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Uncertainty in Artificial Intelligence
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Is Probability Theory Sufficient for Dealing with Uncertainty in AI: A Negative View*

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Abstract

An issue which has become a focus of controversy in recent years is whether or not classical probability theory is sufficient for dealing with uncertainty in AI. The topicality of this issue has grown as a result of the emergence of expert systems as one of the principal areas of activity in AI and the development of methods for evidential reasoning based on the Dempster-Shafer theory and fuzzy logic which extend beyond the current boundaries of probability theory.

A point of view which is articulated in this paper is that the inadequacy of probability theory stems from its lack of expressiveness as a language of uncertainty, especially for describing fuzzy events and fuzzy probabilities. For example, how would one represent the meaning of the proposition p: *it is very likely that Mary is young*, in which *likely* is a fuzzy probability and *young* is a fuzzy predicate? Furthermore, how can one infer from this proposition an answer to the question: What is the likelihood that *Mary is not very young*?

We show through examples that problems of this type -- problems which do not lend themselves to solution by conventional probability-based methods -- can be dealt with effectively through the use of fuzzy logic.

1. The Issue of Adequacy

During the past few years, the question raised in the title of this paper has become a matter of heated debate, especially in the context of dealing with uncertainty in expert systems. There are some who claim, as do some of the authors in this volume, that it is provable that probability theory is the only correct way of dealing with uncertainty and that anything that can be done with other techniques can be done equally well through the use of probability-based methods [47], [44], [31], [33].

There are others, and I am one of them, who dissent from this view and question the long-standing tradition in science to treat any kind of uncertainty -- regardless of its nature -- in probabilistic terms.

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Speaking for myself, what I do not question is the validity of axiomatic approaches such as those described by Lindley [31] and Cox [9] which lead to the conclusion that, from a set of what appear to be reasonable axioms regarding a measure of belief, one is led to the conclusion that such a measure must be probabilistic in nature. What I believe to be the case is that, viewed as a language, classical probability theory is insufficiently expressive to cope with the multiplicity of kinds of uncertainty which one encounters in AI and, more particularly, in expert systems.

More specifically, the main limitation of probability theory in its present form is that, like almost all of mathematics, it is based on two-valued logic. What this means is that all predicates and concepts in probability theory have crisp denotations, implying that an object x is either an instance of a predicate or a concept or it is not. As a case in point, consider one of the most basic concepts in probability theory--the concept of an *event*. An event, E, considered as a measurable subset of the sample space, either occurs or does not occur; it cannot occur to a degree. This restriction rules out events defined by fuzzy predicates like *warm, small, short* and/or fuzzy quantifiers like *most, several, few*. Simple examples of such events are: *tomorrow will be a warm day, finding a few small balls in a box, observing a coin falling heads several times*, etc. Such *fuzzy events* pervade our daily encounters with chance phenomena and shape our intuitive perceptions of likelihood and probability.

Another basic limitation of classical probability theory relates to the presumption that probabilities are real numbers. In reality, most probabilities, regardless of whether they are associated with crisp or fuzzy events, are not known with sufficient precision to be representable as real numbers or, more generally, as second-order probabilities. For example, what is the probability that *Mary will marry a rich man* or *Jane will be divorced from her husband*? Although the theory of subjective probabilities does provide methods for elicitation of subjective numerical probabilities, it does not answer the question of how such probabilities are arrived at in the first place, nor does it come to grips with the issue of representation of imprecisely known probabilities as fuzzy rather than second-order probabilities [37], [10], [55].

A good example of a fuzzy event which is associated with a fuzzy probability is furnished by a recent headline in the *San Francisco Chronicle*, which read: *Experts predict a big San Francisco earthquake unlikely soon*. The fuzzy event in this case is the occurrence of a *big earthquake in San Francisco in the near future*, and its fuzzy probability is *unlikely*. Note that no expert knows enough about earthquakes to be able to interpret the headline in question in quantitative terms like: The probability that, within the next three years, there will be an earthquake in San Francisco of strength seven or more on the Richter scale is 0.01.

Another example: Consider an urn which is known to contain n balls of various sizes, several of which are large. What is the probability that a ball drawn at random is not large?

These and several other examples which will be given in the sequel are intended to mak two points:

1. Classical probability theory is insufficiently expressive to serve as the language o uncertainty in AI.
2. The inexpressiveness of classical probability theory derives from the fact that, as language, it has no facilities for representing the meaning of propositions[1] containing:
   (a) fuzzy predicates such as *small, large, young, safe, much larger than, soon*.
   (b) fuzzy quantifiers such as *most, many, few, several, often, usually*.
   (c) fuzzy probabilities expressed as *likely, unlikely, not very likely*, etc.
   (d) fuzzy possibilities expressed as *quite possible, almost impossible*, etc.
   (e) fuzzy truth values such as *very true, quite true, mostly untrue*.
   (f) predicate modifiers such as *very, quite, extremely, somewhat, slightly*.

Lacking these facilities, one cannot[2] express within the framework of classical theory th meaning of descriptions of facts, rules and events examplified by the following:

p_1: *slimness is attractive.*

p_2: *most small cars are unsafe.*

p_3: *it is very likely that Mary is young.*

p_4: *Brian is much taller than most of his close friends.*

p_5: *an urn contains ten balls of various sizes a few of which are quite large.*

p_6: *if the search is moderately small then exhaustive search is feasible.*

p_7: *if a piece of code is called frequently then it is worth optimizing.*

p_8: *if large oil spill or strong acid spill then emergency is strongly suggested.*[3]

In addition to its inability to represent the meaning of fuzzy facts, rules and events such as those listed above, classical probability theory has no facilities for inference from fuzzy premises. As a case in point, suppose that we want to chain a fuzzy fact of the form

$$X \text{ is } F \ , \qquad (1.1)$$

where X is a variable and F is a fuzzy predicate, e.g.,

X is very small

---

[1] In probabilistic terms, a proposition may be viewed as a description of an event.

[2] What is meant here is that either it cannot be done at all or, if it can be done indirectly, it cannot be done simply.

[3] p_6, p_7 and p_8 are taken from [5]. Although they are not treated as such. most rules in a typical expert system are fuzzy to some degree.

with a fuzzy rule

$$if\ X\ is\ G\ then\ Y\ is\ H \tag{1.2}$$

e.g.,

if X is much smaller than 10 then Y is large,

in which there is a partial match between F and G. The question is: How can one compute the fuzzy value of Y given the fuzzy predicates F, G and H? Classical probability theory does not provide an answer to this question.

In sum, the inadequacy of classical probability theory as a conceptual framework for dealing with uncertainty in AI stems from two facts:

1. The theory does not provide a general computational system for representing the meaning of fuzzy propositions, i.e., propositions containing fuzzy predicates and/or fuzzy quantifiers and/or fuzzy probabilities.
2. The theory does not provide a general computational system for inference from fuzzy propositions.

When we add to probability theory the needed facilities for dealing with fuzzy propositions, we get a subset of fuzzy logic. In what follows, we shall consider several examples of problems which do not lend themselves to solution within the framework of classical probability theory and will show how they can be solved through the use of fuzzy logic. These problems may be regarded as a test-bed for assertions to the effect that anything that can be done with techniques outside of probability theory can be done equally well with techniques that lie within it.

2. Inference

As was alluded to already, to be an effective tool for dealing with uncertainty, a theory must pass two basic tests:

(a) It must provide a system for representing the meaning of various types of propositions relating to uncertain events and uncertain dependencies; and

(b) It must provide a system for inferring from the representations of such propositions.

Classical probability theory passes both tests when the propositions are crisp, and fails in both cases when the propositions are fuzzy. Since classical probability theory is subsumed by fuzzy logic, fuzzy logic passes (a) and (b) for crisp propositions. However, unlike classical probability theory, it also passes (a) and (b) for fuzzy propositions.

In support of this claim, we shall consider several representative problems which are not amenable to solution by conventional probability-based techniques and show how they can be treated within the framework of fuzzy logic. Each of these problems involves inference from one or more fuzzy premises.

1. An urn contains n balls of various sizes. Several of the balls are large. What is the probability that a ball drawn at random is large ? [61]

Solution. Let U denote the urn; let $b_1, \ldots, b_n$ denote the balls in U; and let $\mu_{LARGE}(b_i)$, $i = 1, \ldots, n$, denote the grade of membership of b_i in the fuzzy set *LARGE*.[4]

Using the concept of a sigma-count [51], the number of large balls in U may be expressed as

$$\Sigma Count(LARGE) = \Sigma_i\, \mu_{LARGE}(b_i)\ , \tag{2.1}$$

which means that the count of large balls is the sum of degrees to which each ball in U fits the description *large*.

The proposition *U contains several large balls* may be interpreted as an elastic or, equivalently, fuzzy constraint on $\Sigma Count(LARGE)$. Consequently, if *several* is interpreted as a fuzzy number whose membership function is $\mu_{SEVERAL}$, then the degree to which the constraint is satisfied by the balls in U may be written as

$$\tau = \mu_{SEVERAL}\,(\Sigma_i\, \mu_{LARGE}\,(b_i)\,)\ , \tag{2.2}$$

where τ may be interpreted as the *truth value* of the proposition *several of the balls in U are large* given $\{b_1, \ldots, b_n\}$, or, equivalently, as the *possibility* that U contains the balls $\{b_1, \ldots, b_n\}$ given the proposition *several of the balls are large*.

Using the latter interpretation, we can compute the fuzzy probability that a ball drawn at random is large. Specifically, the probability of drawing b_i is $1/n$, and hence the probability of drawing a large ball is

$$q = \frac{1}{n}\, \Sigma_i\, \mu_{LARGE}\,(b_i)\ . \tag{2.3}$$

From the knowledge that several of the balls are large, we cannot compute q. However, we can compute its *possibility distribution*, that is, for each value of v in [0,1], we can compute the possibility that $q = v$.

For simplicity, let $\mu_i \triangleq \mu_{LARGE}\,(b_i)$, $i = 1, \ldots, n$, where $\triangleq$ stands for *is defined to be*. What we know about the μ_i is their possibility distribution, i.e.,

$$\pi\,(\mu_1, \ldots, \mu_n) = \mu_{SEVERAL}\,(\Sigma_i\, \mu_i)\ . \tag{2.4}$$

At this point, then, the problem is to find the possibility distribution, $\pi\,(q)$, of

$$q = \frac{1}{n}\, \Sigma_i\, \mu_i \tag{2.5}$$

[4] The denotation of a predicate is expressed in uppercase letters

from the knowledge of the possibility distribution of the μ_i.

Using fuzzy logic, the solution of this problem reduces to the solution of the variational problem

$$\pi(q) \triangleq max_{\mu_1, \ldots, \mu_n} \; \pi(\mu_1, \ldots, \mu_n)$$

$$= max_{\mu_1, \ldots, \mu_n} (\mu_{SEVERAL}(\Sigma_i \; \mu_i))$$

subject to

$$q = \frac{1}{n} \Sigma_i \, \mu_i \; .$$

This problem has an obvious solution, namely,

$$\pi(q) = \mu_{SEVERAL} \; (qn) \; , \qquad (2.6)$$

which implies that q may be interpreted as a fuzzy probability which is representable as the fuzzy number

$$q = \frac{1}{n} \; SEVERAL \; . \qquad (2.7)$$

For example, if $n = 10$ and *SEVERAL* is defined as

$$SEVERAL = 0.4/3 + 0.8/4 + 1/5 + 1/6 + 0.6/7 + 0.3/8,$$

where a term such as 0.8/4 signifies that the grade of membership of 4 in *SEVERAL* is 0.8, then the corresponding representation for q is

$$q = 0.4/0.3 + 0.8/0.4 + 1/0.5 + 1/0.6 + 0.6/0.7 + 0.3/0.8.$$

Note that if the number of large balls is a crisp number, say 3, then the probability of drawing a large ball would be 0.3.

2. Given the proposition *r: it is likely that Mary is young*, find the probability that Mary is not young.

Solution. Let p denote the probability density of Mary's age, i.e., $p(u)du$ is the probability that Mary's age lies in the interval $[u, u + du]$, $u \in U$.

The given proposition, r, may be viewed as an elastic constraint on p which defines its possibility distribution. Specifically, in terms of p, the probability of the fuzzy event *Mary is young* may be expressed as [63]

$$Prob \; \{Mary \; is \; young\} = \int_U \mu_{YOUNG} \; (u) \; p(u) \; du \; , \qquad (2.$$

where μ_{YOUNG} is the membership function of the fuzzy set labeled *YOUNG*. This probabi ity is characterized as *likely* by r. Consequently, the possibility distribution of p is give by

$$\pi(p) = \mu_{LIKELY} \; (\int_U \mu_{YOUNG} \; (u) \; p(u) \; du) \; . \qquad (2.$$

Knowing $\pi(p)$, we can compute the probability of the fuzzy event *Mary is not youn* Specifically, we have

$$Prob \; \{Mary \; is \; not \; young\} = \int_U (1 - \mu_{YOUNG} \; (u)) \; p(u) \; du \qquad (2.1$$

$$= 1 - \int_U \mu_{YOUNG} \; (u) \; p(u) \; du$$

$$= 1 - Prob \; \{Mary \; is \; young\} \; .$$

Furthermore, the probability that *Mary is young* is described by the fuzzy probabili *LIKELY*. Consequently, from (2.10) it follows at once that

$$Prob \; \{Mary \; is \; not \; young\} = 1 \ominus LIKELY \; , \qquad (2.1$$

where $\ominus$ represents the operation of subtraction in fuzzy arithmetic [29]. The fuz number $1 \ominus$ *LIKELY* may be read as *UNLIKELY*, where *UNLIKELY* is the antonym *LIKELY* (Fig. 1).

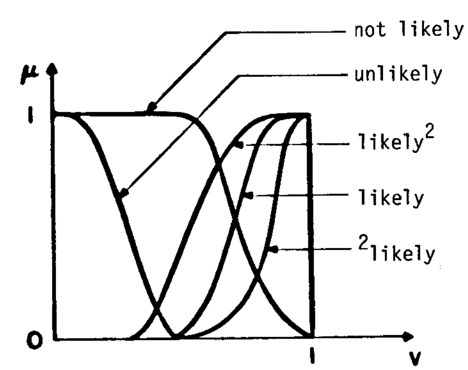

Fig. 1. Possibility distribution of fuzzy probabilities.

²*likely* represents *very likely*.

In conclusion, from the proposition *r: it is likely that Mary is young*, we can infer -- through the use of fuzzy logic -- that *s: it is unlikely that Mary is not young*. Note that this conclusion is in accord with our intuition.

3. Given the proposition *r: most Swedes are tall*, find the fraction of Swedes who are very tall.

Solution. Let $p(u)$ denote the probability density of the height of Swedes. As in the preceeding example, the meaning of the proposition *most Swedes are tall* may be represented as an elastic constraint on *p*. More specifically, the constraint in question defines the possibility distribution of *p* through the expression

$$\pi(p) = \int_U \mu_{TALL}(u)\, p(u)\, du \quad . \qquad (2.12)$$

Now, assuming that the predicate modifier *very* acts as an intensifier [52], i.e.,

$$\mu_{VERY\ TALL}(u) = (\mu_{TALL}(u))^2 \quad , \qquad (2.13)$$

the fraction of Swedes who are very tall may be expressed as

$$q = \int_U \mu^2_{TALL}(u)\, p(u)\, du \quad . \qquad (2.14)$$

Consequently, the determination of q reduces to the solution of the variational problem

$$\pi_q(v) = max_p \left(\int_U \mu_{TALL}(u)\, p(u)\, du\right) \qquad (2.15)$$

subject to

$$v = \int_U \mu^2_{TALL}(u)\, p(u)\, du \quad .$$

As shown in [67], the solution of this problem is given by

$$q = MOST^2 \quad , \qquad (2.16)$$

where $MOST^2$ is the product of the fuzzy number *MOST* with itself in fuzzy arithmetic (Fig. 2). More explicitly,

$$\mu_{MOST^2}(u) = \mu_{MOST}(\sqrt{u}) \quad . \qquad (2.17)$$

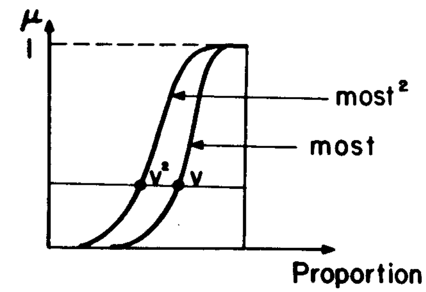

Fig. 2. Possibility distributions of *most* and $most^2$.

Thus, from the proposition *most Swedes are tall*, we can infer -- through the use of fuzz logic -- that $most^2$ *Swedes are very tall*.

4. Consider the question stated in Section 1, in which the problem is how to chain a fuzz fact of the form

$$X \ is \ F \quad , \qquad (2.1$$

where *X* is a variable taking values in *U* and *F* is a possibility distribution in *U* whic constrains *X*, with a fuzzy rule of the form

$$\text{if } X \ is \ G \ then \ Y \ is \ H \quad , \qquad (2.1$$

where *Y* is a variable taking values in *V*, and *G* and *H* are possibility distributions in and *V*, respectively.

A basic rule of inference is fuzzy logic which is applicable to this problem is the *genera ized modus ponens* [53]. Specifically, it can be shown that the elastic constraints on *X* ar *Y* defined by (2.18) and (2.19) induce a constraint on *Y* which may be expressed as th proposition

$$Y \ is \ R \quad , \qquad (2.2$$

in which R is a possibility distribution in *V* given by

$$R = F \circ (G' \oplus H) \quad . \qquad (2.2$$

In this expression, G' is the complement of G, $\oplus$ is the bounded sum and o is the operation of composition [53]. In terms of the possibility distribution functions of F, G, H and R, (2.21) may be expressed more explicitly as

$$\pi_R(v) = \vee_u (\pi_F(u) \wedge (1 - \pi_G(u) + \pi_H(u))) \; . \quad (2.22)$$

This expression for the possibility distribution function of R answers the question in Section 1.

5. From p_1, p_2 and p_3 defined below, compute the likelihood that Maria is not old.

p_1: *it is unlikely that Maria is very young*

p_2: *it is likely that Maria is young*

p_3: *it is very unlikely that Maria is old*

q : *How likely is it that Maria is not old?*

Solution. To find the answer to the posed question, we shall reduce the stated problem to the solution of a nonlinear program [65].

First, each of the premises is translated into a constraint on the probability density, p, of Maria's age. Thus, as in Examples 2 and 3, we have

p_1: *it is unlikely that Maria is very young* →

$$\pi_1(p) = \mu_{LIKELY}\left(1 - \int_0^{100} \mu^2_{YOUNG}(u)\, p(u)\, du\right) \quad (2.23)$$

$$\pi_2(p) = \mu_{LIKELY}\left(\int_0^{100} \mu_{YOUNG}(u)\, p(u)\, du\right) \quad (2.24)$$

$$\pi_3(p) = \mu_{LIKELY^2}\left(1 - \int_0^{100} \mu_{OLD}(u)\, p(u)\, du\right) \; , \quad (2.25)$$

where $\int_0^{100} \mu_{YOUNG}(u)\, p(u)\, du$ represents the probability of the fuzzy event *Maria is young*, with the understanding that the range of the variable *Age(Maria)* is the interval [0,100].

Next, we must translate the answer to the posed question, which we assume to be of the form *it is* λ *that Maria is not old*, where λ is a fuzzy probability. Thus

$$q \rightarrow \pi_q(p) = \mu_\lambda \quad (2.26)$$

where μ_λ is the unknown membership function of λ.

Finally, by using the conjunction of π_1, π_2 and π_3, the problem in question is reduced to the solution of the nonlinear program

$$\mu_\lambda(v) \triangleq max_p \{ (\mu_{LIKELY}(1 - \int_0^{100} \mu^2_{YOUNG}(u)\, p(u)\, du) \quad (2.27)$$

$$\wedge\, \mu_{LIKELY}(\int_0^{100} \mu_{YOUNG}(u)\, p(u)\, du)$$

$$\wedge\, \mu^2_{LIKELY}(1 - \int_0^{100} \mu_{OLD}(u)\, p(u)\, du))\}$$

subject to

$$v = \int_0^{100} (1 - \mu_{OLD}(u)\, p(u)\, du \; ,$$

where v is the numerical probability of the fuzzy event *Maria is not old.*

Concluding Remark

The above examples are merely a small sample of problems which do not lend themselves to solution by conventional probabilistic methods. What these examples are intended to demonstrate is that classical probability theory makes us provision for inference from fuzzy data, and that to deal with such data it is necessary to employ the conceptual framework of fuzzy logic. In general, the employment of fuzzy logic for purposes of inference requires the solution of a nonlinear program which involves the possibility distributions induced by both the premises and the query.

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Uncertainty in Artificial Intelligence
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CONFIDENCE FACTORS, EMPIRICISM AND THE DEMPSTER-SHAFE THEORY OF EVIDENCE

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The issue of confidence factors in Knowledge Based Systen has become increasingly important and Dempster-Shafer (D theory has become increasingly popular as a basis for thes factors. This paper discusses the need for an empiric interpretation of any theory of confidence factors applied Knowledge Based Systems and describes an empiric interpretation of DS theory suggesting that the theory h been extensively misinterpreted. For the essentially syntact DS theory, a model is developed based on sample spaces, t traditional semantic model of probability theory. This model used to show that, if belief functions are based on reasonab accurate sampling or observation of a sample space, then t beliefs and upper probabilities as computed according to D theory cannot be interpreted as frequency ratios. Since ma proposed applications of DS theory use belief functions situations with statistically derived evidence (Wesley [1]) a seem to appeal to statistical intuition to provide interpretation of the results as has Garvey [2], it may argued that DS theory has often been misapplied.

The success of the scientific approach is generally attributed philosophers such as Popper to its insistence on empirical verification theories (Davis [3]). Stated from a different point of view, theories whi do not make empirically verifiable predictions about reality are n scientific. When one builds Knowledge Based Systems for applications a includes the use of confidence factors, these confidence facto presumably are present to make some statement about the real world. such Knowledge Based Systems are to be considered scientific, we mu face the problem of empirically testing these statements. Argumen advanced in support of the various theories of confidence factors a almost never empirically testable. Some arguments often presented