A. Einstein, "Principle of conservation of motion..." 1906

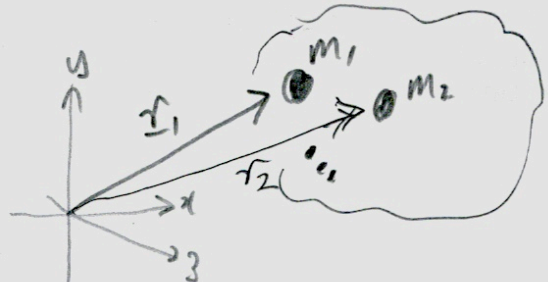
§1 A special case ... New derivation of $E=mc^2$

Overall

conservation of motion of center of gravity $\Longrightarrow$ Energy S has inertia S/c^2

NB Einstein uses V for our c

Center of gravity for system of masses $m_1, m_2, \ldots$



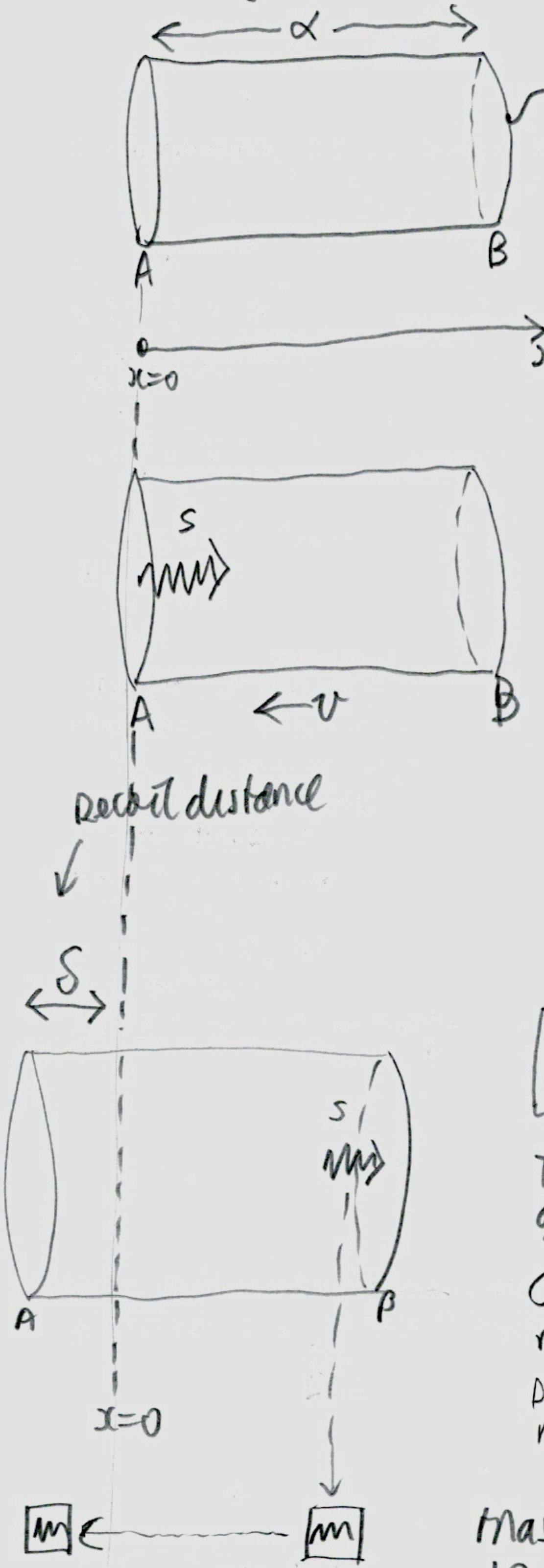
$$\underset{\sim}{R} = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2 + \cdots}{m_1 + m_2 + \cdots} \qquad \text{x-component } R_x = \frac{m_1 r_{1x} + m_2 r_{2x} + \cdots}{m_1 + m_2 + \cdots}$$

Theorem of ordinary mechanics : For an isolated system, center of gravity remains at rest or in uniform motion

It cannot accelerate from rest and then decelerate back to rest

* This will happen, unless energy S has inertia S/c^2 *

Stationary cylinder of mass M



$\boxed{\text{time} = 0}$

Radiant energy S emitted at A towards B
Radiation carries momentum S/c in $+x$ dir'n
∴ Mass gains negative momentum $-S/c$
∴ mass moves in $-x$ direction at v:

$$Mv = S/c$$

$$v = \frac{S}{Mc}$$

This assumes $S/c^2 \ll M$ so that
1. $v \ll c$ and $\frac{M}{\sqrt{1-v^2/c^2}} \approx 1$
2. Mass loss from M is negligible

$\frac{S}{Mc^2} = \frac{v}{c} \ll 1$

$\boxed{\text{time} = \frac{\alpha}{c}}$

Time of flight $= \frac{\alpha-\delta}{c} \approx \frac{\alpha}{c}$ assuming δ is v. small

Cylinder comes to rest on reabsorbing S.

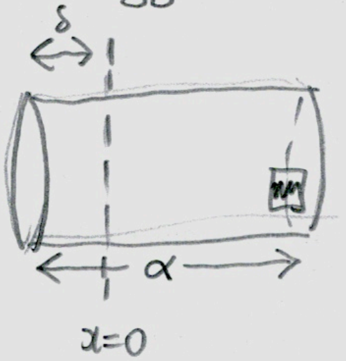
Distance moved $\delta = \frac{\alpha}{c} \cdot \frac{S}{Mc} = \frac{S}{Mc^2}\alpha$

Massless device carries energy S back to A to complete cycle.

$\boxed{\text{Net effect: center of gravity of isolated system has moved itself by distance } \delta}$

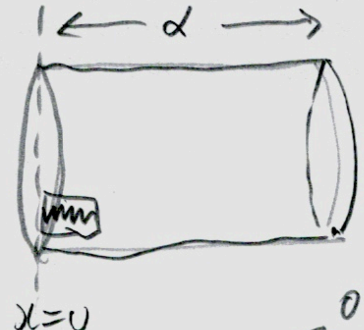
Solution: Energy S has mass S/c^2, so center of gravity never moved from initial position. $\Rightarrow$ when device moves energy S from B to A: Reaction force on cylinder moves it back to initial position

Energy S located at B



set I = mass of energy S

Energy S located at A



center of gravity about $x=0$: $M\left(\frac{\alpha}{2}-\delta\right)+I(\alpha-\delta)$

center of gravity about $x=0$: $M\frac{\alpha}{2} + \cancel{I \cdot 0}^{\,0}$

set equal and solve for I

$$M\left(\frac{\alpha}{2}-\delta\right)+I(\alpha-\delta) = M\frac{\alpha}{2}$$

$$\underbrace{I(\alpha-\delta)}_{\text{Approx } \alpha \text{ since } \delta \ll \alpha} = M\delta = M\overbrace{\frac{S\alpha}{Mc^2}}^{\delta}$$

$$\therefore \boxed{I = S/c^2}$$

Doc. 35

THE PRINCIPLE OF CONSERVATION OF MOTION OF THE CENTER OF GRAVITY AND THE INERTIA OF ENERGY

by A. Einstein

[*Annalen der Physik* 20 (1906): 627-633]

In a paper published last year[1] I showed that Maxwell's electromagnetic equations in conjunction with the principle of relativity and the principle of energy conservation led to the conclusion that the mass of a body changes with the change in its energy content, no matter what kind of change of energy this may be. It turned out that to an energy change of magnitude ΔE there must correspond a change of mass of the same sign and of magnitude $\Delta E/V^2$, where V denotes the velocity of light.

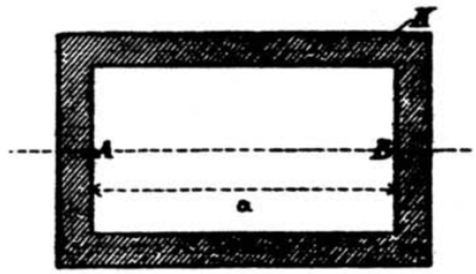
In the present paper I want to show that the above theorem is the necessary and sufficient condition for the law of the conservation of motion of the center of gravity to be valid (at least in first approximation) also for systems in which not only mechanical, but also electromagnetic processes take place. Although the simple formal considerations that have to be carried out to prove this statement are in the main already contained in a work by H. Poincaré[2], for the sake of clarity I shall not base myself upon that work.

§*1*. *A special case*

Let K be a stationary rigid hollow cylinder freely floating in space. Let there be in A an arrangement for sending a certain amount S of radiating energy through the cavity to B. During the emission of this quantity of radiation a radiation pressure acts upon the left interior wall of the tube K, imparting to the latter a certain velocity that is directed to the left. If the hollow cylinder's mass is M, then this velocity equals $\frac{1}{V} \cdot \frac{S}{M}$, as can be proved easily from the laws of radiation pressure, where V denotes the

[1] [1]A. Einstein, *Ann. d. Phys.* 18 (1905): 639.

[2] [2]H. Poincaré, in *Lorentz-Festschrift* (1900): 252-278.



velocity of light. K will maintain this velocity until the radiation complex, whose spatial extension is very small in comparison with the cavity of K, gets absorbed by B. The duration of the hollow cylinder's motion is (apart from terms of higher order) equal to a/V, if a denotes the distance from A to B. After absorption of the radiation complex by B, the body K is again at rest. During the radiation process under consideration, K has shifted a distance of

$$\delta = \frac{1}{V}\frac{S}{M} \cdot \frac{a}{V}$$

to the left.

In the cavity of K, let us have a body k (imagined as massless for the sake of simplicity) next to a (likewise massless) mechanism that can move the body k, which shall first be located in B, back and forth between B and A. After the amount of radiation S has been absorbed by B, this amount of energy shall be transferred to k, and then k moved to A. Finally, the amount of energy S shall again be taken up in A by the hollow cylinder K, and k shall be moved back to B again. The whole system has now undergone a complete cyclic process, which one can imagine to be repeated arbitrarily often.

If one assumes that the carrier body k remains massless even after it has absorbed the amount of energy S, then one also has to assume that the return transport of the amount of energy S is not associated with a change in position of the hollow cylinder K. Thus the only outcome of the entire cyclic process is a shift δ of the whole system to the left; by repeating the cyclic process, one can make this shift as large as desired. We thus arrive at the result that an initially stationary system can change the position of its center of gravity arbitrarily greatly without having external forces acting upon it, and without undergoing any permanent change.

It is clear that the result does not contain any inner contradictions; however, it does contradict the laws of mechanics, according to which a body originally at rest cannot perform a translational motion if no other bodies act upon it.

However, if one assumes that any energy E possesses the inertia E/V^2, then the contradiction with the principles of mechanics disappears. For according to this assumption the carrier body has a mass S/V^2 while it transports the energy amount S from B to A; and since the center of gravity of *the entire system* must be at rest during that process according to the center-of-mass theorem, the cylinder K undergoes during it a total shift S' to the right, amounting to

$$\delta' = \alpha \cdot \frac{S}{V^2} \cdot \frac{1}{M} \cdot$$

Comparison with the result found above shows that (at least in first approximation) $\delta = \delta'$, i.e., that the position of the system is the same before and after the cyclic process. This eliminates the contradiction with the principles of mechanics.

§2. *On the principle of the conservation of the motion of the center of gravity*

We consider a system of n discrete material points with masses $m_1, m_2 \ldots m_n$ and center of gravity coordinates $x_1 \ldots z_n$. With respect to thermal and electric phenomena, these material points are not to be conceived as elementary structures (atoms, molecules), but as bodies in the usual sense of small dimensions, whose energy is not determined by the velocity of the center of gravity. These masses could act on each other through electromagnetic processes as well as through conservative forces (i.e., gravity, rigid connections); however, we shall assume that both the potential energy of the conservative forces and the kinetic energy of the motion of the center of gravity of the masses are infinitesimally small relative to the "internal" energy of the masses $m_1 \ldots m_n$.

Assume that the Maxwell-Lorentz equations