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Large-Scale Structure: Four Claims

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Chapter for a book provisionally titled

The Large-Scale Structure of Inductive Inference

1. Introduction

The previous chapter recounted how the material theory of induction treats relations of inductive support individually. That is, to what extent does this specific item of evidence support that proposition? If we think of inductive inference formally, this purely local examination might be sufficient. For all we need for a valid inference, according to a formal theory, is that the evidence and the supported proposition fit appropriately into the empty slots of some licit schema. This local appraisal is incomplete, however, when inductive inference is understood materially. For in this approach, there is no fixed repertoire of warranted schemas that is applicable in all domains. In their place, (true) background facts in each domain warrant the inductive inferences supported in that domain. It follows that the affirmation that some inductive inference is licit requires a further affirmation of the truth of the background fact or facts that warrant the inference. These last facts are themselves contingent and, in the fullest account, must also be secured inductively with appropriate evidence.

Thus, when understood materially, the cogency of inductive inferences and relations of inductive support cannot be appraised fully in isolation. They must be appraised within the context of a larger ecology of relations of inductive support. This book investigates how that larger ecology is configured. This chapter lays the foundation of the material analysis of this large-scale structure. It consists of the following four claims. They will be introduced and defended in this chapter:

1. Relations of inductive support have a non-hierarchical structure.

2. Hypotheses, initially without known support, are used to erect non-hierarchical structures.

3. Locally deductive relations of support can be combined to produce an inductive totality.

4. There are self-supporting inductive structures.

The defense of these four claims will employ extended examples drawn from the history of science. Providing a sufficiently detailed account of these examples within the confines of this chapter is impractical. My approach is to give these accounts in later chapters in Part II, with one chapter devoted to each of the case studies. Their results will be recalled in this chapter briefly only in so far as they are needed.

In this chapter, Section 2 argues for the first and most important of the foundational claims listed above, the non-hierarchical structure of relations of inductive support. It addresses a supposition that relations of inductive support in science or in individual sciences are unidirectional, always proceeding from the less to the more general. Under this supposition, these relations of support are akin the relations of support among the successive courses of stones in a tower. Each course is supported only by those beneath it. In its place is a conception of greatly tangled relations of support that cross over one another, failing to respect any orderly hierarchy. They are akin to the relations of support in an arch or vaulted ceiling. Each stone is supported by those beneath it and many others, above it and elsewhere distributed over the whole structure. That relations of inductive support form such a massively entangled system is the most prominent feature of the large-scale structure of relations of inductive support according to the material theory. Many further features will depend upon it.

Section 3 asks how these entangled structures can be discovered. A central result of the material theory is that we need first to know something before we can infer inductively. For otherwise we have no secure warranting facts for inductive inferences. If we initially know nothing in some domain, how can we ever learn inductively generalities of infinite scope in the domain? An examination of episodes of scientific discovery gives the answer of the second claim: we proceed by hypothesis. That is, we introduce as hypotheses the facts that would be needed to warrant suitable inductive inferences; and then we make the inferences. In proceeding this way, however, we take on the obligation eventually to return to the hypotheses and provide independent support for them. Only then are our inductive inferences properly secured. The arches or vaulted ceilings of the analogy cannot be constructed simply by piling one stone upon another. To build them, we prop up some stones provisionally by scaffolding, complete the construction and only then can the scaffolding be removed to reveal a structure, all of whose stones are properly supported by masonry. This use of hypotheses is distinct from their use in hypothetico-deductive confirmation. There, they are introduced in order to be confirmed themselves. Here they are introduced to mediate in the confirmation of other propositions.

Section 4 analyzes the intriguing possibility asserted in the third claim that is found repeatedly realized in cases of inductive support in science. In many, the component relations among propositions are individually deductive, even though their combined import is inductive. The section will recall some examples that show how combinations of deductive relations among propositions can, overall, have inductive import.

As a prelude to discussion of the fourth claim, Section 5 characterizes a mature science as inductively rigid. That means that each proposition of the mature science enjoys strong inductive support from the evidence, such that the evidence admits no alternatives. Section 6 develops the fourth claim of a self-supporting inductive structure. It is one in which each proposition is supported inductively by evidence in the structure through warranting propositions also in the structure. It is then argued that the inductive rigidity of a mature science entails that it is a self-supporting inductive structure. For, according to the material theory of induction, the inductive inferences of a mature science are warranted by further propositions, whose factual truth must in turn be supported. If all these facts are collected, we have a self-supporting inductive structure. A concluding subsection considers non-empirical conditions that might be supposed to have a role in establishing the inductive rigidity of a mature science. Each can succeed, it is argued, only if it is a contingent proposition, itself subject to inductive scrutiny within the science.

Section 7 provides a brief preview of what is to come.

2. Non-Hierarchical Relations of Inductive Support

Relations of inductive support have a non-hierarchical structure.

2.1 The Hierarchical Conception: The Tower

The original and simplest notion of inductive inference is the notion of generalization from instances. It is codified in the schema of enumerative induction and employed in embellished form by time-honored procedures such as Bacon's tables and Mill's methods. It promotes an oversimplified image of science as an accumulation of generalizations of successively broader scope.

Here is how it looks. In biology, we might start with the particular observations of the flora and fauna of Europe and form generalizations over them. We then expand our inductive base with particular observations of the flora and fauna of the Middle East, Africa and Asia. Generalizations concerning them are combined with the earlier generalizations concerning European flora and fauna. We then expand our inductive base even further by introducing knowledge of biological species in the Americas and then the Antipodes. New generalizations concerning them are combined with those achieved earlier to yield generalization of still greater scope.

We can find similar structures in other sciences. In physical astronomy, we note with Newton that all bodies on earth gravitate; and that all celestial bodies gravitate. We combine the two generalizations to arrive at the greater generalization that all matter gravitates. We note that our moon and the moons visible to us are near spherical, so we infer that all moons are near spherical. We infer the same for planets and then eventually for suns and stars.

The result is a stratification of the propositions of a science according to their generality. At the bottom are the least general, the particular facts, commonly conceived as facts of experience or possible experience. As we ascend the hierarchy, we pass to generalizations from them; and then generalizations from them; and so on. The generalizations of the higher layers are supported inductively by those of the lower layers. We descend in the hierarchy by making deductive inferences. They take us from generalizations, higher in the hierarchy, to those lower.

This hierarchy is analogous to the structural support relations among stones in a tower, shown in Figure 1. The first course of stones sits on firm ground. It supports the next course of stones, which supports the one above it; and so on to the top of the tower. The firm ground is analogous to experience. It supports the simplest propositions of experience, which are commonly conceived as propositions about particulars. Each course of stones structurally supports those above it, just as generalizations lower in the hierarchy inductively support those higher up.

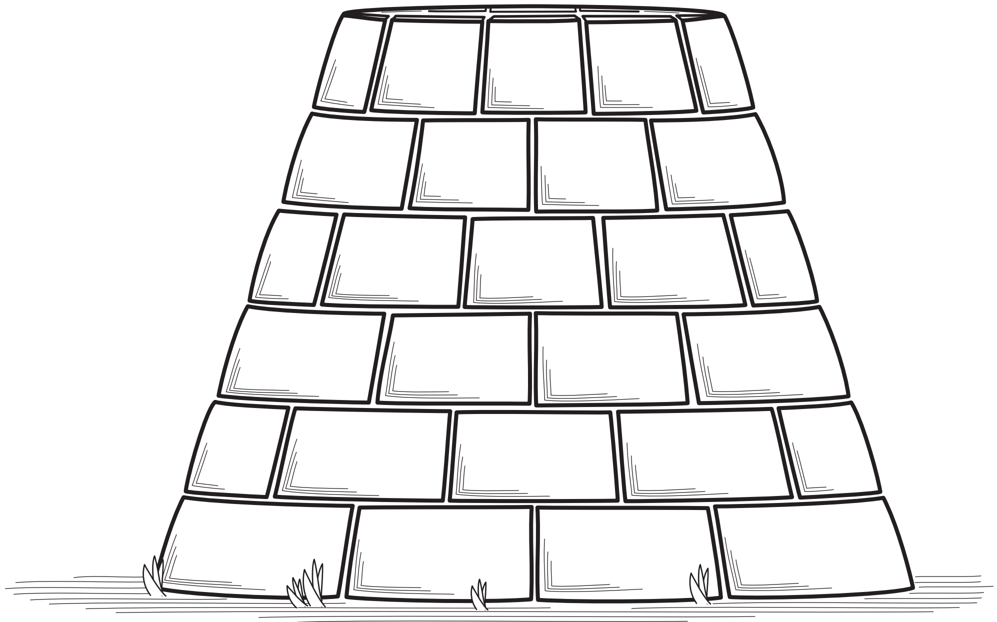

Figure 1. A Tower

While a hierarchical structure of this sort sometimes appears in science, overall it is a poor representation of the organization of propositions in science and the inductive relations among them. It fails for at least two reasons. First (to be developed in Section 2.2), contrary to the tacit supposition, relations of inductive support do not respect the hierarchy of generality. Second (to be developed in Section 2.3), the propositions of science are sufficiently varied in content that their strict partitioning and ordering by generality is unsustainable.

2.2 Relations of Inductive Support do not Respect the Hierarchy

The hierarchical presumption is that relations of inductive support are unidirectional: they proceed from the less to the more general. A closer examination of the relations of inductive support within a science shows that this unidirectionality is not respected. Relations of support typically cross over one another. Speaking now only loosely of comparisons of greater and lesser generality, propositions that at one level of generality can be supported by a combination of propositions of lesser, equal or greater generality. The relations are commonly so tangled that no simple ordering of their direction by generality amongst the propositions of a science is possible.

We shall see more examples below of this lack of respect in this chapter. It is worth pausing here to visit an especially striking example. It is provided in the Chapter 7, "The Recession of the Nebulae." In 1929, Edwin Hubble announced the result that would become the observational foundation of modern cosmological models. Nebulae[1] recede from us with velocities linearly proportional to their distances. Superficially, his analysis looks like the simplest of generalizations. He reported as data the velocities of recession of individual nebulae, as inferred from red shifts in their light, and the distances to these nebulae. This is the level of lesser generality in the hierarchy. He then formed a generalization over all nebulae: their velocities of recession vary linearly with their distances. This generalization resides in a higher level of greater generality in the hierarchy.

Hubble's generalization, it would seem, proceeded as we may naively expect, unidirectionally up the hierarchy. The linear relation of velocity and distance of a finite number of particular galaxies is generalized to a linear relation among all the velocities and distances of all the galaxies. As the later chapter shows, Hubble's actual inferences were far more complicated and were quite unconstrained by this hierarchy. Most troublesome of several problems was that Hubble lacked almost half the requisite independent distance measurements. His data set reported velocities for 46 nebulae, but included independently derived distance estimates for only 24 of them. Hubble was, however, determined to include all 46 nebulae in his analysis and employed inductive stratagems of some ingenuity and complexity to proceed. In one prominent case, he *assumed* the generality of a linear relationship between the velocities and distances and used it to infer to the unknown distances. This inference mixed elements from the less general and more general levels to infer propositions in the less general level. He could then test that the inference was successful by using the inferred distances to recover the absolute magnitudes of the nebulae concerned. He checked that these inferred absolute magnitudes conformed with other nebulae of independently known absolute magnitudes.

2.3 The Hierarchy of Generalizations is Unsustainable.

The second false presumption in the hierarchical conception is that it is possible everywhere to partition and order the propositions of a science by generality. While something like this may be possible in simpler contexts, the presumed partitioning and ordering becomes

[1] Hubble's "extragalactic nebulae" or just "nebulae" are, of course, now called "galaxies."

impossible to maintain as the propositions of science become more abstract and remote from the specific propositions of observation and experiment. No simple sequence of successive generalizations takes us from the chemical reactions observed in a laboratory to the bonding theory of the complex molecules of organic chemistry; or from the observed emission spectra of gases to the quantum mechanics of the electrons of atoms; or from the motions of the planets to the curved spacetime geometry of general relativity. The inductive pathways from simpler observations and experimental results to the completed theories are sufficiently convoluted that there is no evident basis for comparisons of generality among the intermediate propositions.

For example, ordinary Newtonian mechanics in its various parts treats the distribution of stresses in bodies, the motion of terrestrial projectiles, the flow of fluids, the motions of planets and much more. How do we rank their many propositions according to their generality? Is the theory of the distribution of the many stress forces in a complicated architectural structure more general than the analysis of the few gravitational forces acting in a simple problem in orbital mechanics? Or is the latter more general since it treats not just forces but the motions they produce? In chemistry, the energy states of a single hydrogen atom are treated by quantum mechanics. Prior to its quantum treatment, the chemistry of hydrogen is treated by a simple phenomenological theory that tells us that gaseous hydrogen consists of molecules in which two hydrogen atoms bond. Is the phenomenological theory of the hydrogen molecule more general because it treats bonded hydrogen, whereas the quantum theory of individual atoms does not? Or is the quantum treatment of the hydrogen atom more general since it is part of the more advanced quantum treatment of chemical bonding in which the energy levels of the hydrogen atom play a central role? These questions, and many more like them across the sciences, admit no well-founded answers.

2.4 The Arch

There is no overall partitioning and ordering of the propositions of science by generality. Even when such local orderings appear, relations of inductive support do not respect them. Instead relations of inductive support are distributed over the propositions of science in a massively entangled network. The simplest instances of this entangled network arise in a crossing over of relations of support whenever we have properties that are highly correlated. Then a proposition concerning one property can provide support for others at what we might loosely judge to be a comparable level of generality; and those others can provide support in reverse for the original proposition. These relations of support are warranted in turn by the more general proposition of the correlation itself.

For example, stars may vary in many properties, including their effective temperatures, masses, sizes and elemental spectral lines. A class O star in the Harvard spectral classification system is a rare star type, characterized by very high effective temperature of the order of 30,000K or greater. Many other properties of stars are strongly correlated with this temperature. A class O star will also have a very large mass and a very large luminosity.

Exactly because all these properties are highly correlated and otherwise unusual, finding one of them in some new star is strong evidence for each of the others. For example, finding that a newly observed star has a very high effective temperature greater than 30,000K is strong evidence that the star is very massive. The converse holds: finding that the star is very massive is strong evidence that it has a very high effective temperature. This crossing over of evidential support can be continued for other pairings of properties of class O stars.

There is an architectural analogy to this pair of propositions, each of which provides inductive warrant for the other. It replaces the analogy to the tower. It is an arch, shown in Figure 2.

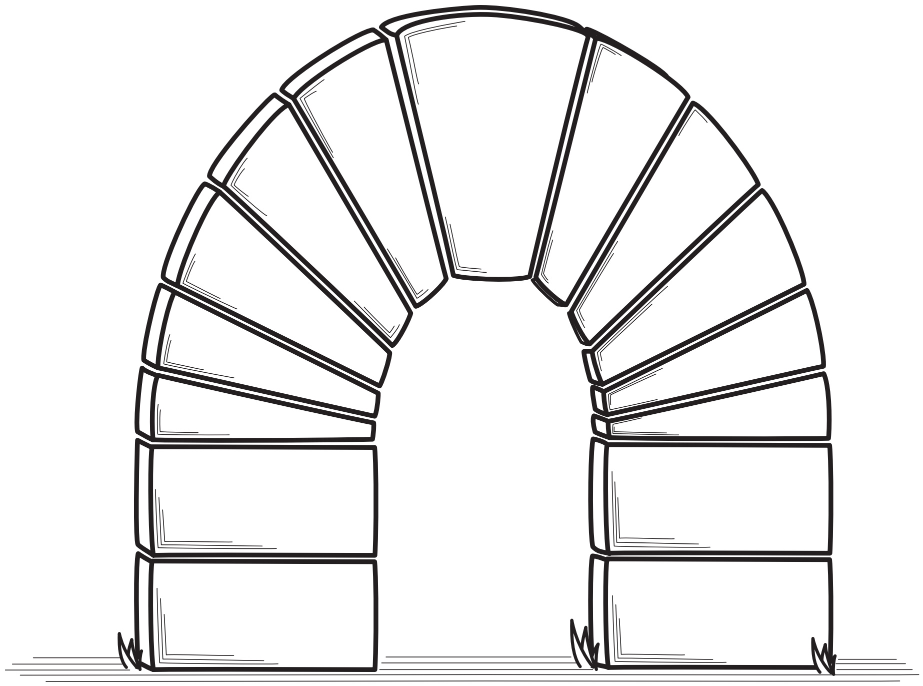

Figure 2. An Arch

Each side of the arch rests on the firm ground of experience. However none of the stones higher in the arch is merely supported by the stones beneath it. They are also supported by stones still higher in the arch and ultimately by the stones of the other side. One side of the arch, if built without the other, would simple fall down. The two sides mutually support one another.

2.5 Arches Illustrated

Later chapters describe more examples of this arch-like crossing over of relations of inductive support. Chapter 8, “Newton on Universal Gravitation,” describes two cases of pairs of propositions that mutually support each other. The first arises in Newton’s “moon test.” There he argues for the identity of the force of gravity and the celestial force that holds the moon in its orbit around the earth. The observational evidence is the observed accelerations of the moon towards the earth and falling bodies at the surface of the earth. Newton computes the acceleration the celestial force would yield if it acted at the earth’s surface, while strengthening according to an inverse square law. He finds that acceleration to match the observed acceleration of fall bodies at the earth’s surface.

Consider the proposition that the celestial force on the moon strengthens according to an inverse square law with distance. In this first inference, it is used as an inferential warrant in arriving at the identity of the celestial and terrestrial forces. This usage can be reversed. The proposition of the identity of the celestial and terrestrial forces can also be used as a warrant. Then one can infer from the observed motions that the celestial-gravitational force acting on the moon strengthens with distance according to an inverse square law.

That is, the proposition of the identity of celestial and gravitational force and the proposition of the inverse square law mutually support one another.

In a second example in Newton's account, Newton fits elliptical orbits to the observed positions of the planets. The inference from these positions to their specific elliptical orbits is warranted by the proposition that the planets are acted on by an inverse square law of gravity. Excluding perturbations, that law entails that planets move in conic sections: ellipses, hyperbolas or parabolas. However a second argument reverses the proposition that warrants the proposition supported. The key warranting fact is that the elliptical orbits are re-entrant. Each planetary year, a planet follows the same elliptical orbit. This re-entrance, Newton shows, can only arise with an inverse square law of gravity. Taken together, we find the specific elliptical orbits of the planets support the inverse square law; and the inverse square law supports the specific elliptical orbits of the planets.

Radiocarbon dating of artifacts provides another illustration of this crossing over of relations of support. It is described in Chapter 10, "Mutually Supporting Evidence in Radiocarbon Dating." In the simplest description, there are two sorts of propositions concerning the dating of artifacts. The "H" propositions date them by the traditional methods of historical analysis and archaeology. The "R" propositions date them by estimating how long was taken for their content of the radioactively unstable isotope of ^{14}C to decay to the measured levels. The R propositions depend on an accurate knowledge of the original content of ^{14}C captured in artifacts at their formation in different epochs. This knowledge is provided by H propositions: the dating of artifacts by traditional methods. Here, H propositions provide evidential support for R propositions. However the reverse can also happen. Are we sure that no error has crept into the historical methods used to arrive at a traditionally established dating? Then radiocarbon dating can reassure us or correct us. Now R propositions are providing evidential support for H propositions.

Details of more examples of mutually supporting pairs of hypotheses can be found in other chapters. The Chapter 11, "The Determination of Atomic Weights," we see how

Avogadro's hypothesis and the Law of Dulong and Petit supported each other in chemical investigations of the early nineteenth century. This same relation of mutual support later arose among the chemists' version of Avogadro's hypothesis and the physicists' version of the hypothesis within the kinetic theory of gases. In Chapter 9, "Mutually Supporting Evidence in Atomic Spectra," we find the Ritz combination principle providing support for the quantum theory. Then, later, the quantum theory provides support for a corrected version of the Ritz combination principle.

2.6 The Vaulted Ceiling

The examples above of pairs of mutually supporting propositions are exceptional for their simplicity. It is far more common for these relations of mutual support to be embedded within a much larger network of inductive relations of support in a science. The Newtonian example is not of an *isolated* structure since the various hypotheses in it figure in relations of support for other propositions in science.[2] In general, relations of support cross over one another in many different ways and at many different levels. One then finds that even a small part of science can be part of a prodigious array of relations of support connecting it with neighboring sciences and then beyond them to the farthest reaches of science.

The analogy to a single arch does not capture this richness. An analogy to a dome is a little better. Stones in each part of the dome depend for their support on stones in many other parts. A still better analogy is to a massively complicated vaulted ceiling, as shown in Figure 3. It consists of many interconnected domes and arches. The integrity of the entire structure depends on the mutual support of all its parts.

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[2] For example, an inverse square law is presumed in the computations associated with Cavendish type experiments that determine the magnitude of the gravitational constant G. The law is also used to infer that spherical planets act gravitationally as if their masses were concentrated at their centers, to infer that certain comets move on hyperbolas and compute the behavior of terrestrial tides.

Figure 3. A Vaulted Ceiling[3]

This interconnectedness of relations of inductive support provides mature science with its monolithic structure. One cannot reverse one part without destabilizing the remainder of the structure. A vivid example of an effort to reverse one part comes with the persistent creationist efforts to remove evolutionary theory from biology. The problem they face is that evolutionary theory is inductively entangled with the other sciences. In their challenge to evolutionary theory, the creationists find they need to impugn the great age of the earth in favor of a much younger earth, whose age is determined from biblical scholarship. Hence, they must impugn modern uniformitarian geology. It is based on an old earth whose major geological features were formed slowly over eons. They must impugn the radiological methods used to date both organic artifacts and rocks, which will ultimately lead to conflicts with radiochemistry. They must also dispute standard cosmology since it also calls for an ancient earth. This then forces them to question observational and theoretical astronomy and the physics on which it depends.

The size of the network of support relations in mature sciences leads to a combinatorial explosion in the number of support relations that directly or indirectly bear upon the propositions

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[3] Image: John D. Norton, Commons Room, Cathedral of Learning, University of Pittsburgh

of the component sciences. This effect gives depth to the inductive security of each part. A fully worked out example would help us to see this security more clearly. Unfortunately, displaying the complexity of such a network in all its detail is an immense task too large for this chapter or this book. However, we can get a good sense of the density and richness of these structures by visiting just small pieces of it in the examples developed in the chapters that follow.

2.7 Vaulted Ceilings Illustrated

Chapter 11, "The Determination of Atomic Weights," recounts the immense difficulties faced by the chemists in the early nineteenth century in determining relative weights of atoms. The problem had arisen in Dalton's *New System of Chemical Philosophy* of 1808 and 1810. He knew, for example, that 8 grams of oxygen combines with one gram of hydrogen to make water. To infer from this that the molecular formula of water is H_2O, Dalton would need to know that an oxygen atom is 16 times as massive as a hydrogen atom. Dalton had no table of atomic weights to consult and no way to determine them, so he just assumed that the ratio was eight to one. The result was that, famously, he arrived at the molecular formula for water of HO. Dalton was trapped in a circularity: to know the correct molecular formulae, he needed to know the relative weights of atoms; but he could only learn the relative weights of atoms from the molecular formulae.

One might imagine that this circularity was easily broken. It was not. The task required the efforts of chemists over roughly a half century. The chapter recounts Cannizzaro's celebrated solution, which he circulated at the 1860 Karlsruhe conference of chemists. He relied on Avogadro's hypothesis, the law of Dulong and Petit and an extensive set of measurements of the physical properties of a wide range of substances to determine their molecular formulae. The determinations were quite complicated and I have done my best to present them in Chapter 11. For our purposes here, the key fact was that the molecular formulae were not just determined but overdetermined. That meant that some subset of them could be used to provide inductive support from some other part; and vice versa.

For example, once Cannizzaro had determined that hydrogen and oxygen gases are diatomic, H_2 and O_2, his gas density data enabled him to fix the molecular formula for water as H_2O. Or, he could start with this molecular formula for water and find that oxygen and hydrogen are diatomic. This is just a glimpse of a massive tangle of relations of inductive support in

Cannizzaro's analysis. For example, that hydrogen gas is diatomic entered into similar overdetermined relations of support concerning compounds of the halogens: chlorine, bromine and iodine.

Chapter 9, "Mutually Supporting Evidence in Atomic Spectra," provides another illustration of this sort of tangle of relations of inductive support. Energetically excited hydrogen gas emits light. It emits only very specific frequencies of light whose measurement became an important project for spectroscopists in the late nineteenth and early twentieth centuries. Those frequencies divided into well-structured sets of lines, found in different parts of the electromagnetic spectrum: the infrared, the visible and the ultraviolet. These sets or "series" were named after the spectroscopists who measured them: the Lyman, Balmer, Paschen, Brackett and Pfund series.

The series were connected by a simple arithmetic relationship first noted by Rydberg but exploited by Ritz in 1908 as his "principle of combination." The key fact was that the lines of all the series could be generated by taking the arithmetic differences of a set of terms. For Ritz, this fact provided a useful heuristic. He could apply his combination principle to the lines of a known series and predict a new, hitherto unobserved series. The approach proved successful and, immediately, Ritz could report a new line conforming with his prediction.

For our purposes, what is important is that full set of lines in all these series is overdetermined, once one adopts Ritz's principle. That means that one can take the lines of one series and, from them, infer to the existence another series. What results is a tangle of relations of inductive support. This structure is, fortunately, much easier to comprehend, as the chapter shows, since it is recoverable by simple arithmetic additions and subtractions.

2.8 The Firm Ground of Experience

In the arch and vaulted ceiling analogy, the ground that supports the masonry corresponds to the empirical basis of the science. This basis does not depend on any, simple-minded, strict distinction between observational and theoretical propositions, for I follow the now common view that a clear distinction between them cannot be made. Rather I mean by it what is commonly taken in a present science as its supporting empirical facts. These can be very far removed from direct human observations.

For example, one of the most stable and most important observational facts supporting modern cosmology is that space is filled with a 2.7K background of thermal radiation. This simple sounding fact was only secured over decades after extraordinary efforts, some of which are recounting in Chapter 9, "Inference to the Best Explanation: Examples," of *The Material Theory of Induction*. Among the difficulties faced, to establish a thermal character in a radiation field, one must have measurements made at many different frequencies. Only then can the energy distribution characteristic of thermal radiation be established.

A related observational fact of modern cosmology is that galaxies are observed to recede from us with a velocity that increases linearly with distance. While the observation is now routinely reported without much hesitation in modern treatments, it was subject to a searching critique in the later 20th century by Halton Arp. He argued that the red shift in light from the galaxies could not be interpreted as resulting from a velocity of recession since objects with very different red shifts appeared to be connected spatially. A quite extensive debate was needed to refute his hesitations. For details, see Norton (manuscript).

The analysis of just what might be meant by the empirical facts of a science is a project that goes beyond present concerns. My view is that Nora Boyd's (2018, 2018a) analysis provides the best, modern treatment. She allows that all such empirical facts are entangled with theory However, she argues, these facts can still be used to decide among competing theories through a process of winding back to the provenance of the facts. When we seek to use some empirical fact to decide between two theories, we wind back through the various stages of the formation of the fact. If sufficient data has been preserved, we eventually come to a point at which enough of the theoretical encumbrance has been removed for the fact to provide a neutral basis of comparison for the two theories.

3. The Role of Hypotheses in Discovery of Inductive Relations of Support

Hypotheses, initially without known support, are used to erect non-hierarchical structures

3.1 The Discovery Problem

The discussion of the last section concerns relations of inductive support, independent of human knowledge of them. A further question of great importance is how we can learn these relations. For only then do they assist us in our inductive exploration of the world. If the totality of facts connected by relations of inductive support were delivered to us as a completed whole, it would be a straightforward matter to check that all the requisite relations of inductive support obtain. This is a science fiction scenario. It is what would happen were we to stumble onto a copy of the fictional *Encyclopedia Galactica* of some advanced alien civilization. In it, entire sciences hitherto unknown to us would be delivered to us in their totality.

In real life, our explorations proceed more haltingly. The guiding rule of the material theory of induction is: "You must already know something to be able to infer inductively." For we cannot know that some inductive inference is licit unless we are assured of the truth of the warranting fact. Yet if we are in the early stages of investigation in some new field, we commonly know rather little and it is likely too little to proceed with assured inductive inferences of any great reach.

This is a problem faced by all new sciences. The strategy that has been used almost universally is to proceed provisionally. We may not know which are the general facts of some domain, but we can sometime determine which propositions are plausible candidates for the facts that would warrant the inductive inferences sought. To use a familiar term, these plausible propositions are "hypotheses." We can then proceed provisionally under the supposition that our hypothesis is a fact and infer to the propositions it would warrant, were it a fact. The key element is that the supposition is provisional. Conclusions drawn or inductively supported using it themselves have provisional status only. They will remain so until we find inductive support for the warranting hypothesis. We have incurred an inductive debt in proceeding to the conclusions and they are properly secured only when that inductive debt is discharged by finding support for the warranting hypothesis.

Hypotheses have a natural analog in the procedures for building arches, domes and vaulted ceilings. A masonry arch, dome or vaulted ceiling cannot be built simply by piling stones, one upon another. For as soon as a few stones have been placed, the highest ones would be without adequate support and would fall. The standard procedure is to use scaffolding, known technically as "centring." As shown in Figure 4, it consists traditionally of a wooden framework.

The stones are set on top of the framework. Prior to the completion of an arch, these stones are not properly supported by the other stones of the arch. Their support is only provisional, since the wooden centring will eventually be removed. Here they are analogous to hypotheses whose support is also only provisional. When all the stones of the arch have been placed, the centring can be removed. For now the remaining stones of the arch fully support each other. This final stage of construction is analogous to the discharging of the evidential debt taken by introducing the hypothesis. As the full investigation is completed, further inductive support, anchored eventually in experience, is provided for it.

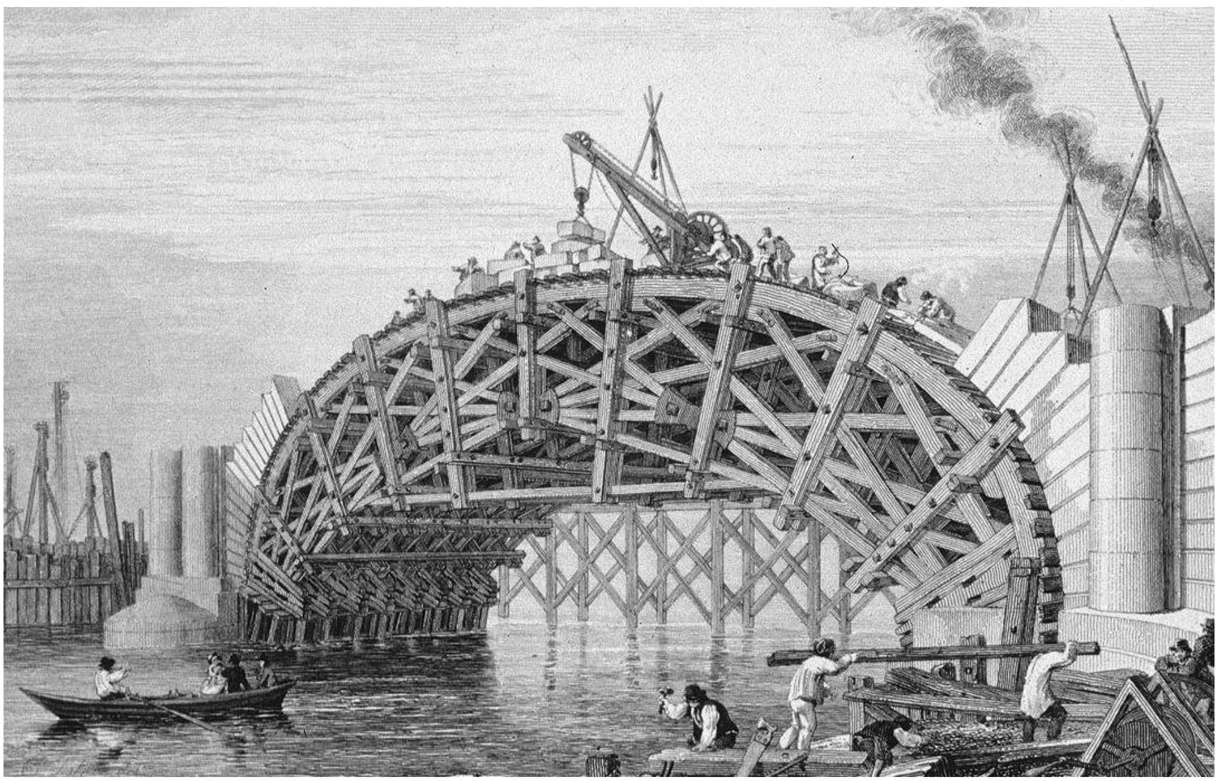

Figure 4. Wooden Centring used in the Construction of the Waterloo Bridge

3.2 Hypotheses Illustrated

The chapters that follow provide illustrations of this use of hypotheses. In several of them, the use of hypotheses is invited by a specific problem. Scientists find themselves trapped in an evidential circle. Commonly there are two related quantities to be determined. To find one, the scientists need to know the second. But, it seems initially, that they cannot know the second unless they already know the first. They are trapped. A suitably chosen hypothesis is used routinely to break the circle.

Chapter 12, “The Use of Hypotheses in Determining Distances in Our Planetary System,” is an extended study of this use of hypotheses. Consider the earliest efforts to determine distances to celestial bodies. The moon subtends an angle of about 1/2 degree in our visual field. If we knew the diameter of the moon, simple geometry would then let us compute the distance to the moon. However, we do not know the diameter of the moon precisely because we do not know how far distant it is from us. Determining its distance and diameter forms the troublesome evidential circle. The sun also subtends an angle of about 1/2 degree in our visual field. Determining its distance from us is blocked by the same evidential circle. Determining distances to the planets is even harder since naked eye astronomy cannot resolve their disks. They are just points of light in the sky.

The chapter recounts how ancient and later astronomers sought to break out of this evidential circle by ingenious geometrical triangulations, or, as it is known in the astronomical context, measuring parallax. These efforts met with limited success. Ancient astronomers were unable to measure the tiny parallactic angles accurately enough. In the seventeenth century, using telescopic aids, a fairly good parallactic measurement of the distance to Mars was achieved. However, even with telescopic aids, *direct* parallactic measurements of the key earth-sun distance were not achieved as late as the nineteenth century.

From the outset, to fill the gaps, hypotheses were called into service. They were not used to fix the distances directly, but only to provide hypothetical estimates of the ratios of the distances. Then all that was needed was a single distance determination, such as the distance to the moon or to Mars, and the remaining distances could be computed from the ratios. What makes this case study revealing is that, in addition to a success story, it recounts failures. They arose when independent evidential support could not be secured for the hypotheses and they were eventually rejected. The chapter recounts three attempts.

The earliest were Pythagorean/Platonic proposals that recovered the ratios from musical harmonies and simple arithmetic relations. A later proposal was incorporated into Ptolemy’s geocentric cosmology. He proposed a plausible distance ordering for the celestial bodies and recovered the ratios of their distances from the further hypothesis that were packed together as closely as the geometry of his system allowed. Neither Pythagorean nor Ptolemaic proposals were able to secure independent evidence. Their inductive debt was not discharged and they were abandoned.

They were replaced by the Copernican, heliocentric hypothesis. Through it, the ratios of the planetary orbital distances were readily recoverable from terrestrial measurements. Unlike the earlier systems, the Copernican hypotheses gained evidential support both from within and without. Most important was its conformity with Newton's mechanics. Newton had used the more fully developed heliocentric astronomy of his time as an essential premise of his argument for universal gravitation. In another example of the crossing over of relations of inductive support, the direction of inductive support was reversed. Newton's mechanics soon became strong evidence for the details of Copernican astronomy.[4]

The dependence of solar system distance measurements on the heliocentric theory persisted. The most accurate estimates of the key earth-sun distance in the eighteenth and nineteenth centuries came from careful measurements of the transits of Venus across the face of the sun. The earth-sun distance could then be recovered from them by geometric triangulations. These calculations still relied upon the heliocentric theory's determination of the ratios of the orbits of the earth and Venus.

Further illustrations of the use of hypotheses to break evidential impasses have already appeared in examples earlier in this chapter. We saw how Dalton was trapped in an evidential circle concerning atomic weights and molecular formulae. It was broken through two hypotheses: Avogadro's hypothesis and the law of Dulong and Petit. The evidential debt incurred in supposing them was eventually discharged through the mutual support of these two hypotheses and the support provided for them from the emergence of the statistical mechanical treatment of gases in physics.

We also saw that Hubble was stymied in his efforts to use the data from all 46 nebulae for which he had measurements by a lack of independent distance measurement for 22 of them. Chapter 7, "The Recession of the Nebulae," recounts how Hubble was still able to incorporate

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[4] The inversion in this relationship is seen most clearly in the ability of the Newtonian system to provide corrections to the heliocentric astronomy of Newton's time. The planets do not orbit in ellipses but in precessing ellipses. What came to be known as Kepler's third harmonic law was corrected to accommodate the finite mass of the sun.

these 22 nebulae in his analysis by means of hypotheses that gave him indirect indications of their distances. At various stages of his analysis he hypothesized that the linear relationship among the other 24 nebulae held also for these 22; that the absolute magnitude of the brightest star in each nebula is the same; and that the range of absolute magnitudes of nebulae in a cluster is confined to a small range common to all nebulae.

In the early twentieth century analysis of atomic spectra, we saw how the discovery of new series was advanced by the Ritz combination principle. It was introduced as an hypothesis. It gained the requisite independent evidential support with the emergence of modern quantum theory, where it was recovered as a consequence of Bohr's atomic theory.

These last illustrations have been mostly of successes. This happy outcome is not assured. A prominent example of a failure is provided by the steady state cosmology of the mid twentieth century. It was based on the hypothesis of the "perfect cosmological principle," which was first advanced by Bondi and Gold (1948). According to it, the universe is homogeneous on the large scale, not just spatially but over time as well. The way we see the universe now, on the large scale, is the way it has always been and will always be. A quite definite cosmology now follows. Most striking of its features is the continuous creation of matter. For unless matter is continually created throughout space, the expansion of the universe would lead to a dilution of its average matter density and violate the perfect cosmological principle. The steady state cosmologists took on a quite massive evidential debt in hypothesizing the perfect cosmological principle. They were never able to establish independent evidence for the hypothesis; they were never able to repay the debt. Most notable was the failure of the steady state theorists to accommodate Penzias and Wilson's 1965 discovery of the cosmic background radiation, while the competing "big bang" or "primeval fireball" hypothesis eventually proved to accommodate it handily.[5]

3.3 This is NOT Hypothetico-Deductive Confirmation

This use of hypotheses may appear similar to the hypothetico-deductive approach to confirmation. These are accounts of confirmation based on the principle that an hypothesis is

[5] For a brief account of this last competition, see Chapter 9 "Inference to the Best Explanation: Examples" in *The Material Theory of Induction*.

inductively supported when it successfully entails true evidence deductively.[6] The essential difference lies in the goal of introducing the hypotheses into an evidential analysis. In hypothetico-deductive confirmation, hypotheses are introduced so that the evidence can confirm them according to the hypothetico-deductive principle. In the applications within the material theory, hypotheses are introduced to mediate in the confirmation of *other* propositions. The confirmation of the hypothesis is a task reserved for later investigations. The hypothesis is expected to be confirmed not hypothetico-deductively, but by other inductive inferences with their own material warranting facts.

4. Deductive Inferences in Inductive Structures

Locally deductive relations of support can be combined to produce an inductive totality.

4.1 Inferences that Are or Are Nearly Deductive

There is a striking feature of many of the inferences in this text and in the earlier text, *The Material Theory of Induction*. While the inferences contribute to relations of inductive support, many of them are close to being deductive inferences or may actually be deductive inferences. That is, when combined with the warranting fact, the inference *from* the evidence *to* the conclusion to be supported is often deductive. The direction of the inference here is important. It is not merely the deductive inferences of hypothetico-deductive support. For in the latter, the deduction passes from the hypothesis or theory to the evidence. That direction has now been reversed.

Here are some examples. Chapter one of *The Material Theory of Induction* recalled Curie's inference from the crystallographic properties of the few samples of radium chloride at her disposal. She inferred to the generality of these crystallographic properties. I identified the warrant for her inference as:

> (Weakened Haüy's Principle) *Generally*, each crystalline substance has a single characteristic crystallographic form.

[6] For an elaboration on this principle and the extensive problems associated with it, see Norton (2005).

When this weakened principle is used to warrant Curie's inference, it is the qualification "Generally" that makes the inference inductive. For it accommodates the possibility of polymorphism, that one crystalline substance may manifest in more than one crystallographic form. The inductive risk taken by Curie is quite small, especially if we assume that her generalization was tacitly limited to crystals of radium chloride prepared under conditions comparable to those in her laboratory.[7] If we drop this qualification and revert to Haüy's original conception, the warranting fact would be:

> (Haüy's Principle) Each crystalline substance has a single characteristic crystallographic form.

Under this warrant, Curie's inference would be a deduction.

Chapter two of *The Material Theory of Induction* recounted Galileo's inference concerning his law of fall. He had found that, in equal time intervals, a body in free fall successively covers distances in the ratios of 1 to 3 to 5 to 7. He generalized this sequence of ratios to the sequence of odd numbers. In this inference, I argued Galileo had used the warranting fact that the ratios of 1 to 3 to 5 to 7 were present no matter the time interval used in measurement. It then followed, deductively, that the only possible general law was of the sequence of odd numbers. Indeed the deductive inference needs as a premise only the ratio of 1 to 3 and its invariance under a change of the unit of time.

There are, it turns out, other well-recognized, historically important examples in which the inference from evidence to our theories is deductive. These cases have been codified as "demonstrative inductions." Their inferences are demonstrative in the sense that they are deductions. However they are called "inductions" to reflect an older usage of the term as referring to inferences from particulars to generalities. My contribution to this literature in Norton (1993) was to trace how quantum discontinuity was established in the early decades of the twentieth century. The essential datum was Planck's 1900 formula for the distribution of energy over the different frequencies of black body radiation. In the early analysis, it was shown that assuming discontinuities in energies enabled one to deduce the Planck formula. Poincaré and Ehrenfest soon showed that the direction of deduction could be reversed. With suitable

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[7] I thank Pat Corvini for emphasizing this point to me.

background facts, it was possible to deduce quantum discontinuity from the evidence of the Planck formula.

4.2 Support that is Locally Deductive, but Globally Inductive

In deductive inferences, the conclusions are at best logically equivalent deductively to the premises or logically weaker than them. So it appears that deductive or near deductive inferences to our conclusions cannot give what we seek from inductive investigations. We seek an expansion of our knowledge. These deductive inferences are merely rearranging and returning to us all or part of what we have already supposed.

This pessimistic expectation is not realized, however, once we recall that relations of support within inductive structures are not hierarchical but massively entangled. That enables the entangled relations of deductive support to combine to provide inductive support in the overall structure. This circumstance arises when we have sets of propositions that mutually support each other, deductively. Nonetheless, accepting the totality is to accept propositions logically stronger than the evidence.

Striking examples of this combination of deductions arise in Newton's arguments for universal gravitation and his inverse square law of gravity. They have already been sketched above and a more detailed exposition is provided in Chapter 8, "Newton on Universal Gravitation." To recall, the first example arises in Newton's "moon test." In it, he shows that terrestrial gravity is the same force as the celestial force holding the moon in its orbit around the earth. To show it, Newton reckoned that, if the force acting on the moon strengthens with the inverse square of distance as the earth is approached, it would accelerate terrestrial bodies with just the accelerations actually found at the earth's surface. The logic of the moon test involves two hypotheses:

$H_{\text{inv. square}}$: The celestial force acting on the moon is strengthened by an inverse square law with distance at the earth's surface.

H_{identity}: Terrestrial gravitation and the lunar celestial force are the same.

In the context of Newton's moon test, drawing on the evidence of the accelerations of the moon and terrestrial bodies in free fall towards the earth, each of these hypotheses can be deduced from the other. That is, each hypothesis provides a warrant for a deductive inference from the evidence to the other hypothesis. The two hypotheses combined are the result of the moon test analysis. Their conjunction is inductively supported by the evidence of lunar and terrestrial accelerations.

The second example has a similar structure. The most basic results of Newton's celestial mechanics reside in two hypotheses:

$H_{ellipses}$: The planets move in their specific elliptical orbits.

$H_{inv.\ square}$: The Planets are attracted to the sun by a force that varies with the inverse square of distance.

Against the background of the observed positions of the planets and the laws of Newton's mechanics, each hypothesis could be deduced from the other. Indeed Newton employed a quite subtle variant of the usual way of inferring among these two hypotheses. In the case of the near circular orbits of the planets, he needed only the datum that the planetary orbits are re-entrant. That is, in a planetary year, each planet returns to its starting point. He could then show that this re-entrance was a sensitive test for deviations from the inverse square law. The observed exactness of the re-entrance entailed the exactness of the inverse square law. Once again, the overall inductive import of the analysis was that the evidence of the observed positions of the planets supported inductively the conjunction of the two hypotheses.

Chapter 9, "Mutually Supporting Evidence in Atomic Spectra," provides another example with a similar structure. It was noted above that the Ritz combination principle enables inferences of support among the different series of the hydrogen spectrum. As the chapter details, these inferences are deductive. Using the Ritz combination principle as a premise, from the Balmer series, we can deduce the Paschen, Bracket and Pfund series. These deductions can be reversed as well. Adding the premise of only a single line from the Balmer series, we can deduce the entire, infinite Balmer series from the Paschen series. There are infinitely many series in the hydrogen spectrum, although only finitely many have been observed. The series are closely connected by further deductive relations such that we can infer deductively from any series to any other by means of the Ritz combination principle and, if needed, the additional premise of a finite set of suitably selected lines. While these interrelations are deductive, the final import is inductive. The Ritz combination principle and the finitely many spectral lines observed provide inductive support for the entire system of infinitely many series, each with infinitely many lines.

There might be, for some, an air of paradox in the idea that we can combine deductive relations to yield a structure with inductive import. That impression is mistaken. These cases are actually more secure inductively than many considered in earlier sections. In those earlier cases, inductive relations of support are combined to produce structures with overall inductive import. Inductive risk is introduced in both the component relations of inductive support and in the combined structure. If those component relations of support are deductive, this first source of inductive risk is eliminated.

5 The Maturity of a Science

5.1 Inductive Rigidity

A preparation for the discussion of the fourth and final claim is the characterization of what constitutes mature sciences. They are characterized by an inductive rigidity. That is, each proposition of the science is well-supported evidentially, so that a change in the proposition is not allowed by the evidence for the science. There is no assurance that a science can achieve maturity. In the early stages of the development of a science, important propositions are entertained hypothetically. They are not fixed rigidly. As the development continues, further relations of inductive support are found, the hypotheses gain evidential support and their provisional status is discharged. If this process proceeds to completion, the science achieves maturity such that each of its propositions is well-supported.

Once this maturity is achieved, the inductive rigidity of a mature science is widely recognized amongst its practitioners. Challenges to the science are treated as tiresome, moribund exercises. A skeptic may doubt some proposition in a mature science. In response, someone competent in the science would be able to display the evidence that supports the proposition. In the case of special relativity, this is a dialog with which I have some personal experience. The theory has been routinely challenged by critics since its inception over a century ago. Many of its foundational propositions have, at one time or another, been disputed, unsuccessfully. The light postulate of the theory asserts that all inertially moving observers find the same speed "*c*" for light in vacuo. It is initially a puzzling postulate. Imagine an inertially moving observer who is chasing at high speed after a light signal that moves at *c*. That observer will not find the light signal slowed from *c*, even in the slightest. This perplexing result makes the postulate a favored target. However, that postulate has direct support from de Sitter's 1913 analysis of light emitted from distant double stars. Its deeper support derives from the Lorentz covariance recoverable from Maxwell's electrodynamics. That dynamics is in turn supported by a plethora of individual experiments in electricity and magnetism.[8]

This maturity is a goal that proponents of a theory strive to achieve; and standard text-book sciences commonly come very close to achieving it. It is not uncommon, however, for the full achievement of the goal to be incomplete in parts of the theory. There, propositions may achieve general acceptance while lacking proper support. The falsification of such a proposition is usually associated with great excitement and even a momentary sense of crisis. However, precisely because the falsified propositions never were strongly supported, their failure can be absorbed into theory.

On September 19, 1957, Francis Crick announced what came to be called the "central dogma" of molecular biology. It speaks, in various forms of a unidirectional synthesis pathway within cells from DNA to RNA to proteins. The reverse pathway is prohibited. While the dogma was widely adopted, there was little real evidence for it. It was a simple and comfortable idea that fitted with a denial of the Lamarckian inheritance of acquired characteristics.[9] When it was discovered that certain viruses could implement the reversed pathway from RNA to DNA, the result was readily incorporated into molecular biology. *Nature* (Anon., 1970) published an excited editorial "Central Dogma Reversed."

In the course of the twentieth century, many new particles were discovered. It was routinely assumed that the laws governing them would respect parity. That is, they would not distinguish left from right. In retrospect, there was no good evidence for this assumption other than it had become routine in the physical laws discovered earlier. Then, in 1964, Cronin and Fitch discovered experimentally that the weak interaction in particle physics can violate charge-parity conservation. In another example, the hard-to-detect neutrinos had long been attributed a zero rest mass. This had seemed a reasonable assumption. The early determinations of the neutrino rest mass pointed to a quantity that was in the neighborhood of zero. However, as neutrino physics developed, it became clear that a very small mass had to be attributed to neutrinos. That would enable the process of neutrino oscillation in which neutrinos migrate over the three different flavors in which neutrinos manifest. This oscillation explained experimental

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[8] For historical details, see Norton (2014).

[9] Here I rely on Cobb (2017).

and observational anomalies, most notably a dearth of measured electron neutrinos emitted by the sun. (For a review, see Gonzalez-Garcia, 2003.)

In these last cases, anomalous evidence could be absorbed into the existing theories since the propositions that they contradicted lacked the strength of evidential support of other parts of the theory. Had these other better-supported parts been contradicted, the outcome would have been more troublesome. For a well-supported proposition is tightly bound with so much more of the theory. Should it fail, it will bring down much more of the theory with it. While particle physics could absorb non-zero neutrino masses, matters would have been quite different had the OPERA Collaboration (2011) measurement proved correct. Their measurements, they announced, appeared to show that neutrinos were propagating faster than light. Their correctness would have destabilized particle physics. It would have contradicted a fundamental posit of the governing quantum field theory, the locality of quantum field operators. Particle physics was saved, for now.

The inductive rigidity of a mature science does not make the science incorrigible. It is simply a statement of the best that can be gleaned from the evidence. No matter how strong the inductive support of a science, some inductive risk is associated with it. When incontrovertible evidence does emerge that contradicts a well-supported proposition within a mature theory, the result can and usually is a breakdown of the theory. Rigid steel beams have some elasticity, but they will snap if over extended. What ensues is a revolution in science, such as has been a popular topic of investigation in history of science.

These revolutions commonly occur when the science is extended beyond domains in which it was first developed and in which its evidential base is found. Newton's seventeenth century mechanics was developed on an evidential base of slow-moving objects, such as falling stones and orbiting planets. Special relativity emerged when developments in nineteenth century electrodynamics gave reliable results concerning much faster propagations at the speed of light. Special relativity, in turn, fails when we move to domains of intense gravitation, as Einstein found through his general theory of relativity. All these superseded theories, however, remain evidentially well-supported as long as we consider only the evidence of the domains for which they were devised. While general relativity and relativistic cosmology now tells us that Euclidean geometry may fail when applied to spaces of cosmic extent, Pythagoras' ancient theorem remains as reliable as it ever was for the builders of houses, castles and skyscrapers.

5.2 A Distributed Vindication

While the inductive rigidity of a mature science is a commonplace for its practitioners, its demonstration would be a massive task. The network of interrelated propositions is enormous for any real science. A full display of the evidence and inductive relations supporting each goes well beyond what is possible in a book chapter. Indeed, for a well-developed science of great scope, displaying this rigidity in all detail would likely be beyond the capacities of any single author. Rather, the requisite knowledge, while likely not fully known to any one scientist, is distributed over the full community.

This distribution is illustrated by our proper confidence in the laws of conservation of energy and momentum; and our expectation that no proposal for a perpetual motion machine can succeed. Given the variety of types of proposals that have been advanced over the centuries, a full inventory of the evidence against them would be prohibitively long. In each case, it is not enough merely to assert generically that the conservation of energy and momentum prohibits the operation of the machine. A full analysis requires us to display where the details of the mechanism proposed conflicts with other propositions in established science.[10] Different proposals will call on expertises in the different sciences in which the proposals are formulated. We can be confident however that, for each new proposal, there is an expert in the community familiar with the pertinent science and able to respond.

A recent illustration is the "EmDrive" proposal for spaceship propulsion that was brought to the attention of a larger scientific community by a *New Scientist* article of 2006 (Mullins, 2006). It consists of microwaves in a chamber such that, it is proposed, the forces exerted by the microwaves in many directions on the chamber walls do not entirely cancel out. They leave a small net force that can propel the chamber. In this, it is unlike any other propulsion scheme known. For all known schemes produce propulsion by driving some form of matter in the opposite direction to the thrust sought. A rocket expels hot gases. An airplane projects a current of air or hot gases behind it. A ship's propeller projects a stream of water behind it. The forward force on the rocket, airplane or ship is balanced by an equal and opposite, reaction force on the driven matter, as required by Newton's third law of motion. This driven matter carries rearward

[10] For a history of these proposals, see Ord-Hume (1977).

momentum. The conservation of momentum then assures us that the rocket, airplane or ship gains forward momentum in the opposite direction. That is what accelerates it.

The EmDrive violates the conservation of momentum. It is a closed device that is supposed to set itself into motion, without ejected matter or a reaction force. While the proposal is *prima facie* extremely implausible, interest in it has proven remarkably stable and is matched only by the tenacity of skeptical critics. Part of the positive interest lies in wishful thinking. If it works, it is a device that could power starships! Another reason for its endurance lies in the small magnitude of the force predicted. Detecting it requires the most delicate experiments. As critics have pointed out, such experiments can easily produce spurious results, if all confounding effects[11] are not properly controlled.

The resulting literature here is too extensive to survey. Recounting one exchange, however, is sufficient to illustrate how the distribution of expertise works. Harold White and his collaborators of the NASA Johnson Space Center are proponents of these microwave propulsion systems. In a technical paper, White and March (2012) proposed that the reactionless thrust might arise through the Casimir force of the quantum vacuum. This is specialized physics. As White and March acknowledge in their introductory paragraph, classical electrodynamics precludes a reactionless force. Indeed, that classical electrodynamics conserves momentum is a result readily accessible to anyone with a serious, college level course in electrodynamics. The Casimir effect, however, is more arcane. It is a force produced by quantum fields in a vacuum. Its basic mechanism is not so obscure. However, it is more demanding to develop a theoretical analysis of it that would securely preclude the reactionless force proposed by White and March. Such an analysis is within the expertise of Trevor Lafleur, a physicist specializing in plasma physics. His analysis (Lafleur, 2014) finds no basis for the reactionless force in the quantum vacuum.

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[11] Such confounders can be subtle. For example, Tajmar et al. (2018) report such a confounder in the coupling between electrical cables in the experimental set-up and the earth's magnetic field.

6. Inductively Self-Supporting Structures

There are self-supporting inductive structures.

6.1 Closure: That is All There Is.

A self-supporting inductive structure is a set of propositions such that: each proposition in the set is well supported evidentially; the evidence supporting them is in the set of propositions; and the propositions that warrant the relations of inductive support are also propositions within the set.

We have already seen such self-supporting inductive systems in the small. If we take the backgrounds propositions among which they proceed as fixed, they are found in the examples above of pairs of hypotheses that are mutually supporting; and of networks of inductive support such that the relations of support cross over one another in a bewildering tangle. The more difficult problem and the more interesting one is whether such systems arise on the large scale and whether they are embodied by our mature sciences. I will argue in the subsection below that, if a mature science is properly characterized by the rigidity described in the last section, then the material theory entails that it is a self-supporting inductive structure.

Before proceeding, it will be helpful to address directly the sense that such structures are paradoxical. They may sound akin to lifting oneself into the air by pulling on one's own bootstraps. However, there is no paradox. If one can affirm that each proposition in the set is, individually, well-supported in virtue of other propositions in the set, then there is nothing more that can be asked. The analogy to pulling oneself up by one's own bootstraps fails.[12] A better architectural analogy is to some elaborate sculpture, whose total stability appears impossible, but yet it still stands. A simple example is the tensegrity icosahedron of Figure 5:

[12] In the imagined scenario, we hover in midair by pulling on our bootstraps. There is an imbalance of forces on our boots. The upward force from the bootstraps in tension is balanced by the downward force from the corresponding compression in our legs. What is left unbalanced is the force of gravity, which pulls our boots and also us down. Our boots are not well supported.

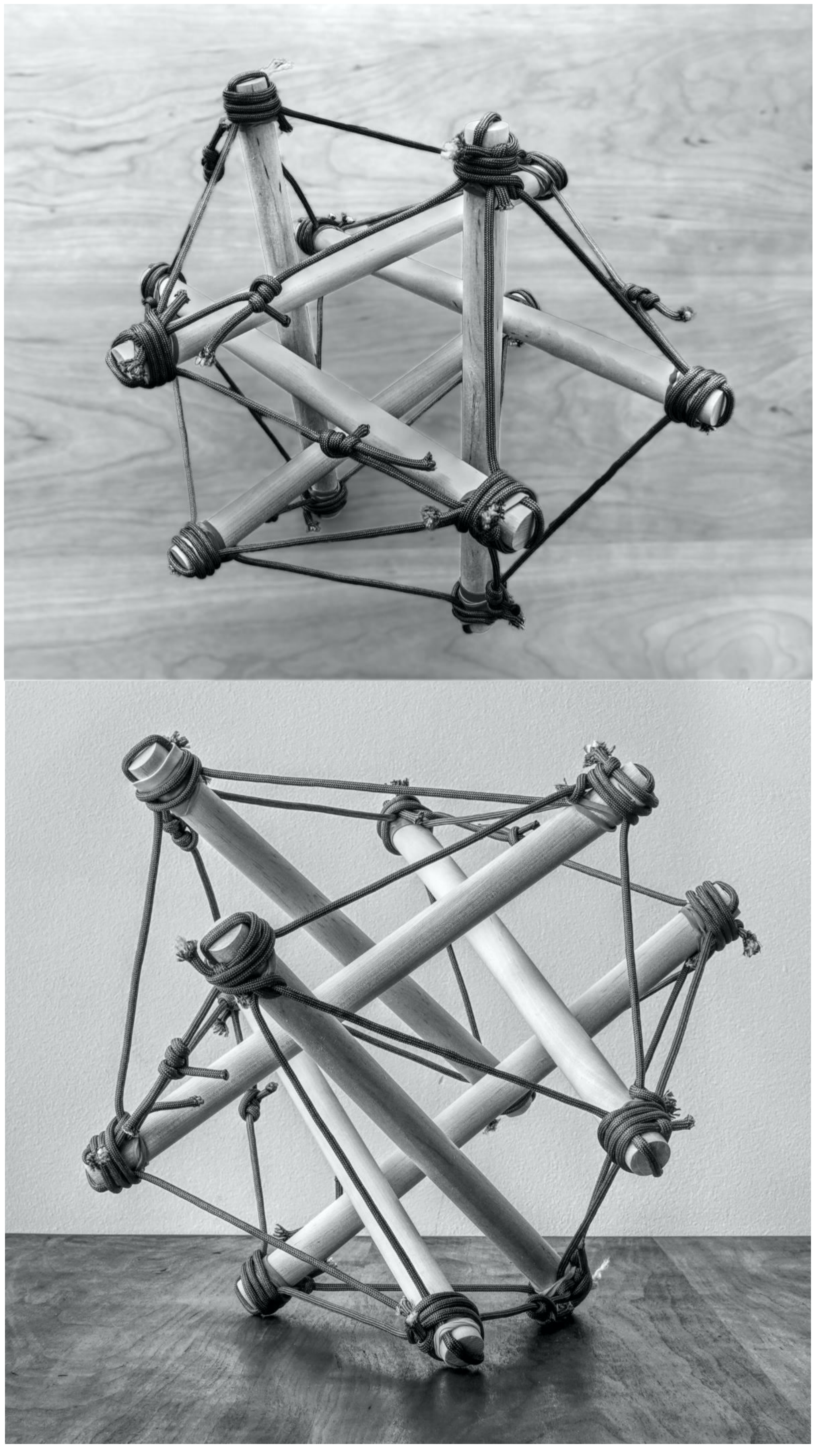

Figure 5. Plan and Elevation of a Tensegrity Icosahedron[13]

[13] Model and photographs, John D. Norton.

On a superficial description, it seems impossible that such a tensegrity structure can stand. There are six rods connected only by cords in tension. One end of each of three rods rests on the table surface. All the remaining rods and their parts are held suspended above the table surface. No rod directly touches any other rod. Their *sole* connections are through cords in tension. Such a structure, it would seem, must collapse into a pile of rods and cords. Must not a rod, supported only by cords in tension, anchor those chords on another rod that is still higher in the structure; and must not that rod be held by cords tied to another still higher rod; and so on in an infinite regress? Yet there are only six rods; and it stands.

On closer examination, we can inspect any rod individually and affirm that it is supported securely by cords attached to both ends. That is true for any rod we examine. That is all that is needed for the structure to stand. We need no additional, holistic condition, beyond the condition that each rod individually is supported.

It is the same with self-supporting inductive structures. If we can affirm that each proposition individually is well supported inductively, nothing further need be demanded. Of course, if we were tacitly to assume a hierarchical structure for relations of inductive support, then these self-supporting inductive structures are impossible. For then at least some of the propositions needed to warrant all the inductive inferences in the structure could not themselves be inductively supported within a finite structure. An infinite regress would ensue. However, as argued in detail above, this hierarchical assumption is incorrect.

One might still harbor reservations. These self-supporting inductive structures necessarily harbor circularities in the relations of support. That these circularities are benign is argued at length in the following Chapter 3, "Circularity." Or one might accept that such structures exist, but that they make the import of evidence equivocal since our evidence might support many such systems. In Chapter 4, "The Uniqueness of Domain-Specific Inductive Logics," it is argued that a mechanism, native to the material theory of induction, precludes this danger.

6.2 Mature Sciences are Self-supporting Inductive Structures

The material theory of induction entails that a mature science is a self-supporting inductive structure. This follows from the inductive rigidity presumed above as the defining characteristic of a mature science. That is, each proposition of the science enjoys strong support from the evidence. According to the material theory, the warrants for the inductive relations of support for each proposition reside in other propositions. If those propositions lie within the science, by the assumption of rigidity, they are well supported. If one of them lies outside the science in a neighboring science, then rigidity can only be preserved if that proposition is in turn well supported in that science. In this latter case, the original structure may not be self-supporting inductively until it is expanded to enclose the neighboring science. Many such expansions may be needed. However, if the mature science is inductively rigid, a self-supporting inductive structure must eventually be found.[14]

This consideration, in my view, is enough to establish that mature sciences, suitably expanded to their neighbors as needed, are self-supporting inductive structures. It may be helpful, however, to revisit some of the main themes of the material theory of induction to secure this conclusion. Consider how else the inductive rigidity of a mature science may come about. If inductive support is not supplied by warranting material facts, we might look to universal inductive rules or perhaps other related notions. The basic argument for the material theory of induction precludes them. This argument was elaborated in Chapter 2 of the *The Material Theory of Induction* and reviewed briefly in Chapter 1 above. Since inductive inferences are ampliative, they return conclusions of greater strength logically than their premises. It follows that they will fail if they are applied in factually inhospitable domains. Any rule or principle that authorizes an inductive inference or some relation of inductive support depends on that factual circumstance. That fact, however it may be expressed, is the material fact that warrants the induction.

The Material Theory of Induction devoted many chapters to showing how various supposedly universal rules of inductive inference were really just restricted in scope to domains hospitable to them. We need not rehearse those examples here. However, it is worth considering other general rules or principles that one might imagine as having a role in the inductive rigidity of a mature science. It may be tempting to imagine that one needs to suppose some deeper, non-

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[14] In principle, one could imagine that sequences of inductive inferences and warranting propositions form an infinite chain that outstrips finite description. I do not see that, as a matter of inductive logic, such a chain can be dismissed as troublesome without further examination of its details. However, I discount that it arises with our mature science. For, if that were the case, the inductive rigidity of a mature science would not be humanly accessible, contrary to our experience of mature sciences.

empirical truths to serve here. They would be beyond normal evidential scrutiny or may even not need normal evidential scrutiny.

The material theory of induction requires that any such condition must itself turn out to be a contingent fact, subject to evidential scrutiny, for it must be factually adapted to the domain at hand. Otherwise, for the reasons just given, it cannot perform the function of authorizing an inductive inference. The following subsection illustrates this outcome with a few examples.

6.3 Alternative, non-Empirical Bases for Rigidity Rejected

What are some rules, principles or conditions that are possible, non-empirical alternatives to a material warrant for inductive inferences? Natural choices for proposals are Kantian a priori synthetic propositions. They are factual, but require no evidence since their truth can be established independently of experience, supposedly. Since the literature on this one idea could absorb many lifetimes, I dare only express my view that this literature has failed to provide the sort of conditions that would serve the present need. Kant's original proposals did not fare well. It may have been appealing to imagine that, as an a priori certainty, space could never manifest to us other than as Euclidean. However, those who have absorbed the variant spatial geometries brought by general relativity find it otherwise. The geometry of space is not something determinable a priori, but a subject of empirical investigation.

Might we seek such a condition in a principle of causality? It is a Kantian principle and also has an enduring popularity outside Kantian circles. The principle asserts that every effect is brought about in a regular manner by some cause. Might such a supposition be a precondition for science and thus for inductive inferences in science? I have criticized this conception at length elsewhere. See for example Norton (2003, 2016). In short, the problem is that the terms "cause" and "effect" are so poorly specified that the principle is vacuous. Rather, we can always implement the principle in any scenario simply by artful choices for what the terms designate. Things in the world do connect in a myriad of interesting ways. What those ways are cannot be stipulated a priori, but must be discovered empirically.

We might look for these conditions in another direction that I believe appears more promising. It is often found remarkable that mathematical descriptions of the world prove so fertile and powerful. Might the supposition of a mathematical structure of the world be a prior condition necessary at least for the physical sciences? There is much to say on this supposition.

The main point of relevance is that the supposition itself is open to empirical test. We have tested it and found that it applies to a surprisingly large range of phenomena. This means that, in the absence of any deeper, a priori vindication, it is a contingent fact to be learned inductively. In this regard, it is no different from the other warranting facts of the physical sciences. It is not an obstacle to the material warranting of inferences, but a part of it.

As an illustration, take the circumstance that much of modern physics proceeds on the presumption that its basic laws are to be written as differential equations. That fundamental presumption has been challenged by Stephen Wolfram (2002). His "new kind of science" seeks to replace these differential equations in physics by discrete algorithms and cellular automata. It is a most radical proposal. Wolfram has continued to press his approach, but its reception amongst physicists remains poor. Their skepticism is not based on an assertion that, as an a priori matter, the physical world must be governed by differential equations. Rather, as Becker (2020) reports briefly, the doubt is driven by disbelief that Wolfram's methods can recover the present results of physics with the same scope and accuracy. The concern is empirical. The proposal lacks powerful enough inductive support to supplant existing methods.

Nonetheless, we can still ask what are the prospects for an a priori justification of the mathematical character of nature. These prospects are poor, in my view, since it is doubtful that there is a deep truth in the supposed mathematical character of nature. Rather I harbor an enduring concern that our deference for the power of mathematical descriptions is excessive. The supposed truth is empty unless the specific mathematics favored by nature is specified. Yet the only way we know to identify the right mathematics among very many choices is empirical. Thus, I find it hard to be moved by a celebrated and poetic confession attributed to Heinrich Hertz:[15]

> One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser that we are, wiser even than their discoverers, that we get more out of them than was originally put into them.

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[15] As quoted in Bell (1937, p. 16). The quote is unsourced and seems to be the origin of later repetitions. Shour (2021) has recently tracked down the origin of the remark in Hertz's published writings. (I thank Marc Lange for letting me know of Shour's paper.)

On the contrary, I am in awe not at the formulae but at the creativity of mathematicians who formulated them. For new physical theories commonly come in clumsy mathematical clothing. Each new physical theory is taken as a challenge by mathematicians to find formulations in which the new theory looks mathematically simple and natural. The ensuing mathematics fits the world not through some pre-ordained harmony, but merely retrospectively through our ingenious and artful contrivances.[16]

To see the process, one need only recall the inadequacies of geometry as Euclid formulated it for the celestial mechanics of the seventeenth century. Kepler sought to use the Platonic solids in a nestled geometric structure to explain the relative orbital sizes of the planets. Far from reflecting the inner mathematical constitution of the world, we now regard the whole project as dependent on barren mathematical coincidences. One can only wonder at Newton's labors in his *Principia* to develop his celestial mechanics using simple Euclidean geometry that was so poorly suited to the task. The theory becomes so much more elegant and transparent when re-expressed in the later methods of vector calculus, contrived in part precisely for this purpose.

Finally, when explicit attempts to identify these non-empirical conditions fail, one might be tempted by the idea that these conditions are present, but ineffable. They are so deeply enmeshed in our ways of thinking that, it is speculated, we cannot discern them. This appears to me to be the last defense of a failing program. These conditions have powerful consequences in connecting facts and these connections are fully accessible to us. Yet the conditions that spawn them are supposed to be opaque to us. The supposition of their invisibility to us makes them irrelevant to us. All that matters are the contingent conditions they supposedly induce among the facts of the science; and these are just more facts of the science.

7. Conclusion

The four claims defended in this chapter form the basis of the material understanding of the large-scale structure of relations of inductive support. These claims by no means exhaust the questions one might raise about this large-scale structure and the accompanying skeptical

[16] For another expression of this view in counterpoint to Einstein's later Platonism, see Norton (2000, Appendix D).

challenges to the material understanding. Some of these questions and challenges will be raised in the chapters to come in Part I; and the claims defended in this chapter will be used to answer them. We will ask in Chapter 3, if the structure is non-hierarchical, does it harbor circularities? (Yes.) Are they benign? (Yes.) What of uniqueness, we will ask in Chapter 4. That is, can a finite body of empirical evidence, even if extensive, yield a unique, self-supporting structure? (Yes.) Or must we forever contend with multiple, competing self-supporting structures? (No.) Relations of inductive support are non-hierarchical and circular. Does this mean, we will ask in Chapter 5, that the material theory of induction is just a coherentist epistemology? (No) And finally in Chapter 6, what of *the* problem of induction? Is the material theory prone to the traditional problem? (No) Is there an analogous problem residing in a fatal regress of warrants? (No)

These are all good questions and worthy challenges. I will show that the material approach to inductive inference has ample resources for answering them.

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