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The Recession of the Nebulae

John D. Norton[1]
Department of History and Philosophy of Science
University of Pittsburgh

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The Large-Scale Structure of Inductive Inference

1. Introduction

In 1929, the astronomer Edwin Hubble announced what would become the single most important observation of modern cosmology. Hubble reported that the extra-galactic nebulae[2] are receding from us with a velocity proportional to their distance, a result that soon came to be known as "Hubble's law."[3] The establishment of this linear relation would seem to be one of the simplest of generalizations. Hubble needed only to compare the velocities of recession and

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[1] I thank Siska De Baerdemaeker for helpful comments on an earlier draft.

[2] Hubble's "extragalactic nebula" or just "nebula" are the older terms for galaxy. In 1929, the term "galaxy" then referred unambiguously only to our star system, the Milky Way. The Latin *nebula* (plural *nebulae*) means cloud and was used by astronomers of Hubble's time to denote the luminous clouds visible in astronomical telescopes. As he explained in Hubble (1936, pp. 16-17), some of these clouds proved to be gas and dust within our Milky Way. These he called "galactic nebulae." Others were more distant star systems in their own right—"extragalactic nebulae"— which he would just call "nebulae." Hubble defended his reluctance to label these other nebulae as "galaxies" in Hubble (1936 p. 18): "The term *nebula* offers the values of tradition; the term *galaxies*, the glamour… of romance."

[3] In 2018, the members of International Astronomical Union voted to rename the law the "Hubble–Lemaître law."

distances to a selection of nebulae, note their linear relation and declare the result. This is how Hubble's affirmation of the linear relationship is often reported in summary. McKenzie's *Major Achievements of Science* of 1960 (1960, p. 333) describes it as:

> In 1929 Hubble compared Slipher's determinations of the recession of the nebulae with his own determinations of distances and he discovered a simple relation now called Hubble's law, that the velocity is proportional to the distance.

This simple determination would seem to be a good illustration of a natural hierarchical structure for inductive support. In it, inductive inferences may only proceed from a lower, more particular level to a higher, more general level:

Inductive Hierarchy

Lower level: velocity and distance assignments to particular nebulae.

Higher level: general relation connecting the velocities and distances of all nebulae.

Hubble's inference, it seems, merely proceeds up the hierarchy. The particulars of a few individual nebulae at the lower level provides inductive support for the general law at the higher level.

However, simple as this inference may seem, Hubble's celebrated paper of 1929 showed no respect for this inductive hierarchy. Rather, a multiplicity of inductive inferences moved up and down the hierarchy in an intricate arrangement of interlocking parts, much like those of complicated geometric puzzle.

To begin, in 1929, Hubble had access to measurements of the velocities of recession of 46 extra-galactic nebulae, but he had independent distance estimates for only 24 of these nebulae. For these 24, in what initially appears as a simple generalization, he found a linear velocity-distance relation within statistical uncertainties. However, the inference was not a simple generalization since the determination of most of the distances among these 24 nebulae depended on assuming hypotheses still in need for further support. They are the hypotheses of *Brightest Star Magnitudes* and *Clustering of Nebular Luminosity* detailed in Section 3 below. These hypotheses cannot be located uniquely in the inductive hierarchy above. In the inferences they are presumed by the distance determinations, so are prior to the lower level, that is, still lower. However the hypotheses accrue support once the inferences of the 1929 paper are complete. That means that they come at the end of the inferential chain, so they should be placed higher in the inductive hierarchy.

The remaining 22 nebulae were more problematic. For them, Hubble only had measurements of velocities and apparent luminosities, but not distances. He was determined somehow to make use of these data. In doing so, he introduced relations of support that further cut across the inductive hierarchy. This happened in two related ways:

First, he averaged the apparent luminosities of the 22 nebulae and computed the average distance associated with them, assuming the *Clustering of Nebular Luminosity* hypothesis and a mean absolute luminosity found in his second, inverted inference (to be described below). The mean velocity and the mean distance fell within the expectations of the linear relation he had found for the first 24 nebulae, providing further support for that relationship.

Second, he inverted the direction of evidential support. He used the velocity-distance relation itself, in conjunction with the velocities of recession of these 22 nebulae, to infer their distances. This inference proceeds down the inductive hierarchy from the higher to the lower level. He then used the distances computed to determine the absolute luminosities of the 22 nebulae. The results provided direct support for the *Clustering of Nebular Luminosity* hypothesis, already used in the earlier analyses.

The overall outcome was a tangle of inductive inferences that failed to respect any simple linear, inductive hierarchy, such as the one indicated above. We shall see that Hubble remarked repeatedly on the agreement among and, later, the consistency of the results of the inferences as providing the strongest support for his general conclusions. His notion of consistency is much stronger than mere logical compatibility. Rather it reflects a conformity of results that could not come about merely by happenstance. That all the pieces fit as they do is unlikely to happen unless they are properly parts of one integrated whole.

In the following, Section 2 with describe how Hubble came to be concerned with the velocities of the nebulae. Section 3 will outline the hypotheses Hubble used in his determinations of the distances to the nebulae. Sections 4 and 5 will review the inference to the linear velocity-distance relation for the first 24 nebulae. Section 6 will review the inverted inferences for the remaining 22 nebulae. Section 7 will reflect briefly on the strength of support Hubble could display in 1929 for the linear relationship. The concluding Section 8 will summarize the interwoven relations of support in Hubble's 1929 paper. An Appendix reviews technical details of the computations relating absolute and apparent nebular luminosities, which are known tersely as "magnitudes."

2. Background to Hubble's Investigations

It is now a commonplace of astronomy that space is filled with many immense star systems akin to our own Milky Way. They are the galaxies, as they are now called, or the extra-galactic nebulae, as Hubble called them. Yet whether the stars were so distributed in space remained unsettled in the early 1920s. A landmark in the decision was a debate held between the astronomers Harlow Shapley and Heber Curtis on April 26, 1920, at the Smithsonian Museum of Natural History. Shapley defended the view that our Milky Way is the unique great star system of the universe. Curtis, however, argued that our Milky Way is just one of many such "island universes,"[4] as they were then called. The matter was settled fairly quickly. According to Trimble (1995, p. 1142), it was Hubble himself who provided a cleaner resolution. Starting with observations in 1923,[5] he was able to discern Cepheid variable stars in two nearby nebulae, most notably Andromeda. As we shall see below, this enabled a determination of the distances to these nebulae. They were located outside our Milky Way, he found.

Our solar system has a motion within the Milky Way. With the recognition that our Milky Way is just one of many nebulae, a prosaic question arises: what is the motion of our solar system with respect to these other nebulae? In his later work, *Realm of the Nebulae* (1936, pp. 106-18), Hubble recalled how the answer to this question developed. The velocities of nebulae relative to the earth were known from red shift measurements in the 1910s. The motion of the solar system was then estimated as around 420 miles per second. The expectation was that, once this motion was subtracted from the motions of the nebulae, those motions would be small and random. In particular, there would be as many velocities of approach as of recession. Using a statistical analysis to average away these random motions, we should recover the motion of our solar system with respect to the mean rest state of the nebulae in our vicinity.

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[4] The cases each made are published in Shapley and Curtis (1921). See Trimble (1995) for further details.

[5] As reported in Hubble (1929a). The results also appeared in a *New York Times* article on December 23, 1924, p. 6, with the headline "Finds Spiral Nebulae are Stellar Systems: Dr Hubbell Confirms View That They Are 'Island Universes' Similar to Our Own"; and were communicated orally by H. N Russell at the December-January, 1924-25 meeting of AAAS. (Anon, 1925).

As early as 1918 it was already clear that the statistical project was not proceeding smoothly. Wirtz (1918) found the need to add a "*k* term" that corresponded to an overall recession of the nebulae. It meant that the motions of the nebulae visible to us were not distributed randomly about some nebular state of rest. In place of the state of rest was some sort of expansion. The *k* term represented a constant motion of recession from our solar system of 656 km/sec. The motions of the individual nebulae were distributed randomly around that constant motion of recession. Wirtz wrote (p. 115)

> If we give this value a verbal interpretation, it is that the system of spiral nebulae disperses [auseinandertreibt] with a speed of 656 km [per sec] in relation to the momentary position of the solar system as a center.

Over the next decade, Wirtz and others refined the correction term, allowing it to be a function of distance from our solar system. Hubble's celebrated paper of 1929 was a direct contribution to this literature. Its first paragraph identifies the issue to be addressed:

> Determinations of the motion of the sun with respect to the extra-galactic nebulae have involved a K term of several hundred kilometers [per second] which appears to be variable. Explanations of this paradox have been sought in a correlation between apparent radial velocities and distances, but so far the results have not been convincing. The present paper is a re-examination of the question, based on only those nebular distances which are believed to be fairly reliable.

The result announced (1929, p. 170-71) was that a statistical fit gave the overall motion of the nebulae as distributed, with some considerable deviations, around a velocity of recession that increases linearly with distance from us. In more detail, the best estimate of the motion of our solar system is 280 km/sec; and, when this is subtracted from the motions of the nebulae, their motions are scattered around an average recessional velocity of 500 km/sec for each million parsec of distance.[6]

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[6] This value of 500 km/sec.Mpc of what we now call the Hubble constant proved to be about an order of magnitude too large as a result of systematic errors in Hubble's determinations of distances. By 1958, the value had been reduced by Sandage (1958) to a more modern value of 75 km/sec.Mpc, which corresponded to a Hubble age of the universe of 1.3x10^9 years.

A prosaic question about the motion of our solar system had led Hubble to the single most important observational result of modern cosmology.

3. The Determination of Distances

To carry out the analysis of his 1929 paper, Hubble needed determinations of both velocities of and distances to the nebulae. For the 46 nebulae of Hubble's analysis, the velocity determinations proved relatively unproblematic. They were determinable from frequency shifts in the spectra of light from the nebulae. The shifts were immediately interpreted as due to radial velocities, that is motions along the lines of sight to each nebula.[7] As Hubble (1936, pp. 102-105) recounts, Vesto Slipher, working at the Lowell Observatory, had begun the arduous work of measuring these shifts in 1912. By 1925, he had provided the velocities of 25 nebulae.

The locus of difficulty in the analysis was the determination of distances. Two means were available for determining these distances. One was the angular size of the nebula. Nearby nebulae are large: Andromeda extends over 3 degrees in the sky, which is six times the extent of the full moon. If we know the absolute size of the nebula in, say, light years, then the distance to the nebula is immediately determined by elementary geometry.

This means of determining distance to the nebulae was *not* mentioned in Hubble's (1929) paper.[8] Rather, Hubble explicitly reports only luminosity-based determinations. They depend on the fact that the intensity of light emitted by a celestial object diminishes with the inverse square of distance. Thus, if we know the absolute magnitude of the object's luminosity, we can

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[7] Slipher (1912, p. 56) wrote: "…whether the velocity-like displacement might not be due to some other cause, but I believe we have at the present no other interpretation for it. Hence we may conclude that the Andromeda Nebula is approaching the solar system with a velocity of about 300 kilometers per second." Hubble (1936, p. 34) held the same view, but more cautiously: "Although no other plausible explanation of redshifts has been found, the interpretation as velocity-shifts may be considered as a theory still to be tested by actual observations."

[8] Hubble and Humason (1931, p. 52) recount that the difficulty with the method is that the brightnesses of the nebulae fade as we move away from their centers, so that different photographic exposures of the same nebula give different sizes.

determine its distance: we compare this absolute magnitude with the apparent magnitude we perceive, either visually or photographically.

The weakness of this approach is that the absolute magnitudes are hard to determine; direct measurements give us only apparent magnitudes. Without some independent means of determining the absolute magnitude, the approach cannot be applied. In his 1929 paper, Hubble relied on three methods of determining absolute magnitudes. They were:

1. Cepheid Variable Stars. Henrietta Leavitt (1912) had reported that certain stars in the Magellenic Clouds varied periodically in magnitude and that there was a definite relationship between the period and the magnitude. Subsequent parallax measurements to other Cepheid variable stars enabled determinations of their distances and thus also their absolute magnitudes. Combining, these results meant that an observation of the period of one these variable stars enabled a determination of its absolute magnitude and thus its distance. Hubble himself used this method in 1923 in his determination of the distance to the nebula Andromeda. The distinctive shape of the curve[9] plotting the change of visual magnitude with time enabled Hubble to identify the variable stars he found in Andromeda as Cepheid variable stars. This was, Hubble (1936, p.16) reported, the first reliable method of determining distances to nebulae. It was also the most reliable of the three methods of the 1929 paper, but could only be applied if a Cepheid variable star could be resolved in the nebula.

2. Brightest Star Magnitude. It seemed reasonable to assume that different nebulae are constituted of the same sorts of stars, with the same range of possible magnitudes. That leads to the expectation that the brightest stars in each nebula have the same absolute magnitude.[10] Hubble (1929, p. 168) offered an absolute magnitude determined photographically of $M = -6.3$. (See the Appendix for a review of the system of units used for apparent and absolute

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[9] Shown in Hubble (1936, p. 95).

[10] Hubble footnoted an earlier paper, Hubble (1926), in which he had already advanced the hypothesis (p. 357-61), although only hesitantly, as a “reasonable assumption, supported by such evidence as is available.” (p. 357)

magnitudes.) This assumption is important in untangling the evidential relations displayed in Hubble's paper. So I will display it as an hypothesis to which we will return:

> *Brightest Star Magnitude*. The brightest stars in each nebula have the same absolute magnitude.

Hubble approached the hypothesis with optimism and caution. He wrote (1929, pp. 168-69):

> The apparent luminosities of the brightest stars in such nebulae are thus criteria which, although rough and to be applied with caution, furnish reasonable estimates of the distances of all extra-galactic systems in which even a few stars can be detected.

The limitation Hubble conceded is that the method could only be applied to nebulae close enough for individual stars to be resolved telescopically. The third method was untroubled by this limitation.

3. Clustering of Nebular Luminosity. Drawing on his earlier survey of nebulae (Hubble, 1926), he suggested that the absolute magnitudes of nebulae were similar in so far as they were distributed randomly but not too distant from their average. The average value offered (p. 169) is a visually determined magnitude of $M = -15.2$. (Recall from the Appendix that smaller magnitudes correspond to greater brightness. A magnitude of minus-15 is very bright.) Actual values, he reported, are "exhibiting a range of four or five magnitudes about [this] average." Once again, this assumption will play an important role in the evidential relations and is displayed:

> *Clustering of Nebular Luminosity*. The absolute magnitudes of nebulae cluster in a small interval of four or five units of magnitude about a single mean common to all nebulae.

Four to five units of magnitude amounts to a considerable error if we are trying to estimate the distance to just one nebula. It is shown in the Appendix that this uncertainty in the absolute magnitude of any particular nebula introduces an uncertainty in the determination of distance of roughly one order of magnitude, that is, the extremes of the full range differ by a factor of 10.

These deviations can be averaged away if we aggregate data from many nebulae, so that we can recover more reliable distance determinations for averages. This is especially helpful in getting a more accurate distance estimate to a cluster of nebulae whose members are assumed to be grouped around the same location in space. Hubble (p. 169) explained that he would use this averaging technique:

> The application of this statistical average [*M* = -15.2] to individual cases can rarely be used to advantage, but where considerable numbers are involved, and especially in the various clusters of nebulae, mean apparent luminosities of the nebulae themselves offer reliable estimates of the mean distances.

Hubble's (1929) says little more on the use of this technique. Hubble and Humason (1931) is a lengthier and more detailed exposition, using considerably more data. There we find how effective the averaging can be. For there they report clusters consisting almost always of several hundred nebulae, up to a maximum of 800.[11]

To determine the distance to some particular nebula in a cluster, they would survey the full range of apparent magnitudes of the nebulae in the cluster. The aggregation of survey data greatly reduces errors. For example, consider a cluster of 400 nebulae whose magnitudes are spread over an interval of 4 or 5 magnitudes around the true mean of -15.2. The spread of the average of the magnitudes of the cluster around that true mean is reduced by a factor of $\sqrt{400} = 20$. We find in the Appendix, that this reduces the interval to 0.25 magnitudes and corresponds to an error in distance estimates where the farthest distance is merely 12% greater than the nearest. This provides a good determination of the absolute magnitude of and distance to a nebula whose brightness matches the average.[12] That distance is then also the estimate of the distance to the particular nebula of interest.

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[11] A table in Hubble and Humason (1931, p. 74) lists the numbers of nebulae in named clusters as Virgo-(500), Pegasus-100, Pisces-20, Cancer-150, Perseus-500, Coma-800, Ursa Major-300 and Leo-400. Whatever hesitation is flagged by the parentheses for the Virgo cluster, Hubble (1936, p. 54) reports "several hundred" nebulae in the Virgo cluster.

[12] Hubble and Humason (1931, p. 56) summarize the strategy as "The mean or most frequent apparent magnitude of the many members [of a cluster] is a good indication of the distance of a cluster, and hence clusters offer the greatest distances that can definitely be assigned to individual objects."

4. From Particulars to Generalities

While 46 nebulae were included in Hubble's (1929) analysis, he was able to estimate individual distances to 24 only. For these 24, Hubble inferred the linear relation between their distances and velocities by directly comparing distances and velocities. He reported the results of two ways of arriving at the linear relation.

The first, most direct way took the velocities and distances of the individual nebulae and used standard statistical methods to find the best fit of a relation written in more modern vector notation as

$$\mathbf{v}_i = \mathbf{r}_i K + \mathbf{V}_0$$

Here $\mathbf{v}_i$ is the vector velocity of the ith nebula located a vector displacement $\mathbf{r}_i$ from us and $\mathbf{V}_0$ is the vector velocity of our solar system. The constant K is now known as the "Hubble constant" and is the parameter of greatest interest to us now. It converts a scalar distance r to a nebula to its scalar velocity of recession $v=Kr$. The velocity $\mathbf{v}_i$ is not the velocity observed from earth through the redshift, for those observations are taken from a vantage point itself moving at $\mathbf{V}_0$. The velocity we observe for the ith nebula is the difference $\mathbf{v}_i$-$\mathbf{V}_0$. Hubble reported that the best fit gave

$$K = 465 \pm 50 \text{km/sec.Mpc} \quad V_0 = 306 \text{ km/sec} \quad A = 286^{\text{o}} \quad D = 40^{\text{o}}$$

The second way proceeded by first reducing the data for the 24 nebulae to 9 groupings and first averaging within each grouping. Hubble indicated only that the groupings were selected "according to proximity in direction and in distance. (p. 170)" Presumably the effect of the averaging was, once again, to reduce the effect of random deviations from linearity, this time prior to finding the statistical best fit of the above relation. The index i would now refer to the ith group. Hubble reported that best fit as

$$K = 513 \pm 60 \text{km/sec.Mpc} \quad V_0 = 247 \text{ km/sec} \quad A = 269^{\text{o}} \quad D = 33^{\text{o}}$$

For his final result, Hubble selected values intermediate between these two sets and rounded them:[13]

$$K = 500 \text{ km/sec.Mpc} \quad V_0 = 280 \text{ km/sec} \quad A = 277^{\text{o}} \quad D = 36^{\text{o}}$$

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[13] Hubble converted the celestial coordinates into galactic coordinates: longitude 32 $^{\text{o}}$, latitude +18$^{\text{o}}$.

Since the solar velocity $\mathbf{V}_0$ is comparable in size to the nebular velocities $\mathbf{v}_i$, Hubble's analysis had to pass through the more indirect route of finding the best fit of the above relation. Merely computing the ratio of observed velocity and distance for each nebula would have omitted the essential correction for the earth's motion. Hubble's Figure 1, redrawn here as Figure 1, gives a sense of the large size of the residuals that deviate from Hubble's best-fit relations. It displays the velocities of nebulae, after the velocity of our solar system has been subtracted, in relation to their distances.

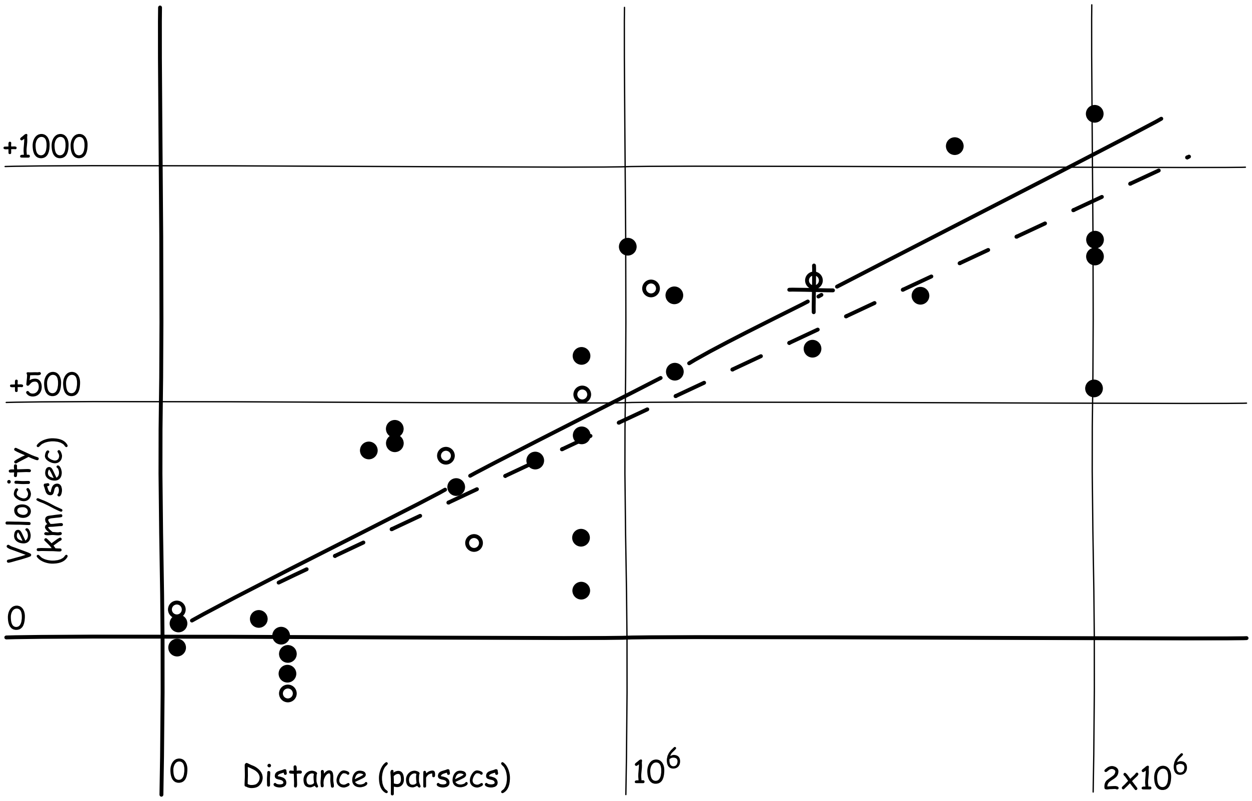

Figure 1. Hubble's "Velocity-Distance Relation among Extra-Galactic Nebulae"

An extended caption explains the data presented. Hubble writes (p. 172):

> The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups;...

There are 24 black discs and they correspond loosely[14] with the data in Table 1 for 24 nebulae whose distance can be determined. Hubble concluded:

> … the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

We will turn to the treatment of these 22 nebulae in Section 6 below.

5. Hubble's Hypotheses

The appearance of this last inference is of a traditional generalization that proceeds from the particulars of the lower level to the covering generality at the higher level of the hierarchy indicated in Section 1 above. The appearance is deceptive, for most of the distance determinations in the particulars depend upon the hypotheses indicated in Section 3 above. Since the subsequent generalizations depended upon them, the generalization is not secure until Hubble provides further evidence in support of the hypotheses. Metaphorically, this stage of Hubble's investigation took on an inductive debt. We shall see that Hubble continues the analysis in a way intended to discharge some of that debt.

The data for these 24 nebulae were presented in tabular form in Table 1 of Hubble's paper, reproduced here at Table 1:

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[14] We should not expect the velocities in the figure to match those of Table 1 up to a constant subtractive factor. The correction for solar motion is a vector subtraction whose scalar effect will vary according to the differences in the directions of the vectors in the subtraction.

	Object	m_s photographic magnitude of brightest stars	r distance[15] in megaparsecs	v velocity km/sec	m_t visual magnitude	M_t absolute visual magnitude computed[16] from r, m_t
1	Small Magellenic	..	0.032	+170	1.5	-16.0
2	Large Magellenic	..	0.034	+290	0.5	-17.2
3	NGC 6822	..	0.214	-130	9.0	-12.7
4	NGC 598	..	0.263	-70	7.0	-15.1
5	NGC 221	..	0.275	-185	8.8	-13.4
6	NGC 224	..	0.275	-220	5.0	-17.2
7	NGC 5457	17.0	0.45	+200	9.9	-13.3
8	NGC 4736	17.3	0.5	+290	8.4	-15.1
9	NGC 5194	17.3	0.5	+270	7.4	-16.1
10	NGC 4449	17.8	0.63	+200	9.5	-14.5
11	NGC4214	18.3	0.8	+300	11.3	-13.2
12	NGC 3031	18.5	0.9	-30	8.3	-16.4
13	NGC 3627	18.5	0.9	+650	9.1	-15.7
14	NGC 4826	18.5	0.9	+150	9.0	-15.7
15	NGC 5236	18.5	0.9	+500	10.4	-14.4
16	NGC 1068	18.7	1.0	+920	9.1	-15.9
17	NGC 5055	19.0	1.1	+450	9.6	-15.6
18	NGC 7331	19.0	1.1	+500	10.4	-14.8
19	NGC 4258	19.5	1.4	+500	8.7	-17.0

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[15] These distances are systematically low. Hubble reports 0.275 Mpc for the distance to nearby Andromeda, whereas the more recent estimate is 0.780 Mpc.

[16] Using formula (A3) of the Appendix. The table has distances in units of megaparsecs, whereas distance in (A3) are entered in parsecs.

20	NGC 4151	20.0	1.7	+960	12.0	-14.2
21	NGC 4382	..	2.0	+500	10.0	-16.5
22	NGC 4472	..	2.0	+850	8.8	-17.7
23	NGC 4486	..	2.0	+800	9.7	-16.8
24	NGC 4649	..	2.0	+1090	9.5	-17.0
	“NGC” = nebula number in the New General Calatog					mean -15.5

Table 1. Hubble’s “Nebulae Whose Distances Have Been Estimated from Stars Involved or From Mean Luminosities in a Cluster.”

To arrive at the distances in this table, Hubble used all three of the methods discussed above. He did not lay out the specifics of the determinations in each case. All the details would be lengthy and not fit into the short announcement Hubble offered. Hubble and Humason's (1931) provides a similar analysis, with more data and details, and has to be considerably lengthier and more complicated in its reporting. In his 1929 report, Hubble limited himself to general statements (p. 170):

> The first seven distances are the most reliable, depending, except for M 32 [=NGC 221] the companion of M31 [=Andromeda, NGC 224], upon extensive investigations of many stars involved.

For Andromeda (M31 = NGC 224), we know from Hubble (1929a) that Hubble used Cepheid variable stars for the distance determination. Presumably the *Brightest Star Magnitude* hypothesis was not used in the distance estimates for these first seven objects, since there are no brightest star magnitude entries for them. Subsequent distance estimates did consider the magnitudes of the brightest stars, since they are given for rows 7 to 20. Hubble continued:

> The next thirteen distances,[17] depending upon the criterion of a uniform upper limit of stellar luminosity, are subject to considerable probable errors but are believed to be the most reasonable values at present available.

The use of mean nebular magnitudes for distance determination is finally mentioned for row 21-24:

> The last four objects appear to be in the Virgo Cluster. The distance assigned to the cluster, 2 x 10^6 parsecs, is derived from the distribution of nebular luminosities, together with luminosities of stars in some of the later-type spirals, and differs somewhat from the Harvard estimate of ten million light years.

Here the *Clustering of Nebular Luminosity* hypothesis was employed. That it had a larger role is suggested by the label given to the table as a whole: it mentions "Distances…From Mean Luminosities in a Cluster."

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[17] Presumably he means "next fourteen": rows 7 to 20.

6. From Generalities to Particulars

Hubble now turned to the remaining 22 nebulae for which velocities were known, but the distances were unknown. He was intent on recovering some evidential import from the data. The data with which Hubble worked is presented in Table 2 and reproduces Hubble's (1929) Table 2. The column v is the velocity determined by red shifts for the nebula with the indicated NGC number. The next column v_s indicates the correction that must be subtracted from the observed velocity to correct for solar motion.

With these data in hand, Hubble proceeded with two approaches. The first was the crudest. It simply worked out the velocity-distance relation for the average behavior of all the 22 nebulae. Since the velocity distance relationship is presumed linear, it should hold for the average of the velocities and distances. Hubble found an average velocity of 745 km/sec and an average distance of 1.4 Mpc. These averaged data then give an estimate for the constant K = 745/1.4 ≈ 530 km/sec.Mpc. Given the magnitude of errors likely (see below), the agreement was likely well within error limits for the value of 500 km/sec.Mpc estimated in the earlier part of the paper.

For our purposes, it is interesting to see that even here Hubble's analysis relied on the *Clustering of Nebular Luminosity* hypothesis. It was not needed to recover the average velocity. That was simply arithmetic.[18] The hypothesis was needed to determine the average distance. According to the hypothesis, the absolute magnitudes of the individual nebulae varied in an interval of 4 to 5 magnitudes about a common mean value. This range would then be reflected in the apparent magnitudes reported in the column m_t of Table 2. However, taking the average of the apparent magnitudes reduces the interval by a factor of $1/\sqrt{22}=1/4.69$ to an interval of roughly the size of a single magnitude. We find in the Appendix that the farthest distance in the associated distance interval is 58% greater than the nearest. The average apparent magnitude of 10.5 is far from the absolute magnitude of -15.3 assumed.[19] The diminution is due entirely to the

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[18] (Average v = 748.4) – (average correction v_s = 2.95) = 745.4 km/sec

[19] This absolute magnitude of -15.3 is recovered from the next stage of calculations on these 22 nebulae.

great distance associated with the average. That distance is computed[20] from (A3) and is 1.445 Mpc.

The more elaborate of the two approaches involved using the velocity-distance relation in reverse. Starting with the corrected velocity, v-v_s, for each of the 22 nebulae, Hubble computed the distance r that the linear velocity distance relation required, where he assumed a value for the K constant of 500 km/sec.Mpc. The results are reported in the r column of Table 2 and conform with the formula $r = (v\text{-}v_s)/500$. Since these distances were computed using the very relation under scrutiny, they could by themselves provide no evidence for the relation. To extract some useful evidential import, Hubble used these distances r to calculate[21] the absolute magnitude M_t of each nebula from the measured, apparent magnitude, m_t. The results are reported in the last column of Table 2. Hubble computed the mean to be -15.3.

What Hubble found notable was that the mean absolute magnitude computed for these 22 nebulae matched almost exactly with the mean -15.5 computed for the first 24 nebulae using their independently known distances. Similarly, their ranges agreed: 4.9 for the 22 nebulae of Table 2[22] and 5 for the 24 nebulae of Table 1. The most direct reading is that the new results from the 22 nebulae provide another instance of the *Clustering of Nebular Luminosity* hypothesis, using the same mean and range as the earlier analysis. This provides direct support for the hypothesis. Hubble was more celebratory and expansive in his assessment (pp. 172-73):

> The two mean magnitudes, - 15.3 and - 15.5, the ranges, 4.9 and 5.0 mag., and the frequency distributions are closely similar for these two entirely independent sets of data; and even the slight difference in mean magnitudes can be attributed to the selected, very bright, nebulae in the Virgo Cluster. This entirely unforced agreement supports the validity of the velocity-distance relation in a very evident matter. Finally, it is worth recording that the frequency distribution of absolute

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[20] That is $\log_{10} d = 0.2(10.5 + 15.3) + 1 = 6.16$, so that $d = 10^{6.16} = 1.445 \times 10^6$ pc.

[21] The calculation employed formula (A3) of the Appendix. Note that d in that formula is in parsecs, whereas r in Table 2 is in megaparsecs.

[22] I find the range to be 4.8, extending from -12.8 for NGC1700 to -17.6 for NGC 4594.

magnitudes in the two tables combined is comparable with those found in the various clusters of nebulae.

	NGC nebula number	v Velocity km/sec	v_s Velocity correction subtracted for solar motion	r Distance Mpc	m_t Apparent magnitude	M_t Absolute magnitude computed from r, m_t
	278	650	-110	1.52	12	-13.9
	404	-25	-65	..	11.1	..
	584	1800	75	3.45	10.9	-16.8
	936	1300	115	2.37	11.1	-15.7
	1023	300	-10	0.62	10.2	-13.8
	1700	800	220	1.16	12.5	-12.8
	2681	700	-10	1.42	10.7	-15
	2683	400	65	0.67	9.9	-14.3
	2841	600	-20	1.24	9.4	-16.1
	3034	290	-105	0.79	9	-15.5
	3115	600	105	1	9.5	-15.5
	3368	940	70	1.74	10	-16.2
	3379	810	65	1.49	9.4	-16.4
	3489	600	50	1.1	11.2	-14
	3521	730	95	1.27	10.1	-15.4
	3623	800	35	1.53	9.9	-16
	4111	800	-95	1.79	10.1	-16.1
	4526	580	-20	1.2	11.1	-14.3
	4565	1100	-75	2.35	11	-15.9
	4594	1140	25	2.23	9.1	-17.6
	5005	900	-130	2.06	11.1	-15.5
	5866	650	-215	1.73	11.7	-14.5
Mean		748.4	2.95		10.5	-15.3

Table 2. Hubble’s “Nebulae Whose Distances are Estimated from Radial Velocities”

7. How Strong Was the Evidence for Linearity?

Our present concern is the tangled structure of the relations of inductive support. While it is independent of this concern, it is worth noting that Hubble's evidence in 1929 for the linear relation was weak. This is so, even though Hubble's (1929) paper is routinely celebrated as the origin of the linear relation between the velocities of recession of the nebulae and their distances. A glance at Figure 1 shows just how weak was the establishment of the linearity. The data points are so broadly scattered about the straight lines fitted that all that can be securely inferred is that the velocities are increasing with the distances. The difficulty is that nebulae close to our Milky Way have particular motions in random directions that are of the order of the overall velocity of recession. These motions confound the linear motion of recession. To reveal the linear relation more clearly requires examination of more distant nebulae for which the particular motions become successively smaller in relation to the velocity of recession.

As long as Hubble's interest lay in the original project of determining the motion of our solar system, the weakness of the evidence for linearity is a smaller concern. We might reasonably expect that other velocity distance relations compatible with the data would only have a minor effect on the estimates of solar motion. The threat, however, is more serious if his paper is to underwrite the founding empirical observation of modern cosmology: the linearity of the velocity-distance relation.

Hubble already had a response to this threat in his 1929 paper. He allowed that his data merely "establish a roughly linear relation." (p. 173). The solution lay in an extension to more distant nebulae and was already underway. He reported a result for NGC 7619, whose distance he estimated at roughly 7 Mpc. That greatly exceeded the distances of 1 or 2 Mpc of nebulae investigated so far. Its speed of recession still fitted well enough with his *K* factor of 500. Shortly after, in joint work, Hubble and Humason (1931) reported on velocities of recession of still more distant nebulae. Their Figure 5. (p. 77) plots data for nebular clusters, one of which is more than 30 Mpc distant. In this plot, the linearity of the 1929 paper survives. Hubble and Humason had become so confident of the linear relationship that they proposed its use to determine distances. It is, they boasted (p. 76)

> … a new method of determining distances of individual objects in which the percentage errors actually diminish with distance.

This remark foreshadows the recent practice of identifying the location of distant galaxies merely by citing their red shift factor directly. Red shift has become the surrogate for distance.

By the time of his more popular work, Hubble (1936), he reasserted his confidence that the linearity of the relation had been vindicated. He wrote of the success of the extension of the investigation to more distant nebulae (pp. 3-4):

> The relation is plausible but not unique. The true relation might be a curve which was nearly linear within the range covered by the observations, but which departed widely from a straight line in the regions beyond the faintest nebulae in the group. This possibility was investigated by extrapolating the adopted relation extending it far out into the hitherto unobserved regions and testing it by new observations. Such a procedure often leads to minor, or even to major, revisions in the relation first selected: it has been said that research proceeds by successive approximations. However, in the investigation of red-shifts, no revision was definitely indicated. The linear relation has survived repeated tests of this nature and is known to hold, at least approximately, as far out into space as the observations can be carried with existing instruments.

8. Conclusion and Summary

The introduction sketched the inductive hierarchy to which one might assume that Hubble's inferences of 1929 conformed. We have now seen that Hubble's inductive inferences did not respect this hierarchy. Rather his inferences are interwoven non-hierarchically through the following sets of propositions:

(a) Sets of velocities of recession assigned to nebulae
(b) Sets of distances assigned to nebulae
(c) Linear relations asserted between their velocities and distances
(d) Hypothesis of *Brightest Star Magnitude*
(e) Hypothesis of *Clustering of Nebular Luminosity*

The inferences were:

(i) In Sections 4 and 5, we saw inferences from the sets of velocity (a) and distance (b) assignments to a linear relationship (c), where many of the distance assignments already presumed the two hypotheses (d) and (e).

(ii) In Section 6 we saw an inference from the means of the velocities (a) and distances (b) to an instance of the linear relationship (c). The determination the mean distance once again presumed hypothesis (e) as well as a mean absolute magnitude for nebulae determined by the inferences of (iv).

(iii) In Section 6, we saw an inference from sets of velocity assignments (a) and the linear relationship (c) to sets of distance assignments (b).

(iv) In Section 6, Hubble proceeded from the distances computed in (iii) and inferred to a set of absolute magnitudes that affirmed hypothesis (e).

The use of the velocity-distance relation in (iii) to infer back to distances became a fixture in astronomy. In his more popular work, Hubble (1936) was confident enough of this inference that he would write (p. 115):

> The velocity-distance relation, once established, could evidently be used as a criterion of distance for *all* nebulae whose velocities were known.

This inference appears initially as the mere recovery of a deductive consequence of the velocity-distance relation. It also has an inductive component. I have emphasized the "all" since all includes the nebulae originally used to establish the velocity-distance relation. We gain inductive support for an independently determined distance to some nebula if we find it conforms with the velocity-distance relation. Alternatively, if conformity fails, we have a check and a correction for the original distance determination.

The cogency of Hubble's inferences required that strong evidential support be provided for hypotheses (d) and (e), else the distance determinations of Hubble's analysis would be compromised. This was an obligation taken very seriously in the later analysis of Hubble and Humason's (1931). Of its 38 pages, 6 were devoted to a section "Upper Limit of Stellar Luminosity as a Criterion of Distance" (pp. 46-51); and another 5 pages were devoted to a section "Total Luminosity of Nebulae as a Criterion of Distance" (pp. 52-56). That is, almost 30% of the paper was spent elaborating and establishing these two hypotheses.

More generally, Hubble repeatedly offered the agreement amongst the results of all these inferences as giving general support to his analysis. We saw already his remark (1929, p. 172-

73): “This entirely unforced agreement supports the validity of the velocity-distance relation in a very evident matter.”[23] Hubble and Humason (1931, p. 43) commence their paper by defending their methods of determining nebular distance, whose initiating assumption is “supported in a general way by the consistency of the results to which it leads.” Later they announce (1931, p. 76): “Since the two investigations were based upon different criteria of distance, the close agreement emphasizes the internal consistency of our present ideas concerning luminosities of nebulae.”

In his more popular narrative (1936, p. 101), Hubble reflected back on the various criteria used to determine nebular distances, including the velocity-distance relation itself and concluded:

> The exploration of the realm of the nebulae was carried out with the aid of these criteria. The early work was justified largely by the internal consistency of the results. The foundations were firmly established, but the super-structure represented considerable extrapolations. These were tested in every way that could be devised, but the tests for the most part concerned internal consistency. The ultimate acceptance of the superstructure was due to the steady accumulation of consistent results rather than to critical and definitive experiments.

A few pages later, Hubble (1936, p. 115) reflected on the use of distances derived from the mean and range of the absolute luminosities in establishing the velocity-distance relation:

> The consistency of these results was additional evidence of the validity of the velocity-distance relation.

The consistency so important to Hubble is not the consistency of logicians. For them, it is a term of art that merely designates a lack of contradiction and can only accord the most feeble of inductive support. Hubble’s consistency is the perfect and intricate meshing of parts, such as is only likely to happen when they have captured some real aspect of the system of interest. Each computation of a distance, magnitude or velocity exposes the analysis to a test. Will the computed quantities agree with those already computed? When they do, the test is passed and the

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[23] Hubble’s (1929) does not provide further evidence explicitly and specifically supporting the *Brightest Star Magnitude* hypothesis. Perhaps this unforced agreement provides independent support for the nebular distances determined using this hypothesis and thus, indirectly, support for the hypothesis itself.

evidential support for the overall investigation grows. The strongest support for his velocity-distance relation, Hubble tells us, is accretion of many such agreements.

Appendix. Luminosity and Magnitude

Hubble's accounts above discuss the brightness of stars and nebulae using the standard system of magnitudes. Hubble's (1929) paper was written for experts, so he had no need there to explain the system. His more popular *Realm of the Nebulae* (1936, pp. 9-13), however, describes the system. The luminosity L of an object is the rate at which it emits luminous energy. Our perception of brightness associates equal increments in brightness to equal multiples of luminosity. Thus, the brightness of an object is given by a logarithmic function of the luminosity. That is, the apparent magnitudes m_1 and m_2 of two objects at the same distance from us are related to their luminosities L_1 and L_2 by

$$m_1 - m_2 = - 2.5 \log_{10} (L_1/L_2) \qquad \text{(A1)}$$

The minus sign in the relation means that a *brighter* object has a *smaller* magnitude.

This particular logarithmic relation was chosen to preserve continuity with the ancient visual system of reporting star brightnesses, already found in Ptolemy's *Almagest*. There, stars were grouped by their brightnesses into six magnitudes. The first magnitude was the brightest and the sixth the dimmest visible. If the associated luminosities are $L_1, L_2, \ldots, L_6$, then stepping through them represents equal increases in apparent brightness as long as

$$L_1/L_2 = L_2/L_3 = L_3/L_4 = L_4/L_5 = L_5/L_6 = 2.5$$

The ratio of 2.5 arises from the stipulation that that the full range of luminosities spans 100 to 1, that is, $L_1/L_6 = 100$. Thus each of the five steps corresponds to a multiplicative factor of $100^{1/5} = 2.512$, which is rounded down to 2.5. The magnitudes are labeled "visual" or "photographic" according to the media with which they are measured. The distinction is important since the two media have different sensitivities to different frequencies of light.

The apparent brightness of an object diminishes with the inverse square of distance from us. If the two objects in formula (A1) were removed to distanced d_1 and d_2 respectively, the ratio (L_1/L_2) must be replaced by the ratio $(L_1/d_1^2) / (L_2/ d_2^2)$. The relation among apparent magnitudes becomes:

$$m_1 - m_2 = - 2.5 \log_{10} (L_1/L_2) (d_2^2/ d_1^2) \qquad \text{(A2)}$$

The absolute magnitude of an object M is stipulated to be the apparent magnitude the object would have were it placed 10 parsecs distant from us.[24] Using only the distance dependency in (A2), it follows that the apparent magnitude m of an object of absolute magnitude M at a distance of d parsecs is[25]

$$m = M + 5 \log_{10} \mathrm{d} - 5 \quad \text{or} \quad \log_{10} \mathrm{d} = 0.2(m - M) + 1 \qquad \text{(A3)}$$

Hubble (1929) supposes that the intrinsic brightnesses of all nebula are within four to five absolute magnitudes of each other. Assuming a mean absolute magnitude for some nebula will lead to errors in distance estimates. To take the most extreme case, an apparent magnitude m may derive from an object with absolute magnitude M_1 at distance d_1; or another object with absolute magnitude M_2 at distance d_2, where $M_1 - M_2 = 5$. Thus we have from (A3) that

$$M_1 + 5 \log_{10} \mathrm{d}_1 = M_2 + 5 \log_{10} \mathrm{d}_2$$

and then

$$5 = M_1 - M_2 = 5 \log_{10} (\mathrm{d}_2/\mathrm{d}_1)$$

It follows that $\log_{10} (\mathrm{d}_2/\mathrm{d}_1) = 1$, so that $\mathrm{d}_2/\mathrm{d}_1 = 10$. That is the uncertainty in the absolute magnitudes of nebulae corresponds to an uncertainty of one order of magnitude in their spatial distances.

If, however, we follow Hubble's technique of averaging, this uncertainty is greatly reduced in estimating the value of the true mean.[26] For a cluster of 400 nebulae, the spread of the mean is reduced by a factor of $1/\sqrt{400} = 1/20 = 0.05$. So the spread is $5 \times 0.05 = 0.25$. Thus we have from (A3) as before

$$0.25 = M_1 - M_2 = 5 \log_{10} (\mathrm{d}_2/\mathrm{d}_1)$$

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[24] A parsec is the distance at which the mean earth-sun distance subtends one second of arc. It is a convenient astronomical unit since distances to nearby stars are revealed by their parallax during the earth's annual motion around the sun. 1 parsec = 3.258 light years. A megaparsec "Mpc" is one million parsecs.

[25] Set $d_2 = 10$ and $d_1 = d$; and note that $\log_{10} (d^2/10^2) = 2 \log_{10} d - 2 \log_{10} 10 = 2 \log_{10} d - 2$.

[26] Assume that we have n=400 independent samples from the same distribution with variance σ^2. The variance of the mean is σ^2/n. Hence the standard deviation is $\sigma/\sqrt{n}$.

We now have for the corresponding distances that $\log_{10} (d_2/d_1) = 0.05$ so that $d_2/d_1 = 1.122$. That is, the farthest distance of the associated interval of distances is merely 12% greater than the nearest.

For a group of 22 nebulae, the spread of the mean reduces by a factor of $1/\sqrt{22}=1/4.69$. If we approximate the spread of 4 to 5 magnitudes to be reduced to one order of magnitude, then we have from (A3) that

$$1 = M_1 - M_2 = 5 \log_{10} (d_2/d_1)$$

We now have $\log_{10} (d_2/d_1) = 0.2$ so that $d_2/d_1 = 1.585$. That is, the farthest distance of the associated interval of distances is 58% greater than the nearest.

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