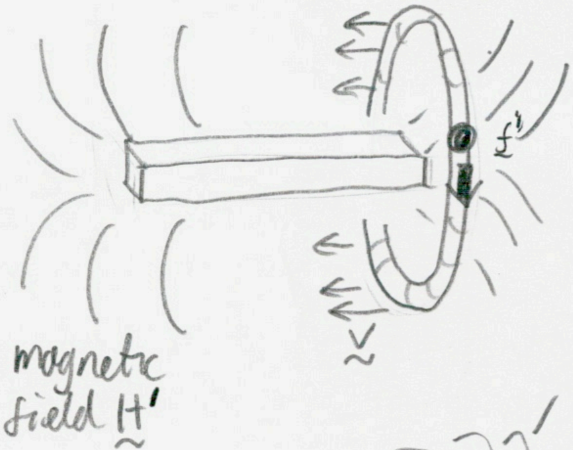
Introduction: magnet & conductor Thought Experiment

Rest frame of magnet

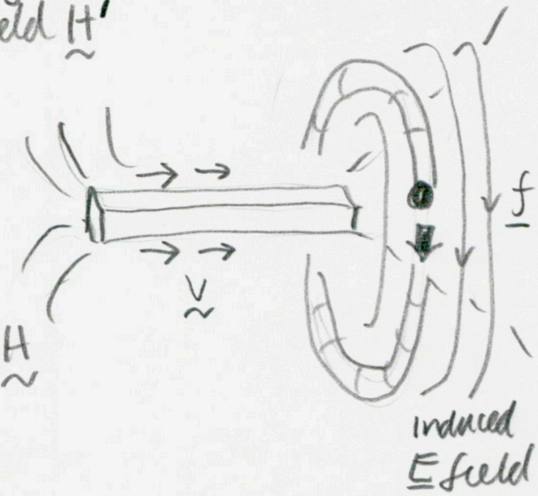


current generating force $\mathbf{f}$ on charge q in conductor:

$$\boxed{\frac{\mathbf{f}'}{q} = \frac{1}{c}\mathbf{V}\times\mathbf{H}'}$$

denote magnet rest frame

Rest frame of conductor



Rules for transforming for rest frame magnet to rest frame conductor

$$\mathbf{H} = \mathbf{H}'$$

$$t = t'$$

$$\mathbf{r} = \mathbf{r}' - \mathbf{V}t'.$$

Hence

$$\frac{\partial}{\partial t'} = \frac{\partial t}{\partial t'}\cdot\frac{\partial}{\partial t} + \frac{\partial \mathbf{r}'}{\partial t'}\cdot\nabla = \frac{\partial}{\partial t} - \mathbf{V}\cdot\nabla$$

$\mathbf{H}'$ field static in magnet rest frame

$$\frac{\partial \mathbf{H}'}{\partial t'} = 0$$

$$\frac{\partial \mathbf{H}}{\partial t} = (\mathbf{V}\cdot\nabla)\mathbf{H}$$

$$= -\nabla\times(\mathbf{V}\times\mathbf{H}) + \mathbf{V}\underbrace{(\nabla\cdot\mathbf{H})}_{0}$$

0 by maxwell's equations $\nabla\cdot\mathbf{H} = 0$

Identity for constant $\mathbf{V}$: $\nabla\times(\mathbf{V}\times\mathbf{H}) = -(\mathbf{V}\cdot\nabla) + \mathbf{V}(\nabla\cdot\mathbf{F})$

maxwell's equation $\nabla\times E = -\frac{1}{c}\frac{\partial H}{\partial t}$

$$\nabla\times\mathbf{E} = \frac{1}{c}\nabla\times(\mathbf{V}\times\mathbf{H})$$

$$\therefore\ \mathbf{E} = \frac{1}{c}(\mathbf{V}\times\mathbf{H}) + \nabla\varphi$$

arbitrary φ

No net contribution to current since for current loop $\oint \nabla\varphi\cdot d\mathbf{r} = 0$

$$\mathbf{E} = \mathbf{f}/q$$

Hence same force as seen in rest frame magnet