§10 Dynamics of $\underbrace{\text{(Slowly Accelerated)}}$ Electron

Why slowly? Eliminate energy loss to radiation

Goal: theory leaves maxwell electrodynamics untouched. But dynamics of ordinary bodies will be affected. mass becomes velocity dependent. Express as "longitudinal", "transverse" mass

Charge ε momentarily at rest in k

$$\mu \frac{d^2\xi}{d\tau^2} = \varepsilon X' \quad \mu \frac{d^2\eta}{d\tau^2} = \varepsilon Y' \quad \mu \frac{d^2\zeta}{\partial \zeta^2} = \varepsilon Z'$$

$$\frac{d^2\xi}{d\tau^2} = \frac{dw_\xi}{d\tau} = \underbrace{\frac{dt}{d\tau}}_{\beta} \cdot \frac{d}{dt}\left(\frac{w_x - v}{1 - \frac{vw_x}{c^2}}\right)$$

$$= \beta \underbrace{\frac{1}{1-\frac{vw_x}{c^2}}}_{\beta^2 \text{ since } w_x = v} \frac{dw_x}{dt} + \beta \underbrace{(w_x - v)}_{0 \text{ since } w_x = v} \frac{d}{dt}\left(\frac{1}{1 - \frac{w_x v}{c^2}}\right)$$

$$= \beta^3 \frac{d^2x}{d\tau^2}$$

$$\frac{d^2\eta}{d\tau^2} = \frac{dw_\eta}{d\tau} = \underbrace{\frac{dt}{d\tau}}_{\beta} \frac{d}{dt}\left(\frac{w_y}{\beta(1-\frac{vw_x}{c^2})}\right) = \underbrace{\frac{\beta}{\beta(1-\frac{vw_x}{c^2})}}_{\beta^2} \cdot \frac{dw_y}{dt} + \beta \overbrace{w_y}^{0} \frac{d}{dt}\left(\frac{1}{\beta()}\right)$$

$$= \beta^2 \frac{d^2y}{d\tau^2} \qquad \text{and similarly for } \frac{d^2\zeta}{d\tau^2}$$

Charge ε moving at $\underline{v} = (v,0,0)$ in K

$$\underbrace{\mu\beta^3}_{\text{longitudinal mass}} \frac{d^2x}{dt^2} = \varepsilon X \quad \mu\beta^2 \frac{d^2y}{dt^2} = \varepsilon\beta\left(Y - \frac{v}{c}N\right)$$

$$\underbrace{\mu\beta^2}_{\text{transverse mass}} \frac{dz}{dt^2} = \varepsilon\beta\left(Z + \frac{v}{c}M\right)$$

Einstein uses

Force = mass × Acceleration $\Longrightarrow$ moving bodies have different masses (resistance to acceleration) in direction of motion & transverse to direction motion

Hence: Longitudinal, Transverse mass

$\Downarrow$

Later:
Better choice is

Force = $\frac{d}{dt}$ (mass × velocity) $\Longrightarrow$ Same mass in both directions

$$\text{mass} = \frac{\text{rest mass}}{\sqrt{1 - v^2/c^2}}$$

Final Results | Kinetic energy of mass at v = Work to (slowly) accelerate charge to v $= \mu c^2 \left\{ \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right\}$

Three properties of electron's motion amenable to experimental test.