Chapter 14

The Material Theory of Induction

1. Introduction

Inductive inference and relations of inductive support play a central role in small-e empiricism. In working with small-e empiricism, one may choose any serviceable account of inductive inference. There is, however, one account that is especially well adapted to small-e empiricism. It is the material theory of induction, as elaborated in my two works, Norton (2021, 2024). It is especially well adapted to small-e empiricism because the theory requires that inductive inferences are themselves to be warranted by background facts; and an empiricist will then seek support for those background facts in experience.

The basis of this unique adaptation is outlined in more detail in Section 2. The main claims of the material theory of induction are summarized in Section 3 and Section 4 provides a brief inventory of applications. This last inventory shows how familiar rules of inductive inference are themselves supported materially by background facts. Section 5 recalls a claim of the material theory, that inductive support in science extends beyond cases readily conforming to specific inductive rules, even if locally limited. The section offers and extended analysis of the historical determination of which are the elements of chemistry

2. Empiricism and the Material Theory

The essential function of any account of inductive inference is to amplify the scope of propositions we have already secured evidentially. Such a theory may allow us to infer from premises of lesser generality to those of greater generality. Or the theory may just tell us that the evidence of such and such propositions provides support, weak or strong, for some proposition of greater scope. However it comes about and whatever form it takes, an account of inductive inference may only bear the name if it authorizes some expansion of what we learn of contingent facts of the world.

In serving in this capacity, an account of inductive inference falls within the scope of empiricism. For an inductive logic allows us to expand what we learn from experience. Empiricism requires that this expansion must itself derive from experience. The expansion cannot rely on an a priori principle for then it contradicts the basic doctrine of empiricism.

For example, it is common in practical science to use a condition of simplicity to expand empirical results. We may have a finite set of data points that represent what we have found empirically in some finite set of cases. We expand to infinitely many cases by plotting a curve through the points, when they are drawn on graph paper. Of all possible curves, it is routine to justify the particular one selected as the simplest. We may imagine that this justification relies on some a priori metaphysics of simplicity. We might then take it as some sort of deep metaphysical truth that the world is simple; and this truth allows the expansion from a finite set of cases to an infinite set.

If this justification were to succeed, it would violate the doctrine of empiricism. It would require us to accept that our empirical explorations need to be supplemented by a special fact about the cosmos not itself derived from experience. Closer examination in Norton (2021, Ch. 6) has sought to show that there is no such additional fact of simplicity, in so far as it is to be conceived as a cosmic fact not itself inferred from experience. The crucial decision of which curve to fit is actually determined by background facts and will differ from case to case. Attempts to formulate a single universally applicable notion of simplicity applicable in all cases either fail empirically or are so qualified to protect them from falsification that they become vacuous.

Curve fitting problems should be treated on a case by case basis, according to the specific background facts of the problem at hand. In many cases, fitting a linear or near linear curve is indicated by what we already know loosely of the system under investigation. For example, in exploring the relationship of the pressure of a gas with its density under ordinary conditions, we antecedently expect some proportionality in the two quantities and that a linear relationship is likely to be a pretty good representation of it over a small range of parameter values. We would be prepared to admit small higher order corrections if the data demanded it. That allows us to explore step-wise up a hierarchy of higher order polynomials—quadratics, cubics, etc.—if they secure a better fit without obvious overfitting.

Would we ever *start* with an infinite order polynomial? In the abstract, one may think not. However, we would so start if the process is known to be cyclic in time, such as planetary orbits or ocean tides. For then we would use a combination of trigonometric functions of time, such as sines and cosines; and they are infinite order polynomials. We might start with a single sine or cosine and then add further sines or cosines to secure a better fit.

The invocation of simplicity is a convenient way to circumventing the need to explain in detail why a particular choice of curve is deemed appropriate. It is tedious in familiar cases to explain why we think it plausible that a linear relation is a good starting point. It may be especially so if our reasons are only vaguely formulated. Even so, our choice of curves must be governed by what we already know factually of the system in question. Those background facts are essential if any curve or hierarchy of curves is to be justified. Without them, all we have is a collection of dots on a page of graph paper. We can have no idea of which curve should connect them or even if a curve should connect them at all. In sum, to require simplicity in curve fitting is a convenient surrogate for the import of background facts that govern the case at hand.

The basic assertion of the material theory of induction is that all successful inductive inferences conform with this pattern. It may appear that that the inductive inference or judgment of inductive support is justified by some general rule of inductive inference. In so far as the induction is successful, the general rule can only be an intermediate whose real inductive power derives from background facts. In this way, according to the material theory of induction, any inductive expansion of experience is in turn warranted by further facts; and those warranting facts are themselves to be secured from experience.

Thus the material theory of induction provides an account in which the totality of what we learn from experience is grounded in the facts of experience themselves. In so doing, it conforms with the requirements of small-e empiricism in the most thorough-going manner.

3. Main Claims of the Material Theory of Induction

Since the two volumes, *The Material Theory of Induction* (2021) and *The Large-Scale Structure of Inductive Inference* (2024), provide a detailed account of the material theory, all that is needed here is a brief inventory of the main claims. The principal claims of the material theory as listed in (2021, Ch. 1) are:

Inductive inferences are warranted by facts not by formal schema.

All induction is local. It is contextual.

Inductive inference is generically variegated and imprecise.

Inductive risk is assessed and controlled by factual investigation.

Inductive inference is material at all levels.

When the material theory is applied to the overall structure of relations of inductive support in a science or science in general, (2024, Ch. 2) lists four principal claims:

- 1. Relations of inductive support have a nonhierarchical structure.
- 2. Hypotheses, initially without known support, are used to erect nonhierarchical structures.
- 3. Locally deductive relations of support can be combined to produce an inductive totality.
- 4. There are self-supporting inductive structures.

Subsequent chapter use these four claims to support further results:

The inevitable circularities in the relations of inductive support in a mature science are benign. (Ch. 3)

The dynamics of inductive inference allow a rich body evidence to single out a mature scientific theory as the uniquely best supported. (Ch. 4)

The traditional problem of induction is dissolved by the material theory, since the theory does not employ universal rules of inductive inference. (Ch. 6)

4. Some Applications

The material theory of induction is offered as the account of all successful inductive inference. As a result, a primary burden of the theory is to demonstrate that it is adequate to existing inductive practices, in so far as they are cogent. In particular, this means that any universal rule of induction in the present literature must have a basis in background facts, in so far as that rule can be used successfully. A major task of *The Material Theory of Induction* was to meet this challenge. It addressed a series of familiar rules of inductive inference and showed how the successes of each were derived from background facts.

Enumerate induction was addressed in Chapter 1. It argued that there is no general license that allows us to infer from particulars to their generalizations. All such inferences, if cogent, are authorized by background facts in the domain of application; and these background

facts specify just which aspects of the particulars may be generalized in each domain. Chapter 3 addresses the "gold standard" of empirical science, the replicability of results. It argues that there is no general principle of replicability. There are exceptions in all combination to the import of successful and failed replication. The individual successes and failures in specific cases, however, can be accounted materially by specific background facts.

Chapter 4 argues that efforts to provide a general schema of analogical inference leads to a regress to ever more complicated rules, as the schema seek to accommodate each new, anomalous example. Analogical inferences are not warranted by such formal rules, but by facts of analogy, where these facts are specific to each case. The general idea of the epistemic potency of what are called "epistemic virtues" or "epistemic values" is sustainable, it is argued in Chapter 5, only in so far as these values are not taken as freely chosen ends in themselves. Rather they can only succeed when they are criteria whose epistemic potency derives from background facts. Section 2 above recalled the account given in (2021, Ch. 6-7) of the example of simplicity. It has epistemic import only in so far as it is a surrogate for specific background facts in each domain of application.

Abduction or inference to the best explanation presents special problems for analysis in Chapters 8-9, since there is no sufficiently well-articulated account of explanation that can sustain the epistemic function abduction requires of it. A careful examination of many standard examples of inference to best explanation find that the inductive success in these cases can be recovered from a simple model which assigns no special epistemic powers to explanation. Rather successes derive from the adequacy to the evidence of the favored hypothesis; and the failure of a foil. The latter fails either by being contradicted by the evidence or by accruing an unsustainable inductive debt of unsupported suppositions.

The concluding Chapters 10-15 address probabilistic accounts of inductive inference, such as a Bayesian account. The material theory does authorize probabilistic inductive inferences and probabilistic inductive support. It should do so, since such probabilistic accounts have been immensely successful over a wide range of domains. The material theory does not support the common idea that *any* indefiniteness or uncertainty can be well represented by a probability measure. Instead, the material theory requires that a probability measure can be employed in any particular inductive problems *only if background facts peculiar to the problem warrant the use of the probability measure*. In physics, a physical theory may authorize a probability measure. In

social science applications, probabilities may be introduced by the assumption that some individual has been randomly sampled from a population; or that the individual may be treated as such. If this authorization is missing, the analysis is at serious risk of inductive fallacies, whose spurious results are an artefact of a misapplied probability measure.

This appraisal of probabilistic induction is supported by noting that all arguments for probabilism are necessarily circular. They are deductive inferences whose premises must be logically at least as strong as the conclusion of probabilism, so that the assumption of probabilism must be included already in the premises. To illustrate how other calculi of inductive inference are possible, several chapters describe cases in which the background facts authorize a non-probabilistic inductive logic. They include systems whose chance properties are akin to an infinite lottery machine (Ch. 13), non-probabilistic cases of indeterminism in physics (Ch. 15) and some quantum systems (Ch. 16).

5. Inductive Inferences to the Chemical Elements

The material theory of induction, as developed in the first volume, *Material Theory of Induction*, addressed the rule-based accounts of inductive inference in the present literature. It sought to demonstrate that these rules derived their cogency from background assumptions and that none has universal applicability. Further exploration of inductive inferences within the sciences in the second volume, *The Large-Scale Structure of Inductive Inference*, sought to show the narrowness of conceiving inductive inference in terms of rules. Rather, it found the possibility of a compounded inductive relationship between evidence and theory whose individual parts were not controlled by identifiable rules of induction, even after allowing that such rules might only have local application.

5.1 Two Claims of the Material Theory of Induction

The present section provides an illustration of inductive relations of support with no obvious rule-like structure. Matter theories suppose that ordinary matter is formed from the combination of a small set of fundamental elements. These theories seek to determine which are these elements from our experience of the properties of matter. Since the observed properties of matter in the small number of cases examined in the laboratory cannot deductively fix the list of elements, the project is inductive at the broadest level. These properties can provide massive support for the list; and they have successfully done so without explicit invocation of any rules of

inductive inference. The support is still inductive. In the historical cases examined below, this inductive support arose in conformity with two claims of the large-scale structure of inductive inference, identified above. The first is:

2. Hypotheses, initially without known support, are used to erect nonhierarchical structures.

When a bare statement of the empirical evidence is not by itself rich enough, hypotheses can be introduced to allow the inductive project to proceed. The essential thing is that their use is provisional. Results derived with them are provisional until independent support can be provided for the hypotheses. Figuratively, we might say that the use of any hypotheses in this manner incurs an inductive debt. It must eventually be repaid. If such independent support cannot be secured, that is, if the inductive debt cannot be repaid, the hypothesis must be abandoned and as must also results dependent on it.

The second claim is:

3. Locally deductive relations of support can be combined to produce an inductive totality.

In the simplest case, there are two sets of propositions that are mutually supporting. The first set is deduced from the evidence by supposing the second set; and the second set is deduced from evidence by supposing the first set. The overall effect is that both are supported. Figuratively it is an arch-like structure. Each side of the arch supports the other side; and the whole structure stands. Whereas the individual inferences are deductive, the overall import is inductive.

5.2 Hypotheses for Identifying Elements

There are, broadly speaking, two hypotheses used historically to enable recovery of the elements. The first is:

Inheritance. The properties of compounded matter are inherited from those of its elements.

In the simplest case, the property manifested in the element itself is then manifested in the compound. In more indirect cases, the capacity to manifest some property is conferred by the element, even if that element does not exhibit the property itself. We shall see that this *inheritance* hypothesis ultimately did fail when independent evidence for it was not forthcoming. It was replaced, successfully, by:

Persistence. Elements are immutable participants in chemical processes.

Independent evidence for it could be secured in the form of mutually supporting propositions that instantiate an arch-like structure as indicated above. With its use, the modern table of the chemical elements was discovered and affirmed.

We might now be inclined to use these hypotheses directly to establish some relation of inductive support. That observed samples of matter displays a specified property, we might say, provides some level of evidential support for an element that confers it. Or the persistence of some substance in an observed chemical change is evidence for the substance's elemental character.

In the historical episodes to be examined here, these hypotheses were not used in this more cautious, inductive sense. These earlier eras were epistemically more optimistic. In effect, the hypotheses used were so strengthened that they could directly support deductive inferences to the identify of elements. The strengthen versions, in effect, asserted something like this. If a finite but substantial number of cases in experience conformed with the hypotheses, then the identity of the corresponding elements was established.

5.3 The Aristotelian Elements

The ancient identification of earth, air, fire and water as the elements present in terrestrial matter prevailed for over two millennia. Aristotle derived this list of elements from the properties manifested by terrestrial matter. In his *On Generation and Corruption* (1984, Book II, 330a25–330a29), Aristotle judged there to be four irreducible properties, the pairs hot-cold and moist-dry. He could then use the hypothesis of *inheritance* to recover the four elements from all four compatible combinations of the properties. (330a30–330b21)

For Fire is hot and dry, whereas Air is hot and moist. . .; and Water is cold and moist, while Earth is cold and dry.

These elements, or "simple bodies," as Aristotle called them, are not the fire, air and the like that we commonly encounter. Each is compounded of several elements with one dominating. He wrote: (330b22–330b30)3

In fact, however, fire and air, and each of the bodies we have mentioned, are not simple, but combined. The simple bodies are indeed similar in nature to them, but

¹ The identification of these as the controlling, irreducible properties is another hypothesis whose development and fate will not be pursued here.

not identical with them. Thus the simple body corresponding to fire is firelike, not fire; that which corresponds to air is air-like; and so on with the rest of them. But fire is an excess of heat, just as ice is an excess of cold.

In this analysis, the use of *inheritance* is in the stronger, deductive mode indicated above. The empirical evidence was only from some set of samples of materials, small in relation to the totality of terrestrial matter. They manifest the properties of hot, cold, moist and dry. Yet the conclusion was that all terrestrial matter was derived from the associated four elements with no qualification that the conclusion was merely well supported.

It is a matter of further investigation to determine whether Aristotle understood his use of *inheritance* to conform with the role identified in the material theory of induction for hypotheses. My concern here is the logical structure of the inductive case made for these four elements. There it does serve as an hypothesis that requires further independent support to complete the analysis.

The use of *inheritance* as an essential component of the understanding of elements persisted long after Aristotle's time. In the writings of Paracelsus from the sixteenth century, we find almost exactly the same analysis of elements as in Aristotle's writings. He wrote, for example: (1894, p. 264)

All created things in the universe, then, were born of four mothers, that is to say, of four elements. These four elements, it should be remarked, exactly sufficed for the creation of all things. Neither more nor fewer were requisite. ... The warm is the element of fire. The cold is the element of earth. The moist is the element of water. The dry is the element of air. The next thing to be considered is how each of the aforesaid natures is what it is peculiarly or separately. Fire is only warm, not dry, not moist. Earth is only cold, not dry, not moist. Water is only moist, and not warm, not cold. Air is only dry, and not warm, not cold. Thus it is they are called elements. As did Aristotle, Paracelsus distinguished the matter of our experience from the essential elements (p. 85):

In all things four elements are commingled one with the other, but in each thing one of these four is perfect and fixed. ... Now, the fact is that some persons think the body to be a true element and quality, and that it in some way displays the virtue of a true element. This is because the body, like the three imperfect elements, is tinged

and qualified, each according to its own nature, by the fixed, perfect, and predestinated element, as by its indwelling inhabitant.

I am loathe to offer any synoptic overview of the writing of Paracelsus, from whom the adjective "bombastic" derives. For Paracelsus also endorsed the alchemical *tria prima*. He wrote (1894, p. 284):

The visible and tangible is the body of the universe, consisting of three primals, Sulphur, Mercury, and Salt.

At this stage in the history of matter theory and still for at least a century more, the inductive debt incurred by the Aristotelian analysis was not repaid. *Inheritance* continued to be used without an adequate evidential basis for it. It had become an unquestioned background assumption in need of questioning. This is reflected in Antoine Lavoisier's somber appraisal of past work in chemistry. He lamented: (1790, p. xvii)

Hence it is by no means to be wondered, that, in the science of physics in general, men have often made suppositions instead of forming conclusions. There suppositions, handed down from one age to another, acquire additional weight from the authorities by which they are supported, till at last they are received, even by men of genius as fundamental truths.

He singled out the Aristotelian four elements for pointed criticism and identified its hypothetical character: (p. xxii)

... the fondness for reducing all the bodies in nature to three or four elements proceeds from a prejudice which has descended to us from the Greek Philosophers. The notion of four elements, which, by the variety of their proportions, compose all the known substances in nature, is a mere hypothesis, assumed long before the first principles of experimental philosophy or of chemistry has an existence. In those days, without possessing facts, they framed systems...

5.4 Phlogiston Chemistry

In standard historical narratives, phlogiston chemistry is the villainous old chemistry vanquished heroically by the new oxygen chemistry. The narrative here provides a different perspective. The phlogiston chemistry of the eighteenth century was distinctive in moving away from *inheritance* as the primary mode for identifying chemical elements and replacing it with

empirically grounded considerations of *persistence*. The transition was tacit, but important, and reflected the close engagement of phlogiston chemistry with the details of laboratory experience.

Earlier writing in phlogiston chemistry retained the idea of *inheritance*. Most notably, the combination of phlogiston with a calx produces a metal. The phlogiston endows the matter with metallic properties and the calx with other distinguishing features. For example, one compendium notes: (Lewis, 1754 p. 53)

The phlogistic principle is the same in one metal as in another, in metals as in other bodies, in the mineral as in the vegetable and animal kingdoms. When metals, by the loss of their own phlogiston, have been changed into a calx or vitreous mass; the introduction of any other inflammable matter, from vegetables or animals, Charcoal, Resins, Oils, Fats, &c. instantly restores their metallic appearance, and all their pristine qualities.

The calx differs greatly in different metals: It is on this, that the distinguishing characters of each particular metal depend, the calx of one metal forming always with phlogiston no other than the fame metal again.

We already see here considerations of *persistence* concerning phlogiston that affirms its elemental character. For Lewis notes that a calx can be restored to the metallic state with an infusion of phlogiston derived from a large variety of sources. There is just the one phlogiston passing from compound to compound.

Later treatments of phlogiston in Joseph Priestley's experimental investigations into gases ("airs") focus almost entirely on identifying the role of phlogiston in chemical processes, in conformity with *persistence*. The hypothesis of *inheritance* plays no essential role in his analyses. Priestley's orientation is also notable for its close contact with laboratory experience. He was to be guided by empirical work. Having found that burning charcoal diminishes the ambient air, Priestley conjectured that this was a general process to be found when all bodies, rich in phlogiston, burn. This was a matter worthy of further investigation. He wrote: (1775, p. 133, his emphasis)

Having been led to suspect, from the experiments which I had made with charcoal, that the diminution of air in that case, and perhaps in other cases also, was, in some way or other the consequence of its having more than its usual quantity of phlogiston, it occurred to me, that the calcination of metals, which are generally

supposed to consist of nothing but a metallic earth united to phlogiston, would tend to ascertain the fact, and be a kind of *experimentum crucis* in the case.

Priestley's conjecture, he reported, was affirmed by his subsequent experiments in which he heated metals in a bell jar to burning by means of light focused by mirrors or lenses.

Decades later, a dejected Priestley had to acknowledge that Lavoisier's oxygen theory had fully supplanted the phlogiston theory. He still sought to defend phlogiston theory in his (1803) *Doctrine of Phlogiston Established*. Section 1 was headed: "That Metals are compound Substances, and contain Phlogiston, proved from the Solution of Iron in the vitriolic and marine Acids, and from some other Considerations." His proof depended on reflecting on processes in which the persistence of phlogiston, he urged, played an essential role

5.5 Lavoisier's *Elements of Chemistry*

The overt transition to *persistence* was implemented by Antoine Lavoisier's (1790) celebrated *Elements of Chemistry*. It is apt in a volume on empiricism to note that Lavoisier declared at the outset an allegiance to a strict empiricism. Having lamented errors by earlier chemists, he announced the "only method of preventing such errors": (p.xviii)

We must trust to nothing but facts: These are presented to us by Nature, and cannot deceive. We ought, in every instance, to submit our reasoning to the test of experiment, and never to search for truth but by the natural road of experiment and observation.

In a much-quoted passage, Lavoisier declared overtly that he is abandoning the search for elements in a fundamental sense and restricting his search for a repertoire of elements that persist under the analytic processes then available. They would be the elements revealed by the means upon which Lavoisier insisted, experiment and observation: (p.xxiv, his emphasis)

... if, by the term elements, we mean to express those simple and indivisible atoms of which matter is composed, it is extremely probable we know nothing at all about them; but, if we apply the term *elements*, or *principles of bodies*, to express our idea of the last point which analysis is capable of reaching, we must admit, as elements, all the substances into which we are capable, by any means, to reduce bodies by decomposition.

Lavoisier recognized that the resulting list of elements was tentative and might change with improvements in analytic methods. He continued:

Not that we are entitled to affirm, that these substances we consider as simple may not be compounded of two or even of a greater number of principles; but, since these principles cannot be separated, or rather since we have not hitherto discovered the means of separating them, they act with regard to us as simple substances, and we ought never to suppose them compounded until experiment and observation has proved them to be so.

Lavoisier was prudent to emphasize the tentative character of his list of elements. For he was then unable to separate out elemental sodium and potassium from their compounds. They entered his tables of simple substances as soda (now sodium carbonate) and potash (now potassium carbonate). The limitation was soon lifted when Humphrey Davy separated metallic sodium and potassium electrolytically in 1807.

Lavoisier proceeded to identify the element oxygen using processes similar to those investigated by Priestley. In his Preface, Lavoisier gave a synoptic statement of how the one asyet-unnamed element oxygen participated in the formation of many different metallic calxes and, as he later mistakenly concluded, also in the formation of acids: (1790, p.xxviii)

Metallic substances which have been exposed to the joint action of the air and of fire, lose their metallic lustre, increase in weight, and assume an earthy appearance. In this state, like the acids, they are compounded of a principle which is common to all [oxygen], and one which is peculiar to each.

Here, as in all of Lavoisier's discussion, Lavoisier passes without apparent concern from his identification of transformations observed in the small number of experiments in his laboratory to a general claim of the universality of the transformation. This, I understand, relies on the stronger deductive reading of *persistence*.

In the best know of his many experimental reports, Lavoisier (1790, Ch. III) formed red calx of Mercury by heating metallic Mercury in air and then, with further heat, decomposed it back into metallic Mercury and a gas he identified as oxygen. He understood the calx to be an oxide of Mercury that was decomposed back to elemental Mercury and oxygen gas. Priestley had considered similar processes. He understood the processes to be a reversal of Lavoisier's understanding. For him, Lavoisier's process was the decomposition of Mercury into its calx and phlogiston.

Both are using *persistence*. What persists in the process without decomposition is to be identified as elemental. They each recognized that a single element was passing between the calx and the Mercury. All that remained open was to decide which of the two was decomposing and releasing the element. A rapid consensus emerged that Lavoisier's understanding was better supported by the empirical evidence. For Lavoisier could isolate oxygen gas, where Priestly could not isolate pure phlogiston; and Lavoisier could report that the weight lost by the calx was precisely that gained by the liberated gas.

For completeness, we should note that Lavoisier did not abandon *inheritance* completely. Where he used it, it proved troublesome. That bodies liquefy and expand on heating puzzled him. He wrote (p.4) "It is difficult to comprehend these phenomena without admitting them as the effect of a real and material substance ..." This substance entered his table of simple substances (p.175) as "caloric" along with "light," even though he conceded that we are unable to isolate caloric (p. 19). Thus, oxygen appears everywhere (e.g. p. 52) as "oxygen gas," which is a compound of oxygen and caloric. This was part of his more general decision to employ the term "gas" to designate all such compounds with caloric. He wrote: (p. 50, his emphasis) "*Gas*, therefore, in our nomenclature, becomes a generic term, expressing the fullest degree of saturation in any body with caloric..." A second use of *inheritance* was that oxygen had the power to create acids, so that the name "oxygen" derives from the Greek for acid maker (p. 51).

Neither consideration was fruitful. The idea of caloric as an elemental substance was abandoned with the development of thermodynamics and the kinetic theory of heat later in the nineteenth century. That oxygen was the acid maker left Lavoisier with the insoluble problem of determining just how oxygen could create muriatic or marine acid. We now know it as hydrochloric acid, HCl, and it contains no oxygen.

5.6 The Inductive Relations

By the mid nineteenth century, the idea of *inheritance*, that elements could be identified in compounds by some distinctive property, had been completely rejected. Its rejection had become a standard part of the preamble to introductory chemistry texts.² Miller (1855. p.2)

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² These are the earliest statements of this form that I could find. No doubt earlier statements eluded my search.

remarked in passing on compounds of thirteen non-metallic elements in the opening pages of a lengthy treatise:

These compounds have in general no more resemblance in properties to the elements which have united to form them than a word has to the letters of which it is made up.

A popular introduction (Steele, 1867, p.4) was designed to be accessible to students described as "a large class of pupils in our schools who can pursue this branch [chemistry] only a single term." (The term, we would suppose, is the fourteen weeks of the volume's title.) Its opening pages defined basic terms and reported:

COMPOUNDS are utterly unlike their elements in all their properties. Examples: yellow sulphur and white quicksilver form red vermilion; the inert nitrogen and the oxygen of the air constitute a corrosive acid—aquafortis; charcoal, hydrogen, and nitrogen produce the deadly prussic acid; solid charcoal and sulphur make a colorless liquid; poisonous and offensive chlorine combines with the brilliant metal sodium to form common salt.

The competing hypothesis *persistence* was successful. Its supposition had been used by Lavoisier painstakingly to build up his list of elements and the compounds they formed. Once he had identified oxygen as an element through his examination of the calx of Mercury, he could expand the analysis in his (1790) Chapter VII to all metal calxes. They can be decomposed to metals and oxygen. They are all oxides. Similarly in Chapter VIII he could show that water could be decomposed. It could be made to react with iron to produce a black iron oxide. The oxygen was supplied by the water, which now was revealed to be a compound of oxygen.

Lavoisier's analysis proceeded in this methodical, step by step fashion. He examined an inventory of which transformations were found possible in the chemical laboratories and, using the assumption of *persistence*, identified those components that were never decomposed into parts as elements. No known experiment decomposes oxygen. Therefore, under the authority of *persistence*, it is an element. This and the many other inferences like it are deductive, using the stronger reading of *persistence*. Under it, Lavoisier is entitled to deduce, from his small set of laboratory results, that substances preserved in its processes, such as oxygen, are elements; and these conclusions are to hold universally.

What of the independent support *persistence* itself requires? It results from a simple reversal of the last set of inferences. Once Lavoisier has developed a comprehensive repertoire of elements, compounds and the reactions that connect them, it is a deductive consequence that elements are just those substances that persist without decomposition

These last two sets of inferences form the mutually supporting sets of deductive relations that provide the overall support for Lavoisier's analysis. They are the two mutually supporting sides of the arch. Their overall import is inductive. For the empirical evidence, Lavoisier's laboratory results, are particulars of time and space. The resulting system of elements and compounds are universal in time and space.

Finally, to preclude confusion, the use of hypotheses here does not conform with the now familiar scheme of hypothetico-deductive confirmation. In this latter scheme, an hypothesis is confirmed when we deduce from it predictions verified among the observations. In the schemes reviewed in this section, the deductions proceed in the opposite direction. Lavoisier infers *from* his laboratory evidence of a persisting substance *to* the conclusion that the substance is elemental.

6. Conclusion

Inductive inference and relations of inductive support lie at the heart of small-e empiricism. The advantage of treating them materially is that the material theory of induction is uniquely adapted to the core commitment of empiricism, that we only learn of the world through experience of it. The two combine to form a well-integrated system.

The risk to small-e empiricism of employing a rule-based account of inductive inference is that a rule, employed in a context in which is it not warranted, introduces factual content without proper empirical support. For example, the rule of enumerative induction authorizes us to infer from the proposition that some A's are B, to the proposition that all A's are B. The well-known difficulty is that the rule can only be used successfully for very few of the possible B's in each circumstance. If we apply it with B's outside these very few, we arrive at spurious generalizations about A's.

Enumerative induction is a simple and transparent instance of this failure. It can happen with any rule-based account, if the rule is applied inappropriately, that is, if it is applied without proper material warrant in background facts. For example, if we use probabilities to represent

complete neutrality of evidential support, the "disjunctive-inductive" fallacy follows, as described in Norton (2010). Assume that our evidence is completely neutral over an exhaustive, mutually exclusive set of outcomes, O_1 , ..., O_{100} . The evidence provides nothing of use in discriminating among them. If we represent that neutrality by a roughly uniform probability distribution over all one hundred outcomes, we would then find that the probability is very high that the outcome that obtains is one of the first ninety-nine. That spurious conclusion is an artefact of the misapplication of probabilities. The appropriate assessment should just be the assumption made at the outset, that the evidence gives us no basis for any conclusion over the comparative likelihood of the outcomes, beyond the original complete indefiniteness.

References

- Aristotle (1984), On Generation and Corruption. In The Complete Works of Aristotle. Volume 1. H. H. Joachim, trans. Princeton: Princeton University Press.
- Lavoisier, Antoine-Laurent (1790) *Elements of Chemistry*. Trans. Robert Kerr. Edinburgh: William Creech.
- Lewis, William (1754) The Chemical Works of Caspar Neumann, M.D. London: W. Johnston.
- Miller, William Allen (1855) *Elements of Chemistry: Theoretical and Practical.* Part 1. *Chemical Physics.* London: John W. Parker and son.
- Norton, John D. (2010) "Cosmic Confusions: Not Supporting versus Supporting Not-." *Philosophy of Science*, **77**, pp. 501-23.
- Norton, John D. (2021) *The Material Theory of Induction*. BSPS*Open*/University of Calgary Press.
- Norton, John D. (2024) *The Large-Scale Structure of Inductive Inference*. BSPS*Open/*University of Calgary Press.
- Paracelsus (1894) *The Hermeutic and Alchemical Writings of Aurelous Philippus Theophrastus Bombast of Hohenheim.* Vol II. A. E. Waite, ed. trans., London: James Elliott and Co.
- Priestley, Joseph (1775) *Experiments and Observations on Different Kinds of Airs*. 2nd ed. London: J. Johnson.
- Priestley, Joseph (1803) *The Doctrine of Phlogiston Established, and that of the Composition of Water Refuted.* 2nd ed. Northumberland: P. Byrne.
- Steele, J. Dorman (1867) Fourteen Weeks Course in Chemistry. New York: A. S. Barnes.