to the Prussian Academy (Einstein 1915b). Already in his immediately preceding communication of November 18, 1915, he remarked that through general covariance, "time and space have been robbed of the last trace of objective reality" (Einstein 1915a, p. 831), by which he meant that "the relativity postulate in its most general formulation... turns the space-time coordinates into physically meaningless parameters" (Einstein 1915b, p. 847). This makes it clear that, at this time, in late November, Einstein was in possession of an answer to the hole argument involving essentially the idea that coordinatizations are not sufficient for the individuation of points in the physical space-time. Curiously, however, when he begins informing his correspondents about these developments in late December, he adds, for the first time, the talk of coincidences so characteristic of the familiar form of the point-coincidence argument.

It seems likely to us that Einstein's immediate inspiration for the point-coincidence talk came from the work of Erich Kretschmann. His 1915 essay, "Über die prinzipielle Bestimmbarkeit der berechtigten Bezugssysteme beliebiger Relativitätstheorien," is a lengthy and labored discussion of the determination of coordinate systems in which the notion of spatiotemporal coincidence plays a prominent role. The paper clearly anticipates essential elements of the point-coincidence argument, as Kretschmann himself seemed to think when, in a later publication, he cited his own 1915 paper "for further details" (Kretschmann 1917, p. 576) on the point-coincidence argument, citing Einstein's version of the argument solely for the introduction of the German term "*Koinzidenzen*," replacing Kretschmann's 1915 "*Zusammenfallen*" (see below).[33]

In his 1915 paper, Kretschmann argues that only what he calls "topological" relations in the form of coincidences have empirical significance, since all observation requires that we bring a part of the measuring instrument into contact with the measured object:

> What is observed here—if we neglect, at first, all direct metrical determinations—is only the completely or partially achieved spatiotemporal coincidence [*Zusammenfallen*] or non-coincidence [*Nichtzusammenfallen*] of parts of the measuring instrument with parts of the measured object. Or more generally: topological relations between spatiotemporally extended objects. (Kretschmann 1915, p. 914)

A similar insistence on the observability of coincidences figures prominently in the best-known of Einstein's statements of the point-coincidence argument, where Einstein writes:

> All our space-time verifications invariably amount to a determination of space-time coincidences [*Koinzidenzen*].... Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences [*Koinzidenzen*] between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. (Einstein 1916, p. 117)[34]

There is, to be sure, the one difference noted later by Kretschmann, which is that Einstein uses the term "*Koinzidenzen*," not Kretschmann's "*Zusam-*

menfallen." The former term is more suggestive of the topologist's notion of the intersections of lines at extensionless points, whereas the latter is more suggestive of macroscopic congruences of bodies at the level of observational practice. Thus, Kretschmann can talk more comfortably of "completely or partially achieved coincidences [*Zusammenfallen*]." The similarity is nonetheless striking.

Kretschmann proceeds in the 1915 paper to develop now-familiar ideas concerning coordinate systems. In particular, he urges on the basis of his earlier assertions on coincidences that, "in no case can a soundly based decision be made, through mere observations, between two quantitatively different but topologically equivalent mappings of the world of appearance onto a space-time reference system" (Kretschmann 1915, p. 916). An immediate application of Kretschmann's remark (but *not* offered by Kretschmann) is the case of the two solutions, g_{ik} and g'_{ik} (in the same coordinate system x^m) of the hole argument. They are "two quantitatively different... mappings of the world of appearance onto a [single] space-time coordinate system." Nonetheless, they are "topologically equivalent," since they agree on all point-coincidences, and hence observation allows no soundly based decision between them. But if observation reveals no difference, does there remain any factual difference between them? If we pursue the development of Kretschmann's ideas, we find that whatever differences obtain between the two solutions, g_{ik} and g'_{ik}, must be merely matters of convention: "Insofar as the kinematical assertions of a system of physical laws cannot be reduced to purely topological relations, they are henceforth to be considered as mere—at most methodologically grounded—conventions" (Kretschmann 1915, p. 924).[35]

Of course, there is no reason to think that Kretschmann intended his discussion to be applied to Einstein's hole argument. However, the similarity between Einstein's expositions of the point-coincidence argument and Kretschmann's discussion is so striking that it cannot be (dare we say!) a mere coincidence and must have resulted from some sort of connection between Einstein and Kretschmann. The only question to be resolved is the nature of that connection. What is extremely suggestive is that Kretschmann's paper appeared in an issue of the *Annalen der Physik* that was distributed on December 21, 1915, five days before the earliest of the surviving letters in which Einstein articulates the point-coincidence argument, his letter to Ehrenfest of December 26 (EA 9-363). We are unaware of any similar invocation of point-coincidences in the corpus of Einstein's writings—both published and unpublished—prior to this letter. What is more, when, in a letter of December 14, 1915 (EA 21-610), Einstein informed Moritz Schlick about the exciting developments of November 1915, he remarked only on space and time having lost the last vestige of physical reality, with no mention of point-coincidences. These facts make almost irresistible the conclusion that Einstein read Kretschmann's paper or learned of its content when it appeared, found the ideas on coincidences extremely congenial, and turned to refine and exploit them to explain to his correspondent Ehrenfest where his hole argument had failed.

Other paths of transmission of these ideas between Einstein and Kretschmann are possible, but seem less likely. Kretschmann completed his Ph.D. in 1914 under Max Planck and Heinrich Rubens in Berlin, standing for the Promotionsprüfung on February 5 of that year. But Kretschmann reports that he finished his studies in Berlin in 1912 (see the *Lebenslauf* at the end of Kretschmann 1914), and the manuscript of his 1915 paper was submitted from Königsberg, where he had finished *Gymnasium* in 1906 and where he became a *Privatdozent* in 1920. Were he present in Berlin after Einstein's arrival in April 1914, it is plausible that he might have had some contact with Einstein, through which contact Einstein may have supplied the ideas about coincidences to or learned them from Kretschmann. Whatever contact they may have had in Berlin, however, cannot have been extensive or engaging to Kretschmann as far as Einstein's still incomplete general theory of relativity was concerned. While he was elsewhere rather long-winded, Kretschmann's 1915 paper contains only a brief discussion of Einstein's theory (pp. 977–978), citing just two of the earlier joint publications by Einstein and Grossmann (Einstein and Grossmann 1913, 1914), and omitting the major review article of November 1914 (Einstein 1914b). The discussion is sketchy and fails to make any serious contact with the idea of adapted coordinates, an idea that was a major focus of Einstein's Berlin work on the theory at that time and very relevant to the subject of Kretschmann's paper. Finally, of course, the possibility of such earlier transmission completely fails to explain the extraordinary fact that the point-coincidence argument and mention of space-time coincidences in this general context appear for the first time in a letter of Einstein's of December 26, 1915, only days after the issue of the *Annalen* containing Kretschmann's paper was distributed.[36]

Acknowledgments. Our sincere thanks are due to Rudolf Hertz, Paul Hertz's son, for his invaluable assistance in our research. Thanks are also due to Peter Havas, for having drawn our attention to the notes from Klein's lectures on general relativity during the summer of 1916. The research for this paper was supported in part by grants from the National Science Foundation (no. SES-8421040, DH), the American Philosophical Association (DH), the Deutscher Akademischer Austauschdienst (DH), and the University of Kentucky Research Foundation (DH). We would like to thank the Hebrew University of Jerusalem, which holds the copyright, for permission to quote from Einstein's unpublished letters, and to thank the Niedersächsische Staats- und Universitätsbibliotek, Göttingen, for permission to quote from Felix Klein's unpublished lectures. Items in the Einstein Archive are cited by their numbers in the Control Index.

Notes

[1] To see this, note that the first solution transformed from x^m to $x^{m'}$ has the functional form g'_{ik} of the coordinates $x^{m'}$, which is the same functional form as the components of the second solution in the coordinate system x^m.

[2] For a summary of the mathematical machinery Einstein used to analyze his adapted coordinates, see Norton (1984, section 6).

[3] This letter is dated on the basis of its place in a sequence of letters discussing the shipment of the de Haas's furniture from Berlin to the Netherlands, the shipment being overseen by Einstein.

[4] For more on this visit, see the discussion in Pais 1982, pp. 250 and 259.

[5] Cod. Ms. Klein 21L, p. 63, Niedersächsische Staats- und Landesbibliothek Göttingen.

[6] Cod. Ms. Klein 21L, p. 69, Niedersächsische Staats- und Landesbibliothek Göttingen.

[7] This timing, the fact that Einstein and Hilbert engaged in an intense correspondence through November 1915 and then had a brief falling out after that correspondence, has raised the possibility that Einstein stole the field equations from Hilbert. We do not take this possibility seriously for the reasons given in Norton (1984, pp. 314–315).

[8] See, for example, Einstein to Paul Ehrenfest, December 26, 1915 (EA 9-363), December 29, 1915 (EA 9-365), and January 5, 1916 (EA 9-372), as well as Einstein to Michele Besso, January 3, 1916 (EA 7-272; reprinted in Speziali 1972, pp. 63–64).

[9] Notice that such magnitudes as "time elapsed" are in turn reducible to space-time coincidences. A crude physical time could be measured by an idealized light clock, which is a small rigidly co-moving rod along whose length a light pulse is repeatedly reflected. The time elapsed is measured by the number of collisions of the light pulse with the mirrored ends of the rod.

[10] Hilbert was the titular director of Hertz's dissertation, but Hertz actually did the work under Abraham, who was then *Privatdozent*; see Pyenson 1979b, p. 76.

[11] See Einstein to Hertz, August 14, 1910 (EA 12-195) and August 26, 1910 (EA 12-198). For more on the beginning of their acquaintance, see Stachel et al. 1989, p. 44, and Klein et al. 1993, p. 315.

[12] See the Hertz–Ehrenfest correspondence in the Ehrenfest scientific correspondence in the Archive for the History of Quantum Physics.

[13] See Pyenson 1990, as well as Laub to Einstein, May 16, 1909 (EA 15-465), Einstein to Laub, May 19, 1909 (EA 15-480), and Einstein to Laub, October 11, 1910 (EA 15-489), November 4, 1910 (EA 15-491).

[14] See, for example, Hertz 1923, 1929a, 1929b, 1930, 1936a, 1936b.

[15] See Clark 1971, p. 184. The chief purpose of Einstein's trip was to meet the novelist Romain Rolland at Vevey, this as part of Einstein's efforts to promote international intellectual cooperation in spite of the barriers raise by World War I. For more on the meeting with Rolland and Einstein's related activities, see Nathan and Norden 1968, pp. 12–18. The year could not be 1913, because Einstein was then still in Zurich, and such a trip would not likely have been undertaken in late August 1914, immediately after the outbreak of the war.

[16] See below. In particular, Hertz uses the older "E, F, and G" notation for what we would now call the components of the metric tensor.

[17] Rudolf Hertz (Paul's son), private communication.

[18] To see the correspondence between our account of the hole argument in Section 1 and Hertz's construction, notice that our second solution, g'_{ik}, in the first coordinate system, x^m, corresponds to Hertz's E, F, G in (u, v), while our first solution, g_{ik}, in the second coordinate system, $x^{m'}$, corresponds to Hertz's E^x, F^x, G^x in (u^x, v^x). Of course, there is the inconsequential change of context. Einstein's argument is formulated in a space-time with an indefinite metric, whereas Hertz's argument is formulated for the space of a two-dimensional Gaussian surface.

[19] Obviously, this construction and the point-coincidence argument have the following in common: They pick out a point in the physical space by the intersection of curves with invariant geometrical properties. In Hertz's case, the curves are curves of constant curvature and maximal curvature gradient; in the case of the point-coincidence argument, they are geodesics.

[20] In his *Vorlesungen über die Entwicklung der Mathematik im 19. Jarhundert* (Klein 1927, pp. 147–148), Felix Klein lists Knoblauch 1913 as one of the "great textbooks" appearing around the turn of the century, along with Darboux's *Leçons sur la théorie générale des surfaces* (Darboux 1914–1915) and Bianchi's *Vorlesungen über Differentialgeometrie* (Bianchi 1910). Although first published in 1927, Klein's lectures were delivered in the years 1915 through 1917.

[21] Einstein's replacing of G, the g_{22} component of the metric, by ϕ is explicable in terms of his 1913 theory. In Einstein's 1913 theory, the $g_{\text{"time" "time"}}$ component of the metric in a static field in a suitably adapted coordinate system represents the single gravitational potential of the field, commonly represented by ϕ. Note that the angle brackets indicate a strikeout in Einstein's original.

[22] In a footnote, Kretschmann comments that the possibility of finding "absolute" coordinates, meaning coordinates picked out uniquely by the geometry of the space being thus coordinatized, had been pointed out to him already in a letter from Gustav Mie in February 1916; see Kretschmann 1917, p. 592, n. 1.

[23] For more on Hilbert's introduction of "Gaussian coordinates," see Stachel 1992, pp. 410–412.

[24] The approximate date of Ehrenfest's letter to Einstein can be determined from his remark, in a letter to Lorentz of December 23, 1915, that he had invited Einstein to spend the holidays in Leiden. Einstein's reply to Ehrenfest's thought experiment is contained in the same letter of January 5, 1916 (EA 9-372), in which he explains that the border's being blocked was the reason why he could not have come to Holland at that time. We thank A.J. Kox for making available transcriptions of

the Ehrenfest–Lorentz correspondence, these from his forthcoming edition of the scientific correspondence of Lorentz.

25 The reconstruction of Ehrenfest's thought experiment is based upon Einstein's reply of January 5 (EA 9-372) and on the description found in Ehrenfest's letter to Lorentz of January 9, in which he enclosed Einstein's letter, asking for Lorentz's opinion.

26 See Einstein to Hertz, undated 1915 (EA 12-205), October 1915 (EA 12-206), Hertz to Einstein, October 8, 1915 (EA 12-207), and Einstein to Hertz, October 9, 1915 (EA 12-208). Though the dating of some of these letters is problematic, they seem clearly to form a sequence written over a short period. It should be noted that most of Hertz's are missing, the letter of October 8 having survived because Hertz retained a copy in his files.

27 See Hertz to Hilbert, February 17, 1916 (Cod. Ms. Hilbert 150, Handschriften-abteilung, Niedersächsische Staats- und Universitätsbibliothek Göttingen).

28 Hilbert's only footnote in this section of the paper (Hilbert 1916, p. 61) cites Einstein's most complete version (1914b, p. 1067) of the hole argument.

29 For more on Hilbert and the causality principle in general relativity, see Stachel 1992, pp. 410–412.

30 The third merely allowed invariant character to a fully covariant law, such as the law of conservation of energy-momentum expressed as the vanishing covariant divergence of the stress-energy tensor.

31 For more on Petzoldt and a more detailed bibliography of his writings, see Howard 1992.

32 For the dating of Einstein's postcard to Petzoldt and other details about their relationship, see Howard 1992.

33 For more on Kretschmann's papers, see Norton 1992, pp. 295–301.

34 See Howard 1992, n. 25, for a critical discussion of Friedman's (1983, pp. 22–25) interpretation of this passage as anticipating the verificationist theory of meaning that later became popular among the logical positivists.

35 In a footnote to the word "convention," Kretschmann carefully indicates the precise sense of the word intended. It is to mean that which is not demonstrable through observation, rather than something arrived at by some kind of free agreement.

36 We might also conjecture that Einstein was asked to review the paper by Planck, the editor of *Annalen.* Kretschmann's paper is dated October 15 and was received on October 21. If it was sent out for review, Einstein would have been the obvious reviewer. The short time between submission and publication, October 21 to December 21, suggests that, even though Kretschmann was a first-time author in the *Annalen*, the manuscript was not sent out for review, since a two-month period between submission and publication was more or less normal for established authors (see Pyenson 1983). This would not be surprising, since Planck had supervised Kretschmann's Ph.D., was presumably confident of Kretschmann's scholarship, and possibly already familiar with the work submitted.

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Conservation Laws and Gravitational Waves in General Relativity (1915–1918)

Carlo Cattani and Michelangelo De Maria

1. Introduction

This chapter deals with two closely related debates in general relativity in 1916–1918, one on gravitational waves, the other on the correct formulation of conservation laws. Both issues involve the definition of a quantity representing the stress-energy of the gravitational field. Such definitions were typically proposed in the context of deriving the gravitational field equations from a variational principle. A proper understanding of the debates on gravitational waves and conservation laws therefore requires some discussion of the rather complicated history of attempts to derive gravitational field equations from a variational principle.[1]

We will trace Einstein's work on gravitational waves and his work on conservation laws during the years 1916–1918 in this more complex network. We will look at objections to Einstein's approach from Levi-Civita, Schrödinger, and Bauer; at alternative approaches suggested by Lorentz and Levi-Civita; and at Einstein's response to all of them. In particular, we will examine the 1917 correspondence between Einstein and Levi-Civita. We will see how Levi-Civita's criticism of Einstein's formulation of conservation laws strengthened Einstein in his conviction that physical considerations force one to adopt a noncovariant formulation of conservation laws for matter plus gravitational field.

2. The Importance of the Conservation Laws in Einstein's 1914 Gravitational Theory

In Einstein and Grossmann 1914 and Einstein 1914, Einstein used a variational method to derive the field equations of limited covariance of his

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