Notes by John D. Norton

Does the Inertia of a Body Depend on its Energy Content?

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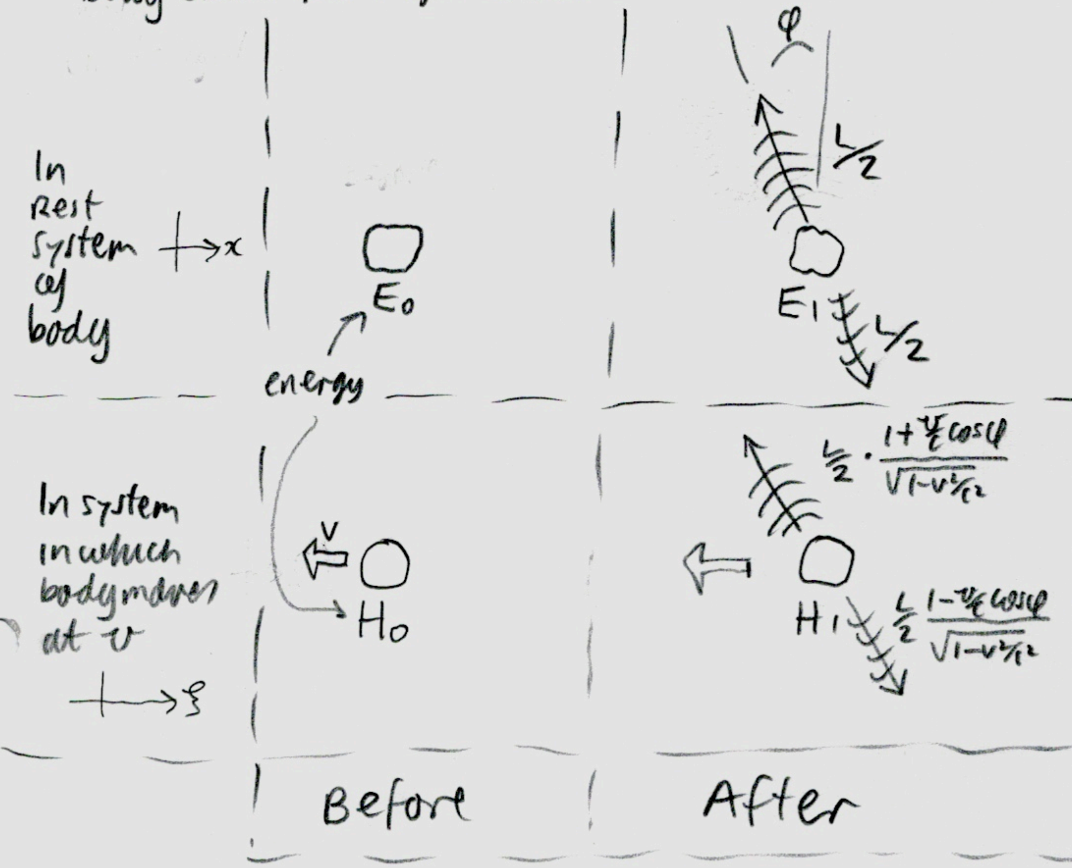
Goal: show: Body gains/loses energy E $\Longrightarrow$ Body gains/loses mass E/c^2 "Inertia of energy"

Body emits plane light waves



$$E_0 = E_1 + L$$

$$H_0 = H_1 + \frac{L}{\sqrt{1-v^2/c^2}}$$

Change in kinetic energy due to emission

$$= \underbrace{(H_0 - E_0)}_{\text{kinetic energy before}} - \underbrace{(H_1 - E_1)}_{\text{kinetic energy after}} = \underbrace{L\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)}_{\frac{1}{2}\left(\frac{L}{c^2}\right)v^2 \text{ for small } v}$$

$\leftarrow$ Newtonian kinetic energy

$\frac{1}{2}\underbrace{\left(\frac{L}{c^2}\right)}$ $\rightarrow$ interpret as loss of mass L/c^2

see my companion article

Much simpler derivation uses momentum balance, momentum $= mv$ } Precluded since AE's longitudinal, transverse mass ruled out mv as conserved momentum

Eliminating the reference to the Newtonian Limit: Rest mass of a body moving at v with kinetic energy $L\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)$

Rest mass m DEFINED as

$$\underset{\text{Force}}{F} = m \frac{d\overset{\text{speed}}{V}}{dt}\Bigg|_{V=0}$$

works for – Longitudinal & transverse mass
– Planck's definition $F = \frac{d}{dt}(m(v)\,v)$

Recover F from expression for kinetic energy

$$\text{"}\underbrace{dE = F.ds}_{\substack{\text{works only for}\\ \text{Planck definition}\\ \text{of force ?}}}\text{"} \Rightarrow \frac{dE}{dt} = FV \Rightarrow F = \frac{1}{V}\frac{dE}{dt}$$

or works at $v=0$ only for other definition??

$$E = L\left(\frac{1}{\sqrt{1-V^2/c^2}} - 1\right) = L\left(\cancel{1} + \frac{1}{2}\frac{V^2}{c^2} + \frac{3}{8}\frac{V^4}{c^4} + \frac{5}{16}\frac{V^6}{c^6} \cdots \cancel{-1}\right)$$

Don't have to use series. Can also differential $\frac{1}{\sqrt{1-V^2/c^2}}$ directly. Just a little more work.

$$F = \frac{1}{V}\frac{dE}{dt} = \frac{1}{V}\cdot L\left(\frac{V}{c^2}\frac{dV}{dt} + \frac{3}{2}\frac{V^3}{c^4}\frac{dV}{dt} + \cdots\right)$$

$$= L\left(\frac{1}{c^2}\frac{dV}{dt} + \frac{3}{2}\frac{V^2}{c^4}\frac{dV}{dt} + \cdots\right)$$

At $V=0$ $\quad F = \frac{L}{c^2}\frac{dV}{dt} \Longrightarrow \boxed{m = L/c^2}$

OR... Take shortcut:

For small V/c, Relativistic dynamics $\approx$ Newtonian dynamics

$$\lim_{V/c \to 0} M_{\text{relativistic}} = M_{\text{Newtonian}}$$

i.e. Let Newtonian mechanics do the job of introducing the definition of mass

$$\text{Kinetic energy} \quad L\left(\frac{1}{\sqrt{1-V^2/c^2}}-1\right) = \underbrace{\frac{1}{2}\frac{L}{c^2}V^2}_{\substack{\text{Newtonian kinetic energy}\\ \text{with mass } L/c^2}} - \frac{1}{8}\frac{L}{c^4}V^4 + \cdots$$

$$\boxed{\frac{1}{\sqrt{1-v^2/c^2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \cdots}$$

the easy way:

$f(x) = \frac{1}{\sqrt{1-x}} = (1-x)^{-\frac{1}{2}}$ $\qquad$ $f(0) = 1$

$f'(x) = \frac{1}{2}(1-x)^{-\frac{3}{2}}$ $\qquad$ $f'(0) = \frac{1}{2}$

$f''(x) = \frac{3}{2.2.}(1-x)^{-\frac{5}{2}}$ $\qquad$ $f''(0) = \frac{3}{2.2}$

$f'''(x) = \frac{3.5}{2.2.2}(1-x)^{-\frac{7}{2}}$ $\qquad$ $f'''(0) = \frac{3.5}{2.2.2}$

$\vdots$ $\qquad$ $\vdots$

$$(1-x)^{-\frac{1}{2}} = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{1}{2!}\cdot\frac{3}{2.2}x^2 + \frac{1}{3!}\frac{3.5}{2.2.2}x^3 + \cdots$$

set $x = v^2/c^2$

$$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \underbrace{\frac{1}{2!}\,\frac{3}{2.2.}}_{\frac{3}{8}}\frac{v^4}{c^4} + \underbrace{\frac{1}{3!}\,\frac{3.5}{2.2.2.}}_{\frac{5}{16}}\frac{v^6}{c^6} + \cdots$$

Newtonian Analog of Result

Define mass m as $\overset{\text{Force}}{F} = m \dfrac{dv}{dt}$ ← speed.

Kinetic energy "$dE = F \cdot ds$" $\Longrightarrow \dfrac{dE}{dt} = F \cdot v = m \dfrac{dv}{dt} v = \dfrac{d}{dt}\left(\tfrac{1}{2} m v^2\right)$

so kinetic energy $E = \tfrac{1}{2} m v^2 + \underbrace{\text{constant}}_{0 \text{ sin } E(0)=0}$

Hence, if kinetic energy is $\tfrac{1}{2} ? v^2$

read off that ? is mass.