

§10 Dynamics of (Slowly Accelerated) Electron

Why slowly? Eliminate energy loss to radiation

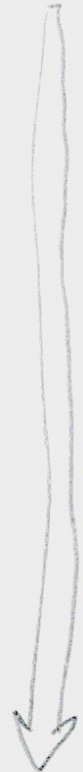
Goal: theory leaves Maxwell electrodynamics untouched. But dynamics of ordinary bodies will be affected. mass becomes velocity dependent. Express as "longitudinal", "transverse" mass

Charge e momentarily at rest in k

$$\mu \frac{d^2 \xi}{d\tau^2} = \epsilon X' \quad \mu \frac{d^2 \eta}{d\tau^2} = \epsilon Y' \quad \mu \frac{d^2 \zeta}{d\tau^2} = \epsilon Z'$$

$$\begin{aligned} \frac{d^2 \xi}{d\tau^2} &= \frac{dw_x}{d\tau} = \frac{dt}{d\tau} \cdot \frac{d}{dt} \left(\frac{w_x - v}{1 - \frac{vw_x}{c^2}} \right) \\ &= \beta \frac{1}{1 - \frac{vw_x}{c^2}} \frac{dw_x}{dt} + \beta \underbrace{(w_x - v)}_0 \frac{d}{dt} \left(\frac{1}{1 - \frac{vw_x}{c^2}} \right) \\ &\quad \underbrace{\beta^2 \text{ since } w_x = v}_{\beta^2 \text{ since } w_x = v} \quad \underbrace{0 \text{ since } w_x = v}_{0 \text{ since } w_x = v} \\ &= \beta^3 \frac{d^2 x}{dt^2} \end{aligned}$$

$$\begin{aligned} \frac{d^2 \eta}{d\tau^2} &= \frac{dw_y}{d\tau} = \frac{dt}{d\tau} \frac{d}{dt} \left(\frac{w_y}{\beta(1 - \frac{vw_x}{c^2})} \right) = \frac{\beta}{\beta(1 - \frac{vw_x}{c^2})} \cdot \frac{dw_y}{dt} + \beta w_y \frac{d}{dt} \left(\frac{1}{\beta(1 - \frac{vw_x}{c^2})} \right) \\ &= \beta^2 \frac{d^2 y}{dt^2} \quad \text{and similarly for } \frac{d^2 \zeta}{d\tau^2} \end{aligned}$$



Charge e moving at $\underline{v} = (v, 0, 0)$ in K

$$\begin{aligned} \underbrace{\mu \beta^3 \frac{d^2 x}{dt^2}}_{\substack{\uparrow \\ \text{longitudinal} \\ \text{mass}}} &= \epsilon X' & \mu \beta^2 \frac{d^2 y}{dt^2} &= \epsilon \beta \left(Y - \frac{v}{c} N \right) \\ \underbrace{\mu \beta^2 \frac{d^2 z}{dt^2}}_{\substack{\uparrow \\ \text{transverse} \\ \text{mass}}} &= \epsilon \beta \left(Z + \frac{v}{c} M \right) \end{aligned}$$

Einstein uses

$$\text{Force} = \text{mass} \times \text{Acceleration} \quad \longrightarrow$$

Moving bodies have different masses (resistance to acceleration) in direction of motion & transverse to direction motion

Hence: Longitudinal, Transverse mass



Later:

Better choice is

$$\text{Force} = \frac{d}{dt} (\text{mass} \times \text{velocity}) \quad \longrightarrow$$

Same mass in both directions

$$\text{mass} = \frac{\text{rest mass}}{\sqrt{1 - v^2/c^2}}$$

Final Results

Kinetic energy of mass at v

= Work to (slowly) accelerate charge to v

$$= mc^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}$$

Three properties of electron's motion amenable to experimental test.