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SIR DAVID BREWSTER, K.H. LL.D. F.R.S.L.&F. &c.

RICHARD TAYLOR, F.L.S. G.S. Astr.S. Nat.H. Mosc. &c.

SIR ROBERT KANE, M.D. M.R.I.A.

WILLIAM FRANCIS, Ph.D. F.L.S. F.R.A.S. F.C.S.

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“Nec araneorum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes.” JUST. LIPS. *Polit. lib. i. cap. 1. Not.*

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I have thus, I trust with perfect impartiality, gone through all the main experimental points of the author's investigations, and upon the whole I can perceive *nothing substantiated* which is positively *irreconcilable with the principle of interference*, while the new modifications of the phenomena here presented, so far as *general* considerations can be relied on, seem sufficiently conformable to the undulatory theory: but as to their more *exact*, or *quantitative* explanation, no definitive opinion can be pronounced, until certain analytical investigations of almost impracticable length and complexity, shall have been gone through, by which alone that theory can be brought into exact and satisfactory comparison with experiment.

II. *On the Dynamical Theory of Heat, with numerical results deduced from Mr. Joule's equivalent of a Thermal Unit, and M. Regnault's Observations on Steam.* By WILLIAM THOMSON, M.A., Fellow of St. Peter's College, Cambridge, and Professor of Natural Philosophy in the University of Glasgow\*.

*Introductory Notice.*

1. SIR HUMPHRY DAVY, by his experiment of melting two pieces of ice by rubbing them together, established the following proposition:—"The phenomena of repulsion are not dependent on a peculiar elastic fluid for their existence, or caloric does not exist." And he concludes that heat consists of a motion excited among the particles of bodies. "To distinguish this motion from others, and to signify the cause of our sensation of heat," and of the expansion or expansive pressure produced in matter by heat, "the name *repulsive* motion has been adopted †."

pendicular to the edge; if circular, tending to the centre; if angular, bisecting it; if concave, spreading out, &c. A representation of the appearance is given in Plate ii. fig. 8 (p. 155). At p. 190, the Editor adds a memorandum found among Dr. Hooke's papers, stating, that on March 18, 1674, he "read a discourse" on several new properties of light; which he sums up as follows:—

"That there is a deflexion of light differing both from reflexion and refraction, and seeming to depend on the unequal density of the constituent parts of the ray, whereby light is dispersed from the place of condensation and rarefied or gradually diverged into a quadrant;" 2ndly, that this takes place "perpendicularly to the edge;" and 3rdly, that "the parts deflected by the greatest angle are the faintest."

I have fully referred to and commented upon Newton's description of the same phenomenon, conveyed in terms so singularly coincident, in my paper before referred to.

\* From the Transactions of the Royal Society of Edinburgh, vol. xx. part 2. Passages added in the present reprint are enclosed in brackets.

† From Davy's first work, entitled "An Essay on Heat, Light, and the Combinations of Light," published in 1799, in "Contributions to Physical

2. The dynamical theory of heat, thus established by Sir Humphry Davy, is extended to radiant heat by the discovery of phænomena, especially those of the polarization of radiant heat, which render it excessively probable that heat propagated through "vacant space," or through diathermanic substances, consists of waves of transverse vibrations in an all-pervading medium.

3. The recent discoveries made by Mayer and Joule\*, of the generation of heat through the friction of fluids in motion, and by the magneto-electric excitation of galvanic currents, would either of them be sufficient to demonstrate the immateriality of heat; and would so afford, if required, a perfect confirmation of Sir Humphry Davy's views.

4. Considering it as thus established, that heat is not a substance, but a dynamical form of mechanical effect, we perceive that there must be an equivalence between mechanical work and heat, as between cause and effect. The first published statement of this principle appears to be in Mayer's *Bemerkungen über die Kräfte der unbelebten Natur*†, which contains some correct views regarding the mutual convertibility of heat and mechanical effect, along with a false analogy between the approach of a weight to the earth and a diminution of the volume of a continuous substance, on which an attempt is founded to find numerically the mechanical equivalent of a given quantity of heat. In a paper published about fourteen months later, "On the Calorific Effects of Magneto-Electricity and the Mechanical Value of Heat‡," Mr. Joule of Manchester expresses very distinctly the consequences regarding the mutual convertibility of heat and mechanical effect which follow from the fact, that heat is not a substance but a state of motion; and investigates on unquestionable principles the "absolute numerical relations," according to which heat is connected with mechanical power; verifying experimentally, that whenever heat is generated from purely mechanical action, and no other effect produced, whether it be by means of the friction of fluids or by the magneto-electric excita-

and Medical Knowledge, principally from the West of England, collected by Thomas Beddoes, M.D.," and republished in Dr. Davy's edition of his brother's collected works, vol. ii. Lond. 1836.

\* In May 1842, Mayer announced in the *Annalen* of Wöhler and Liebig, that he had raised the temperature of water from 12° to 13° Cent. by agitating it. In August 1843, Joule announced to the British Association, "That heat is evolved by the passage of water through narrow tubes;" and that he had "obtained one degree of heat per lb. of water from a mechanical force capable of raising 770 lbs. to the height of one foot;" and that heat is generated when work is spent in turning a magneto-electric machine, or an electro-magnetic engine. (See his paper "On the Calorific Effects of Magneto-Electricity, and on the Mechanical Value of Heat."—*Phil. Mag.*, vol. xxiii. 1843.)

† *Annalen* of Wöhler and Liebig, May 1842.

‡ British Association, August 1843; and *Phil. Mag.*, Sept. 1843.

tion of galvanic currents, the same quantity is generated by the same amount of work spent; and determining the actual amount of work, in foot-pounds, required to generate a unit of heat, which he calls "the mechanical equivalent of heat." Since the publication of that paper, Mr. Joule has made numerous series of experiments for determining with as much accuracy as possible the mechanical equivalent of heat so defined, and has given accounts of them in various communications to the British Association, to the Philosophical Magazine, to the Royal Society, and to the French Institute.

5. Important contributions to the dynamical theory of heat have recently been made by Rankine and Clausius; who, by mathematical reasoning analogous to Carnot's on the motive power of heat, but founded on an axiom contrary to his fundamental axiom, have arrived at some remarkable conclusions. The researches of these authors have been published in the Transactions of this Society, and in Poggendorff's *Annalen*, during the past year; and they are more particularly referred to below in connexion with corresponding parts of the investigations at present laid before the Royal Society.

[Various statements regarding animal heat, and the heat of combustion and chemical combination, are made in the writings of Liebig, (as, for instance, the statement quoted in the foot-note added to § 18 below,) which virtually imply the convertibility of heat into mechanical effect, and which are inconsistent with any other than the dynamical theory of heat.]

6. The object of the present paper is threefold:—

(1.) To show what modifications of the conclusions arrived at by Carnot, and by others who have followed his peculiar mode of reasoning regarding the motive power of heat, must be made when the hypothesis of the dynamical theory, contrary as it is to Carnot's fundamental hypothesis, is adopted.

(2.) To point out the significance in the dynamical theory, of the numerical results deduced from Regnault's observations on steam, and communicated about two years ago to the Society, with an account of Carnot's theory, by the author of the present paper; and to show that by taking these numbers (subject to correction when accurate experimental data regarding the density of saturated steam shall have been afforded), in connexion with Joule's mechanical equivalent of a thermal unit, a complete theory of the motive power of heat, within the temperature limits of the experimental data, is obtained.

(3.) To point out some remarkable relations connecting the physical properties of all substances, established by reasoning analogous to that of Carnot, but founded in part on the contrary principle of the dynamical theory.

PART I.—*Fundamental Principles in the Theory of the Motive Power of Heat.*

7. According to an obvious principle, first introduced, however, into the theory of the motive power of heat by Carnot, mechanical effect produced in any process cannot be said to have been derived from a purely thermal source, unless at the end of the process all the materials used are in precisely the same physical and mechanical circumstances as they were at the beginning. In some conceivable "thermo-dynamic engines," as for instance Faraday's floating magnet, or Barlow's "wheel and axle," made to rotate and perform work uniformly by means of a current continuously excited by heat communicated to two metals in contact, or the thermo-electric rotatory apparatus devised by Marsh, which has been actually constructed; this condition is fulfilled at every instant. On the other hand, in all thermo-dynamic engines, founded on electrical agency, in which discontinuous galvanic currents, or pieces of soft iron in a variable state of magnetization are used, and in all engines founded on the alternate expansions and contractions of media, there are really alterations in the condition of materials; but, in accordance with the principle stated above, these alterations must be strictly periodical. In any such engine, the series of motions performed during a period, at the end of which the materials are restored to precisely the same condition as that in which they existed at the beginning, constitutes what will be called a complete cycle of its operations. Whenever in what follows, *the work done or the mechanical effect produced* by a thermo-dynamic engine is mentioned without qualification, it must be understood that the mechanical effect produced, either in a non-varying engine, or in a complete cycle, or any number of complete cycles of a periodical engine is meant.

8. The *source of heat* will always be supposed to be a hot body at a given constant temperature, put in contact with some part of the engine; and when any part of the engine is to be kept from rising in temperature (which can only be done by drawing off whatever heat is deposited in it), this will be supposed to be done by putting a cold body, which will be called the refrigerator, at a given constant temperature in contact with it.

9. The whole theory of the motive power of heat is founded on the two following propositions, due respectively to Joule, and to Carnot and Clausius.

Prop. I. (Joule).—When equal quantities of mechanical effect are produced by any means whatever from purely thermal sources, or lost in purely thermal effects, equal quantities of heat are put out of existence or are generated.

Prop. II. (Carnot and Clausius).—If an engine be such that, when it is worked backwards, the physical and mechanical agencies in every part of its motions are all reversed, it produces as much mechanical effect as can be produced by any thermodynamic engine, with the same temperatures of source and refrigerator, from a given quantity of heat.

10. The former proposition is shown to be included in the general “principle of mechanical effect,” and is so established beyond all doubt by the following demonstration.

11. By whatever direct effect the heat gained or lost by a body in any conceivable circumstances is tested, the measurement of its quantity may always be founded on a determination of the quantity of some standard substance, which it or any equal quantity of heat could raise from one standard temperature to another; the test of equality between two quantities of heat being their capability of raising equal quantities of any substance from any temperature to the same higher temperature. Now, according to the dynamical theory of heat, the temperature of a substance can only be raised by working upon it in some way so as to produce increased thermal motions within it, besides effecting any modifications in the mutual distances or arrangements of its particles which may accompany a change of temperature. The work necessary to produce this total mechanical effect is of course proportional to the quantity of the substance raised from one standard temperature to another; and therefore when a body, or a group of bodies, or a machine, parts with or receives heat, there is in reality mechanical effect produced from it, or taken into it, to an extent precisely proportional to the quantity of heat which it emits or absorbs. But the work which any external forces do upon it, the work done by its own molecular forces, and the amount by which the half *vis viva* of the thermal motions of all its parts is diminished, must together be equal to the mechanical effect produced from it; and consequently, to the mechanical equivalent of the heat which it emits (which will be positive or negative, according as the sum of those terms is positive or negative). Now let there be either no molecular change or alteration of temperature in any part of the body, or, by a cycle of operations, let the temperature and physical condition be restored exactly to what they were at the beginning; the second and third of the three parts of the work which it has to produce vanish; and we conclude that the heat which it emits or absorbs will be the thermal equivalent of the work done upon it by external forces, or done by it against external forces; which is the proposition to be proved.

12. The demonstration of the second proposition is founded on the following axiom:—

*It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects\*.*

13. To demonstrate the second proposition, let A and B be two thermo-dynamic engines, of which B satisfies the conditions expressed in the enunciation; and let, if possible, A derive more work from a given quantity of heat than B, when their sources and refrigerators are at the same temperatures, respectively. **Then on account of the condition of complete reversibility** in all its operations which it fulfills, B may be worked backwards, and made to restore any quantity of heat to its source, by the expenditure of the amount of work which, by its forward action, it would derive from the same quantity of heat. If, therefore, B be worked backwards, and made to restore to the source of A (which we may suppose to be adjustable to the engine B) as much heat as has been drawn from it during a certain period of the working of A, a smaller amount of work will be spent thus than was gained by the working of A. Hence, if such a series of operations of A forwards and of B backwards be continued, either alternately or simultaneously, there will result a continued production of work without any continued abstraction of heat from the source; and, by Prop. I., it follows that there must be more heat abstracted from the refrigerator by the working of B backwards than is deposited in it by A. Now it is obvious that A might be made to spend part of its work in working B backwards, and the whole might be made self-acting. Also, there being no heat either taken from or given to the source on the whole, all the surrounding bodies and space except the refrigerator might, without interfering with any of the conditions which have been assumed, be made of the same temperature as the source, whatever that may be. We should thus have a self-acting machine, capable of drawing heat constantly from a body surrounded by others at a higher temperature, and converting it into mechanical effect. But this is contrary to the axiom, and therefore we conclude that the hypothesis that A derives more mechanical effect from the same quantity of heat drawn from the source than B, is false. Hence no engine whatever, with source and refrigerator at the same temperatures, can get more work from a given quantity of heat introduced than any engine which satisfies the condition of reversibility, which was to be proved.

No  
definition  
or  
discussion

14. This proposition was first enunciated by Carnot, being the expression of his criterion of a perfect thermo-dynamic en-

\* If this axiom be denied for all temperatures, it would have to be admitted that a self-acting machine might be set to work and produce mechanical effect by cooling the sea or earth, with no limit but the total loss of heat from the earth and sea, or, in reality, from the whole material world.

gine\*. He proved it by demonstrating that a negation of it would require the admission that there might be a self-acting machine constructed which would produce mechanical effect indefinitely, without any source either in heat or the consumption of materials, or any other physical agency; but this demonstration involves, fundamentally, the assumption that, in "a complete cycle of operations," the medium parts with exactly the same quantity of heat as it receives. A very strong expression of doubt regarding the truth of this assumption, as a universal principle, is given by Carnot himself†; and that it is false, where mechanical work is, on the whole, either gained or spent in the operations, may (as I have tried to show above) be considered to be perfectly certain. It must then be admitted that Carnot's original demonstration utterly fails, but we cannot infer that the proposition concluded is false. The truth of the conclusion appeared to me, indeed, so probable, that I took it in connexion with Joule's principle, on account of which Carnot's demonstration of it fails, as the foundation of an investigation of the motive power of heat in air-engines or steam-engines through finite ranges of temperature, and obtained about a year ago results, of which the substance is given in the second part of the paper at present communicated to the Royal Society. It was not until the commencement of the present year that I found the demonstration given above, by which the truth of the proposition is established upon an axiom (§ 12) which I think will be generally admitted. It is with no wish to claim priority that I make these statements, as the merit of first establishing the proposition upon correct principles is entirely due to Clausius, who published his demonstration of it in the month of May last year, in the second part of his paper on the motive power of heat‡. I may be allowed to add, that I have given the demonstration exactly as it occurred to me before I knew that Clausius had either enunciated or demonstrated the proposition. The following is the axiom on which Clausius' demonstration is founded:—

*It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature.*

It is easily shown, that, although this and the axiom I have used are different in form, either is a consequence of the other. The reasoning in each demonstration is strictly analogous to that which Carnot originally gave.

15. A complete theory of the motive power of heat would consist of the application of the two propositions demonstrated above, to every possible method of producing mechanical effect

\* Account of Carnot's Theory, § 13.

† Ibid. § 6.

‡ Poggendorff's *Annalen*, referred to above.

from thermal agency\*. As yet this has not been done for the electrical method, as far as regards the criterion of a perfect engine implied in the second proposition, and probably cannot be done without certain limitations; but the application of the first proposition has been very thoroughly investigated, and verified experimentally by Mr. Joule in his researches "On the Calorific Effects of Magneto-Electricity;" and on it is founded one of his ways of determining experimentally the mechanical equivalent of heat. Thus, from his discovery of the laws of generation of heat in the galvanic circuit†, it follows that when mechanical work by means of a magneto-electric machine is the source of the galvanism, the heat generated in any given portion of the fixed part of the circuit is proportional to the whole work spent; and from his experimental demonstration that heat is developed in any moving part of the circuit at exactly the same rate as if it were at rest, and traversed by a current of the same strength, he is enabled to conclude—

(1.) That heat may be created by working a magneto-electric machine.

(2.) That if the current excited be not allowed to produce any other than thermal effects, the total quantity of heat produced is in all circumstances exactly proportional to the quantity of work spent.

16. Again, the admirable discovery of Peltier, that cold is produced by an electrical current passing from bismuth to antimony, is referred to by Joule‡, as showing how it may be proved

\* "There are at present known two, and only two, distinct ways in which mechanical effect can be obtained from heat. One of these is by the alterations of volume which bodies experience through the action of heat; the other is through the medium of electric agency."—Account of Carnot's Theory, § 4. (Transactions, vol. xvi. part 5.)

† That, in a given fixed part of the circuit, the heat evolved in a given time is proportional to the square of the strength of the current, and for different fixed parts, with the same strength of current, the quantities of heat evolved in equal times are as the resistances. A paper by M. Joule, containing demonstrations of these laws, and of others on the relations of the chemical and thermal agencies concerned, was communicated to the Royal Society on the 17th of December 1840, but was not published in the Transactions. (See abstract containing a statement of the laws quoted above, in the Philosophical Magazine, vol. xviii. p. 308.) It was published in the Philosophical Magazine in October 1841 (vol. xix. p. 260).

‡ [In the introduction to his paper on the Calorific Effects of Magneto-electricity, &c., Phil. Mag. 1843.

I take this opportunity of mentioning that I have only recently become acquainted with Helmholtz's admirable treatise on the principle of mechanical effect (*Ueber die Erhaltung der Kraft*, von Dr. H. Helmholtz. Berlin. G. Reimer, 1847), having seen it for the first time on the 20th of January of this year; and that I should have had occasion to refer to it on this, and on numerous other points of the dynamical theory of heat, the mechanical theory of electrolysis, the theory of electro-magnetic induction, and the

that, when an electrical current is continuously produced from a purely thermal source, the quantities of heat evolved electrically in the different homogeneous parts of the circuit are only compensations for a loss from the junctions of the different metals, or that, when the effect of the current is entirely thermal, there must be just as much heat emitted from the parts not affected by the source as is taken from the source.

17. Lastly\*, when a current produced by thermal agency is made to work an engine and produce mechanical effect, there will be less heat emitted from the parts of the circuit not affected by the source than is taken in from the source, by an amount precisely equivalent to the mechanical effect produced; since Joule demonstrates experimentally, that a current from any kind of source driving an engine, produces in the engine just as much less heat than it would produce in a fixed wire exercising the same resistance as is equivalent to the mechanical effect produced by the engine.

18. The equality of thermal effects, resulting from equal causes through very different means, is beautifully illustrated by the following statement, drawn from Mr. Joule's paper on magneto-electricity †.

mechanical theory of thermo-electric currents, in various papers communicated to the Royal Society of Edinburgh, and to this Magazine, had I been acquainted with it in time.—W. T. March 20, 1852.]

\* This reasoning was suggested to me by the following passage contained in a letter which I received from Mr. Joule on the 8th of July 1847. "In Peltier's experiment on cold produced at the bismuth and antimony solder, we have an instance of the conversion of heat into the mechanical force of the current," which must have been meant as an answer to a remark I had made, that no evidence could be adduced to show that heat is ever put out of existence. I now fully admit the force of that answer; but it would require a proof that there is more heat put out of existence at the heated soldering [or in this and other parts of the circuit] than is created at the cold soldering, [and the remainder of the circuit, when a machine is driven by the current,] to make the "evidence" be *experimental*. That this is the case I think is certain, because the statements of § 16 in the text are demonstrated consequences of the first fundamental proposition; but it is still to be remarked, that neither in this nor in any other case of the production of mechanical effect from purely thermal agency, has the ceasing to exist of an equivalent quantity of heat been demonstrated otherwise than theoretically. It would be a very great step in the experimental illustration (or *verification*, for those who consider such to be necessary) of the dynamical theory of heat, to actually show in any one case a loss of heat; and it might be done by operating through a very considerable range of temperatures with a good air-engine or steam-engine, not allowed to waste its work in friction. As will be seen in Part II. of this paper, no experiment of any kind could show a considerable loss of heat without employing bodies differing considerably in temperature; for instance, a loss of as much as .098, or about one-tenth of the whole heat used, if the temperature of all the bodies used be between 0° and 30° Cent.

[† In this paper reference is made to his previous paper "On the Heat

Let there be three equal and similar galvanic batteries furnished with equal and similar electrodes; let  $A_1$  and  $B_1$  be the terminations of the electrodes (or wires connected with the two poles) of the first battery,  $A_2$  and  $B_2$  the terminations of the corresponding electrodes of the second, and  $A_3$  and  $B_3$  of the third battery. Let  $A_1$  and  $B_1$  be connected with the extremities of a long fixed wire; let  $A_2$  and  $B_2$  be connected with the "poles" of an electrolytic apparatus for the decomposition of water; and let  $A_3$  and  $B_3$  be connected with the *poles* (or *ports* as they might be called) of an electro-magnetic engine. Then if the length of the wire between  $A_1$  and  $B_1$ , and the speed of the engine between  $A_3$  and  $B_3$ , be so adjusted that the strength of the current (which for simplicity we may suppose to be continuous and perfectly uniform in each case) may be the same in the three circuits, there will be more heat given out in any time in the wire between  $A_1$  and  $B_1$  than in the electrolytic apparatus between  $A_2$  and  $B_2$ , or the working engine between  $A_3$  and  $B_3$ . But if the hydrogen were allowed to burn in the oxygen, within the electrolytic vessel, and the engine to waste all its work without producing any other than thermal effects (as it would do, for instance, if all its work were spent in continuously agitating a limited fluid mass), the total heat emitted would be precisely the same in each of these two pieces of apparatus as in the wire between  $A_1$  and  $B_1$ . It is worthy of remark that these propositions are *rigorously* true, being demonstrable consequences of the fundamental principle of the dynamical theory of heat, which have been discovered by Joule, and illustrated and verified most copiously in his experimental researches\*.

19. Both the fundamental propositions may be applied in a

of Electrolysis" (published in vol. vii. part 2, of the second Series of the Literary and Philosophical Society of Manchester) for experimental demonstration of those parts of the theory in which chemical action is concerned.]

\* [I have recently met with the following passage in Liebig's Animal Chemistry (3rd edit. London, 1846, p. 43), in which the dynamical theory of the heat both of combustion and of the galvanic battery is indicated, if not fully expressed:—"When we kindle a fire under a steam-engine, and employ the power obtained to produce heat by friction, it is impossible that the heat thus obtained can ever be greater than that which was required to heat the boiler; and if we use the galvanic current to produce heat, the amount of heat obtained is never in any circumstances greater than we might have by the combustion of the zinc which has been dissolved in the acid."

A paper "On the Heat of Chemical Combination," by Dr. Thomas Woods, published last October in the Philosophical Magazine, contains an independent and direct experimental demonstration of the proposition stated in the text regarding the comparative thermal effects in a fixed metallic wire, and an electrolytic vessel for the decomposition of water, produced by a galvanic current.—W. T. March 20, 1852.]

perfectly rigorous manner to the second of the known methods of producing mechanical effect from thermal agency. This application of the first of the two fundamental propositions has already been published by Rankine and Clausius; and that of the second, as Clausius showed in his published paper, is simply Carnot's unmodified investigation of the relation between the mechanical effect produced and the thermal circumstances from which it originates, in the case of an expansive engine working within an infinitely small range of temperatures. The simplest investigation of the consequences of the first proposition in this application, which has occurred to me, is the following, being merely the modification of an analytical expression of Carnot's axiom regarding the permanence of heat, which was given in my former paper\*, required to make it express, not Carnot's axiom, but Joule's.

20. Let us suppose a mass† of any substance, occupying a volume  $v$ , under a pressure  $p$  uniform in all directions, and at a temperature  $t$ , to expand in volume to  $v + dv$ , and to rise in temperature to  $t + dt$ . The quantity of work which it will produce will be

$$v dv;$$

and the quantity of heat which must be added to it to make its temperature rise during the expansion to  $t + dt$  may be denoted by

$$M dv + N dt.$$

The mechanical equivalent of this is

$$J(M dv + N dt),$$

if  $J$  denote the mechanical equivalent of a unit of heat. Hence the mechanical measure of the total external effect produced in the circumstances is

$$(p - JM) dv - JN dt.$$

The total external effect, after any finite amount of expansion, accompanied by any, continuous change of temperature, has taken place, will consequently be, in mechanical terms,

$$\int \{ (p - JM) dv - JN dt \};$$

where we must suppose  $t$  to vary with  $v$ , so as to be the actual temperature of the medium at each instant, and the integration with reference to  $v$  must be performed between limits corresponding to the initial and final volumes. Now if, at any subsequent time, the volume and temperature of the medium become what they were at the beginning, however arbitrarily they may have been made to vary in the period, the total external effect must,

\* Account of Carnot's Theory, foot-note on § 26.

† This may have parts consisting of different substances, or of the same substance in different states, provided the temperature of all be the same. See below, part 3, § 53-56.

according to Prop. I., amount to nothing; and hence

$$(p - JM)dv - JNd t^*$$

must be the differential of a function of two independent variables, or we must have

$$\frac{d(p - JM)}{dt} = \frac{d(-JN)}{dv}, \dots \dots (1)$$

this being merely the analytical expression of the condition, that the preceding integral may vanish in every case in which the initial and final values of  $v$  and  $t$  are the same, respectively. Observing that  $J$  is an absolute constant, we may put the result into the form

$$\frac{dp}{dt} = J \left( \frac{dM}{dt} - \frac{dN}{dv} \right). \dots \dots (2)$$

This equation expresses, in a perfectly comprehensive manner, the application of the first fundamental proposition to the thermal and mechanical circumstances of any substance whatever, under uniform pressure in all directions, when subjected to any possible variations of temperature, volume and pressure.

21. The corresponding application of the second fundamental proposition is completely expressed by the equation

$$\frac{dp}{dt} = \mu M, \dots \dots (3)$$

where  $\mu$  denotes what is called "Carnot's function," a quantity which has an absolute value, the same for all substances for any given temperature, but which may vary with the temperature in a manner that can only be determined by experiment. To prove this proposition, it may be remarked in the first place that Prop. II. could not be true for every case in which the temperature of the refrigerator differs infinitely little from that of the source, without being true universally. Now, if a substance be allowed first to expand from  $v$  to  $v + dv$ , its temperature being kept constantly  $t$ ; if, secondly, it be allowed to expand further, without either emitting or absorbing heat till its temperature goes down through an infinitely small range, to  $t - \tau$ ; if, thirdly, it be compressed at the constant temperature  $t - \tau$ , so much (actually by an amount differing from  $dv$  by only an infinitely small quantity of the second order), that when, fourthly, the volume is further diminished to  $v$  without the medium's being allowed to either emit or absorb heat, its temperature may be exactly  $t$ ; it may be considered as constituting a thermo-dynamic

[\* The integral function  $\int \{ (JM - p)dv + JNd t \}$  may obviously be called the *mechanical energy* of the fluid mass; as (when the constant of integration is properly assigned) it expresses the whole work the fluid has in it to produce. The consideration of this function is the subject of a short paper communicated to the Royal Society of Edinburgh, Dec. 15, 1851, as an appendix to the paper at present republished.]

engine which fulfills Carnot's condition of complete reversibility. Hence, by Prop. II., it must produce the same amount of work for the same quantity of heat absorbed in the first operation, as any other substance similarly operated upon through the same range of temperatures. But  $\frac{dp}{dt} \tau . dv$  is obviously the whole work done in the complete cycle, and (by the definition of M in § 20)  $Mdv$  is the quantity of heat absorbed in the first operation. Hence the value of

$$\frac{\frac{dp}{dt} \tau . dv}{Mdv}, \text{ or } \frac{\frac{dp}{dt} \tau}{M},$$

must be the same for all substances, with the same values of  $t$  and  $\tau$ ; or, since  $\tau$  is not involved except as a factor, we must have

$$\frac{\frac{dp}{dt}}{M} = \mu, \quad . . . . . (4)$$

where  $\mu$  depends only on  $t$ ; from which we conclude the proposition which was to be proved.

22. The very remarkable theorem that  $\frac{dp}{dt}$  must be the same for all substances at the same temperature, was first given (although not in precisely the same terms) by Carnot, and demonstrated by him, according to the principles he adopted. We have now seen that its truth may be satisfactorily established without adopting the false part of his principles. Hence all Carnot's conclusions, and all conclusions derived by others from his theory, which depend merely on equation (3), require no modification when the dynamical theory is adopted. Thus, all the conclusions contained in Sections I., II., and III., of the Appendix to my Account of Carnot's Theory, and in the paper immediately following it in the Transactions, entitled "Theoretical Considerations on the Effect of Pressure in Lowering the Freezing Point of Water," by my elder brother, still hold. Also, we see that Carnot's expression for the mechanical effect derivable from a given quantity of heat by means of a perfect engine in which the range of temperatures is infinitely small, expresses truly the greatest effect which can possibly be obtained in the circumstances; although it is in reality only an infinitely small fraction of the whole mechanical equivalent of the heat supplied; the remainder being irrecoverably lost to man, and therefore "wasted," although not *annihilated*.

23. On the other hand, the expression for the mechanical effect obtainable from a given quantity of heat entering an engine

from a "source" at a given temperature, when the range down to the temperature of the cold part of the engine or the "refrigerator" is finite, will differ most materially from that of Carnot; since, a finite quantity of mechanical effect being now obtained from a finite quantity of heat entering the engine, a finite fraction of this quantity must be converted from heat into mechanical effect. The investigation of this expression, with numerical determinations founded on the numbers deduced from Regnault's observations on steam, which are shown in Tables I. and II. of my former paper, constitutes the second part of the paper at present communicated.

[To be continued.]

### III. On the Fatty Acid of *Cocculus indicus*.

By Mr. W. CROWDER, Assistant to Dr. Anderson of Edinburgh\*.

**I**N a paper published several years ago in the *Annalen der Chemie und Pharmacie*† upon the substances obtained from *Cocculus indicus*, Dr. Francis pointed out the existence of a fatty acid which had not previously been subjected to investigation, to which he gave the name of stearophanic acid; and after analysing it, its æther and silver salt, he gave the formula  $C^{35}H^{35}O^4$  as representing its constitution. The recent researches of chemists having pointed out that all the fatty acids have the same general formula  $(C^2H^2)^nO^4$ , and the acid under consideration being evidently a member of that series, it seemed probable that its true formula would be  $C^{36}H^{36}O^4$ , with which the results obtained by Dr. Francis, when calculated with the corrected atomic weight of carbon, closely agree. This formula has indeed been since assumed by Dr. Francis himself and by other chemists, as the true expression of his results; and as the investigation of the fatty acids has since the date of his experiments made great strides, I thought it desirable to submit the fat of *Cocculus indicus* to a new examination, the result of which has fully confirmed the correctness of Dr. Francis's numbers, and conclusively established the formula  $C^{36}H^{36}O^4$ , and consequently the identity of the acid with the bassic acid since described by Mr. Hardwicke‡. My experiments were performed in the labora-

\* Communicated by the Author.

† See also Phil. Mag. for September 1842.

‡ [Why, under these circumstances, the author retains the name of bassic acid we are at a loss to understand. The name *stearophanic*, derived from the properties of the substance, is surely preferable to either that of *cocculostearic* suggested by Berzelius, or of *bassic* proposed by Mr. Hardwicke from its occurrence in *Bassia latifolia*; especially as the recent researches of Heintz go to prove that its occurrence is not restricted to the vegetable kingdom, but that it likewise forms one of the constituents of human fat.—W. F.]

abstraction of at least three or four millions of gallons over the area indicated above. As these wells do not affect one another, however, in the slightest degree—and several of them have been flowing uninterruptedly for many years—we may fairly assume that the reservoir is capable of yielding a very much larger supply without detriment to existing interests.

There are perhaps few localities in which the subject of Artesian wells can be better studied than in this valley between Silsoe and Barton-in-the-Clay; the latter village lying under the bold escarpment of the chalk with its projecting spurs and rounded hollow coombes, and the former on the opposing range of the lower greensand. The relative heights, the mineral characters, and the dip of the strata are readily observable; and from various positions the eye can take in at a glance the physical and geological nature of the whole of the surrounding district—the marly impervious valley extending between the sandstone hills and the projecting chalk, and resting upon the under-dipping beds of the former strata.

In these examinations, besides other points of minor importance, five conditions have to be more particularly considered. First, the general levels of the country; secondly, the relative positions, inclination, and thickness of the strata; thirdly, their permeable or impermeable nature; fourthly, the outcrop area and surface configuration of the water-supplying beds; and fifthly, the chemical composition, &c. of the same, as likely to affect or not the quality of the water. Mineralogical characters, therefore, although useless, and even hurtful in their attempted interpretations, in questions of pure or abstract geology, become, in these local and practical investigations, of the highest value.

XIV. *On the Dynamical Theory of Heat, with numerical results deduced from Mr. Joule's equivalent of a Thermal Unit, and M. Regnault's observations on Steam.* By WILLIAM THOMSON, M.A., Fellow of St. Peter's College, Cambridge, and Professor of Natural Philosophy in the University of Glasgow.

[Continued from p. 21.]

PART II.—*On the Motive Power of Heat through Finite Ranges of Temperature.*

24. **I**T is required to determine the quantity of work which a perfect engine, supplied from a source at any temperature,  $S$ , and parting with its waste heat to a refrigerator at any lower temperature,  $T$ , will produce from a given quantity,  $H$ , of heat drawn from the source.

25. We may suppose the engine to consist of an infinite

number of perfect engines, each working within an infinitely small range of temperature, and arranged in a series of which the source of the first is the given source, the refrigerator of the last the given refrigerator, and the refrigerator of each intermediate engine is the source of that which follows it in the series. Each of these engines will, in any time, emit just as much less heat to its refrigerator than is supplied to it from its source, as is the equivalent of the mechanical work which it produces. Hence if  $t$  and  $t + dt$  denote respectively the temperatures of the refrigerator and source of one of the intermediate engines, and if  $q$  denote the quantity of heat which this engine discharges into its refrigerator in any time, and  $q + dq$  the quantity which it draws from its source in the same time, the quantity of work which it produces in that time will be  $Jdq$  according to Prop. I., and it will also be  $q\mu dt$  according to the expression of Prop. II., investigated in § 21; and therefore we must have

$$Jdq = q\mu dt.$$

Hence, supposing that the quantity of heat supplied from the first source, in the time considered is  $H$ , we find by integration

$$\log \frac{H}{q} = \frac{1}{J} \int_t^S \mu dt.$$

But the value of  $q$ , when  $t = T$ , is the final remainder discharged into the refrigerator at the temperature  $T$ ; and therefore, if this be denoted by  $R$ , we have

$$\log \frac{H}{R} = \frac{1}{J} \int_T^S \mu dt \quad \dots \dots \dots (5);$$

from which we deduce

$$R = H\epsilon^{-\frac{1}{J} \int_T^S \mu dt} \quad \dots \dots \dots (6).$$

Now the whole amount of work produced will be the mechanical equivalent of the quantity of heat lost; and, therefore, if this be denoted by  $W$ , we have

$$W = J(H - R) \quad \dots \dots \dots (7),$$

and consequently, by (6),

$$W = JH \left\{ 1 - \epsilon^{-\frac{1}{J} \int_T^S \mu dt} \right\} \quad \dots \dots (8).$$

26. To compare this with the expression  $H \int_T^S \mu dt$ , for the duty indicated by Carnot's theory\*, we may expand the exponential in the preceding equation, by the usual series. We thus

\* "Account," &c., Equation 7, § 31.

find

$$W = \left( 1 - \frac{\theta}{1.2} + \frac{\theta^2}{1.2.3} - \&c. \right) \cdot H \int_T^s \mu dt \quad \left. \vphantom{W} \right\} \quad (9).$$

where

$$\theta = \frac{1}{J} \int_T^s \mu dt$$

This shows that the work really produced, which always falls short of the duty indicated by Carnot's theory, approaches more and more nearly to it as the range is diminished; and ultimately, when the range is infinitely small, is the same as if Carnot's theory required no modification, which agrees with the conclusion stated above in § 22.

27. Again, equation (8) shows that the real duty of a given quantity of heat supplied from the source increases with every increase of the range; but that instead of increasing indefinitely in proportion to  $\int_T^s \mu dt$ , as Carnot's theory makes it do, it never reaches the value  $JH$ , but approximates to this limit, as  $\int_T^s \mu dt$  is increased without limit. Hence Carnot's remark\* regarding the practical advantage that may be anticipated from the use of the air-engine, or from any method by which the range of temperatures may be increased, loses only a part of its importance, while a much more satisfactory view than his of the practical problem is afforded. Thus we see that, although the full equivalent of mechanical effect cannot be obtained even by means of a perfect engine, yet when the actual source of heat is at a high enough temperature above the surrounding objects, we may get more and more nearly the whole of the admitted heat converted into mechanical effect, by simply increasing the effective range of temperature in the engine.

28. The preceding investigation (§ 25) shows that the value of Carnot's function,  $\mu$ , for all temperatures within the range of the engine, and the absolute value of Joule's equivalent,  $J$ , are enough of data to calculate the amount of mechanical effect of a perfect engine of any kind, whether a steam-engine, an air-engine, or even a thermo-electric engine; since, according to the axiom stated in § 12, and the demonstration of Prop. II., no inanimate material agency could produce more mechanical effect from a given quantity of heat, with a given available range of temperatures, than an engine satisfying the criterion stated in the enunciation of the proposition.

29. The mechanical equivalent of a thermal unit Fahrenheit, or the quantity of heat necessary to raise the temperature of a

\* "Account, &c." Appendix, Section IV.

pound of water from  $32^{\circ}$  to  $33^{\circ}$  Fahr., has been determined by Joule in foot-pounds at Manchester, and the value which he gives as his best determination is 772.69. Mr. Rankine takes, as the result of Joule's determination, 772, which he estimates must be within  $\frac{1}{300}$  of its own amount, of the truth. If we take  $772\frac{2}{9}$  as the number, we find, by multiplying it by  $\frac{9}{5}$ , 1390 as the equivalent of the thermal unit Centigrade, which is taken as the value of  $J$  in the numerical applications contained in the present paper.

30. With regard to the determination of the values of  $\mu$  for different temperatures, it is to be remarked that equation (4) shows that this might be done by experiments upon any substance whatever of indestructible texture, and indicates exactly the experimental data required in each case. For instance, by first supposing the medium to be air; and again, by supposing it to consist partly of liquid water and partly of saturated vapour, we deduce, as is shown in Part III. of this paper, the two expressions (6), given in § 30 of my former paper ("Account of Carnot's Theory"), for the value of  $\mu$  at any temperature. As yet no experiments have been made upon air which afford the required data for calculating the value of  $\mu$  through any extensive range of temperature; but for temperatures between  $50^{\circ}$  and  $60^{\circ}$  Fahr., Joule's experiments\* on the heat evolved by the expenditure of a given amount of work on the compression of air kept at a constant temperature, afford the most direct data for this object which have yet been obtained; since, if  $Q$  be the quantity of heat evolved by the compression of a fluid subject to "the gaseous laws" of expansion and compressibility,  $W$  the amount of mechanical work spent, and  $t$  the constant temperature of the fluid, we have by (11) of § 49 of my former paper,

$$\mu = \frac{W.E}{Q(1 + Et)}, \dots \dots \dots (10)$$

which is in reality a simple consequence of the other expression for  $\mu$  in terms of data with reference to air. Remarks upon the determination of  $\mu$  by such experiments, and by another class of experiments on air originated by Joule, are reserved for a separate communication, which I hope to be able to make to the Royal Society on another occasion.

31. The second of the expressions (6), in § 30 of my former paper, or the equivalent expression (32), given below in the present paper, shows that  $\mu$  may be determined for any temperature from determinations for that temperature of—

\* "On the Changes of Temperature produced by the Rarefaction and Condensation of Air," *Phil. Mag.* vol. xxvi. May 1845.

- (1.) The rate of variation with the temperature, of the pressure of saturated steam.
- (2.) The latent heat of a given weight of saturated steam.
- (3.) The volume of a given weight of saturated steam.
- (4.) The volume of a given weight of water.

The last mentioned of these elements may, on account of the manner in which it enters the formula, be taken as constant, without producing any appreciable effect on the probable accuracy of the result.

32. Regnault's observations have supplied the first of the data with very great accuracy for all temperatures between  $-32^{\circ}$  Cent. and  $230^{\circ}$ .

33. As regards the second of the data, it must be remarked that all experimenters, from Watt, who first made experiments on the subject, to Regnault, whose determinations are the most accurate and extensive that have yet been made, appear to have either explicitly or tacitly assumed the same principle as that of Carnot which is overturned by the dynamical theory of heat; inasmuch as they have defined the "total heat of steam" as the quantity of heat required, to convert a unit of weight of water at  $0^{\circ}$ , into steam in the particular state considered. Thus Regnault, setting out with this definition for "the total heat of saturated steam," gives experimental determinations of it for the entire range of temperatures from  $0^{\circ}$  to  $230^{\circ}$ ; and he deduces the "latent heat of saturated steam" at any temperature, from the "total heat," so determined, by subtracting from it the quantity of heat necessary to raise the liquid to that temperature. Now, according to the dynamical theory, the quantity of heat expressed by the preceding definition depends on the manner (which may be infinitely varied) in which the specified change of state is effected; differing in different cases by the thermal equivalents of the differences of the external mechanical effect produced in the expansion. For instance, the final quantity of heat required to evaporate a quantity of water at  $0^{\circ}$ , and then, keeping it always in the state of saturated vapour\*, bring it to the temperature  $100^{\circ}$ , cannot be so much as three-fourths of the quantity required, first, to raise the temperature of the liquid to

\* See below (Part III. § 58), where the "negative" specific heat of saturated steam is investigated. If the mean value of this quantity between  $0^{\circ}$  and  $100^{\circ}$  were  $-1.5$  (and it cannot differ much from this) there would be 150 units of heat emitted by a pound of saturated vapour in having its temperature raised (by compression) from  $0^{\circ}$  to  $100^{\circ}$ . The latent heat of the vapour at  $0^{\circ}$  being 606.5, the final quantity of heat required to convert a pound of water at  $0^{\circ}$  into saturated steam at  $100^{\circ}$ , in the first of the ways mentioned in the text, would consequently be 456.5, which is only about  $\frac{3}{4}$  of the quantity 637 found as "the total heat" of the saturated vapour at  $100^{\circ}$ , by Regnault.

100°, and then evaporate it at that temperature; and yet either quantity is expressed by what is generally received as a *definition* of the "total heat" of the saturated vapour. To find what it is that is really determined as "total heat" of saturated steam in Regnault's researches, it is only necessary to remark, that the measurement actually made is of the quantity of heat emitted by a certain weight of water in passing through a calorimetric apparatus, which it enters as saturated steam, and leaves in the liquid state, the result being reduced to what would have been found if the final temperature of the water had been exactly 0°. For there being no external mechanical effect produced (other than that of sound, which it is to be presumed is quite inappreciable), the only external effect is the emission of heat. This must, therefore, according to the fundamental proposition of the dynamical theory, be independent of the intermediate agencies. It follows that, however the steam may rush through the calorimeter, and at whatever reduced pressure it may actually be condensed\*, the heat emitted externally must be exactly the same as if the condensation took place under the full pressure of the entering saturated steam; and we conclude that *the total heat*, as actually determined from his experiments by Regnault, is the quantity of heat that would be required, first to raise the liquid to the specified temperature, and then to evaporate it at that temperature; and that the principle on which he determines the latent heat is correct. Hence, through the range of his experiments, that is from 0° to 230°, we may consider the second of

\* If the steam have to rush through a long fine tube, or through a small aperture within the calorimetric apparatus, its pressure will be diminished before it is condensed; and there will, therefore, in two parts of the calorimeter be saturated steam at different temperatures (as, for instance, would be the case if steam from a high pressure boiler were distilled into the open air); yet, on account of the heat developed by the fluid friction, which would be precisely the equivalent of the mechanical effect of the expansion wasted in the rushing, the heat measured by the calorimeter would be precisely the same as if the condensation took place at a pressure not appreciably lower than that of the entering steam. The circumstances of such a case have been overlooked by Clausius (Poggendorff's *Annalen*, 1850, No. 4, p. 510), when he expresses with some doubt the opinion that the latent heat of saturated steam will be truly found from Regnault's "total heat," by deducting "the sensible heat"; and gives as a reason that, in the actual experiments, the condensation must have taken place "under the same pressure, or nearly under the same pressure," as the evaporation. The question is not, *Did the condensation take place at a lower pressure than that of the entering steam?* but, *Did Regnault make the steam work an engine in passing through the calorimeter, or was there so much noise of steam rushing through it as to convert an appreciable portion of the total heat into external mechanical effect?* And a negative answer to this is a sufficient reason for adopting *with certainty* the opinion that the principle of his determination of the latent heat is correct.

the data required for the calculation of  $\mu$  as being supplied in a complete and satisfactory manner.

34. There remains only the third of the data, or the volume of a given weight of saturated steam, for which accurate experiments through an extensive range are wanting; and no experimental researches bearing on the subject having been made since the time when my former paper was written, I see no reason for supposing that the values of  $\mu$  which I then gave are not the most probable that can be obtained in the present state of science; and, on the understanding stated in § 33 of that paper, that accurate experimental determinations of the densities of saturated steam at different temperatures may indicate considerable errors in the densities which have been assumed according to the "gaseous laws," and may consequently render considerable alterations in my results necessary, I shall still continue to use Table I. of that paper, which shows the values of  $\mu$  for the temperatures  $\frac{1}{2}$ ,  $1\frac{1}{2}$ ,  $2\frac{1}{2}$  . . .  $230\frac{1}{2}$ , or, the mean values of  $\mu$  for each of the 230 successive Centigrade degrees of the air-thermometer above the freezing-point, as the basis of numerical applications of the theory. It may be added, that any experimental researches sufficiently trustworthy in point of accuracy, yet to be made, either on air or any other substance, which may lead to values of  $\mu$  differing from those, must be admitted as proving a discrepancy between the true densities of saturated steam, and those which have been assumed\*.

35. Table II. of my former paper, which shows the values of  $\int_0^t \mu dt$  for  $t=1, t=2, t=3, \dots t=231$ , renders the calculation of the mechanical effect derivable from a given quantity of heat by means of a perfect engine, with any given range included between the limits 0 and 231, extremely easy; since the quantity to be divided by  $J$ † in the index of the exponential in the

\* I cannot see that any hypothesis, such as that adopted by Clausius fundamentally in his investigations on this subject, and leading, as he shows to determinations of the densities of saturated steam at different temperatures, which indicate enormous deviations from the gaseous laws of variation with temperature and pressure, is more probable, or is probably nearer the truth, than that the density of saturated steam does follow these laws as it is usually assumed to do. In the present state of science it would perhaps be wrong to say that either hypothesis is more probable than the other [or that the rigorous truth of either hypothesis is probable at all].

† It ought to be remarked, that as the unit of force implied in the determinations of  $\mu$  is the weight of a pound of matter at Paris, and the unit of force in terms of which  $J$  is expressed is the weight of a pound at Manchester, these numbers ought in strictness to be modified so as to express the values in terms of a common unit of force; but as the force of gravity at Paris differs by less than  $\frac{1}{10000}$  of its own value from the force of gravity at Manchester, this correction will be much less than the probable errors from other sources, and may therefore be neglected.

expression (8) will be found by subtracting the number in that table corresponding to the value of T, from that corresponding to the value of S. [Tables I. and II. of the former paper are reprinted here, for the sake of convenience in referring to them.]

*Tables extracted from "Account of Carnot's Theory," Trans. R. S. Ed. vol. xvi. part 5.*

*Explanation of Table I.*

The mean values of  $\mu$  for the first, for the eleventh, for the twenty-first, and so on, up to the 231st\* degree of the air thermometer, have been calculated in the manner explained in the preceding paragraphs. These, and interpolated results, which must agree with what would have been obtained by direct calculation from Regnault's data to three significant places of figures (and even for the temperatures between  $0^\circ$  and  $100^\circ$ , the experimental data do not justify us in relying on any of the results to a greater degree of accuracy), are exhibited in Table I.

*To find the amount of mechanical effect due to a unit of heat, descending from a body at a temperature S to a body at T, if these numbers be integers, we have merely to add the values of  $\mu$  in Table I. corresponding to the successive numbers,*

$$T+1, T+2, \dots S-2, S-1.$$

*Explanation of Table II.*

The calculation of the mechanical effect in any case, which might always be effected in the manner described in § 37 (with the proper modification for fractions of degrees when necessary), is much simplified by the use of Table II., where the first number of Table I., the sum of the first and second, the sum of the first three, the sum of the first four, and so on, are successively exhibited. The sums thus tabulated are the values of the integrals

$$\int_0^1 \mu dt, \int_0^2 \mu dt, \int_0^3 \mu dt, \dots \int_0^{231} \mu dt;$$

and if we denote  $\int_0^t \mu dt$  by the letter M, Table II. may be regarded as a table of the values of M.

*To find the amount of mechanical effect due to a unit of heat descending from a body at a temperature S to a body at T, if these numbers be integers, we have merely to subtract the value of M, for the number T+1, from the value for the number S, given in Table II.*

\* In strictness, the 230th is the last degree for which the experimental data are complete; but the data for the 231st may readily be assumed in a sufficiently satisfactory manner.

Table I\*.—Mean Values of  $\mu$  for the successive Degrees of the Air Thermometer from 0° to 230°.

	$\mu$ .		$\mu$ .		$\mu$ .		$\mu$ .		$\mu$ .
1	4.960	48	4.366	94	3.889	140	3.549	186	3.309
2	4.946	49	4.355	95	3.880	141	3.543	187	3.304
3	4.932	50	4.343	96	3.871	142	3.537	188	3.300
4	4.918	51	4.331	97	3.863	143	3.531	189	3.295
5	4.905	52	4.319	98	3.854	144	3.525	190	3.291
6	4.892	53	4.308	99	3.845	145	3.519	191	3.287
7	4.878	54	4.296	100	3.837	146	3.513	192	3.282
8	4.865	55	4.285	101	3.829	147	3.507	193	3.278
9	4.852	56	4.273	102	3.820	148	3.501	194	3.274
10	4.839	57	4.262	103	3.812	149	3.495	195	3.269
11	4.826	58	4.250	104	3.804	150	3.490	196	3.265
12	4.812	59	4.239	105	3.796	151	3.484	197	3.261
13	4.799	60	4.227	106	3.788	152	3.479	198	3.257
14	4.786	61	4.216	107	3.780	153	3.473	199	3.253
15	4.773	62	4.205	108	3.772	154	3.468	200	3.249
16	4.760	63	4.194	109	3.764	155	3.462	201	3.245
17	4.747	64	4.183	110	3.757	156	3.457	202	3.241
18	4.735	65	4.172	111	3.749	157	3.451	203	3.237
19	4.722	66	4.161	112	3.741	158	3.446	204	3.233
20	4.709	67	4.150	113	3.734	159	3.440	205	3.229
21	4.697	68	4.140	114	3.726	160	3.435	206	3.225
22	4.684	69	4.129	115	3.719	161	3.430	207	3.221
23	4.672	70	4.119	116	3.712	162	3.424	208	3.217
24	4.659	71	4.109	117	3.704	163	3.419	209	3.213
25	4.646	72	4.098	118	3.697	164	3.414	210	3.210
26	4.634	73	4.088	119	3.689	165	3.409	211	3.206
27	4.621	74	4.078	120	3.682	166	3.404	212	3.202
28	4.609	75	4.067	121	3.675	167	3.399	213	3.198
29	4.596	76	4.057	122	3.668	168	3.394	214	3.195
30	4.584	77	4.047	123	3.661	169	3.389	215	3.191
31	4.572	78	4.037	124	3.654	170	3.384	216	3.188
32	4.559	79	4.028	125	3.647	171	3.380	217	3.184
33	4.547	80	4.018	126	3.640	172	3.375	218	3.180
34	4.535	81	4.009	127	3.633	173	3.370	219	3.177
35	4.522	82	3.999	128	3.627	174	3.365	220	3.173
36	4.510	83	3.990	129	3.620	175	3.361	221	3.169
37	4.498	84	3.980	130	3.614	176	3.356	222	3.165
38	4.486	85	3.971	131	3.607	177	3.351	223	3.162
39	4.474	86	3.961	132	3.601	178	3.346	224	3.158
40	4.462	87	3.952	133	3.594	179	3.342	225	3.155
41	4.450	88	3.943	134	3.586	180	3.337	226	3.151
42	4.438	89	3.934	135	3.579	181	3.332	227	3.148
43	4.426	90	3.925	136	3.573	182	3.328	228	3.144
44	4.414	91	3.916	137	3.567	183	3.323	229	3.141
45	4.402	92	3.907	138	3.561	184	3.318	230	3.137
46	4.390	93	3.898	139	3.555	185	3.314	231	3.134
47	4.378								

\* The numbers here tabulated may also be regarded as the active values of  $\mu$  for  $t=\frac{1}{2}$ ,  $t=1\frac{1}{2}$ ,  $t=2\frac{1}{2}$ ,  $t=3\frac{1}{2}$ , &c.

Table II.—Mechanical Effect in Foot-Pounds due to a Thermic Unit Centigrade, passing from a body, at any Temperature less than 230° to a body at 0°.

Superior limit of temperature.	Mechanical effect.								
°	ft.-lbs.								
1	4-960	48	223-487	94	412-545	140	582-981	186	740-310
2	9-906	49	227-842	95	416-425	141	586-524	187	743-614
3	14-838	50	232-185	96	420-296	142	590-061	188	746-914
4	19-756	51	236-516	97	424-159	143	593-592	189	750-209
5	24-661	52	240-835	98	428-013	144	597-117	190	753-500
6	29-553	53	245-143	99	431-858	145	600-636	191	756-787
7	34-431	54	249-439	100	435-695	146	604-099	192	760-069
8	39-296	55	253-724	101	439-524	147	607-656	193	763-347
9	44-143	56	257-997	102	443-344	148	611-157	194	766-621
10	48-987	57	262-259	103	447-156	149	614-652	195	769-890
11	53-813	58	266-509	104	450-960	150	618-142	196	773-155
12	58-625	59	270-748	105	454-756	151	621-626	197	776-416
13	63-424	60	274-975	106	458-544	152	625-105	198	779-673
14	68-210	61	279-191	107	462-324	153	628-578	199	782-926
15	72-983	62	283-396	108	466-096	154	632-046	200	786-175
16	77-743	63	287-590	109	469-860	155	635-508	201	789-420
17	82-490	64	291-773	110	473-617	156	638-965	202	792-661
18	87-225	65	295-945	111	477-366	157	642-416	203	795-898
19	91-947	66	300-106	112	481-107	158	645-862	204	799-131
20	96-656	67	304-256	113	484-841	159	649-302	205	802-360
21	101-353	68	308-396	114	488-567	160	652-737	206	805-585
22	106-037	69	312-525	115	492-286	161	656-167	207	808-806
23	110-709	70	316-644	116	495-998	162	659-591	208	812-023
24	115-368	71	320-752	117	499-702	163	663-010	209	815-236
25	120-014	72	324-851	118	503-399	164	666-424	210	818-446
26	124-648	73	328-939	119	507-088	165	669-833	211	821-652
27	129-269	74	333-017	120	510-770	166	673-237	212	824-854
28	133-878	75	337-084	121	514-445	167	676-636	213	828-052
29	138-474	76	341-141	122	518-113	168	680-030	214	831-247
30	143-058	77	345-188	123	521-774	169	683-419	215	834-438
31	147-630	78	349-225	124	525-428	170	686-803	216	837-626
32	152-189	79	353-253	125	529-075	171	690-183	217	840-810
33	156-736	80	357-271	126	532-715	172	693-558	218	843-990
34	161-271	81	361-280	127	536-348	173	696-928	219	847-167
35	165-793	82	365-279	128	539-975	174	700-293	220	850-340
36	170-303	83	369-269	129	543-595	175	703-654	221	853-509
37	174-801	84	373-249	130	547-209	176	707-010	222	856-674
38	179-287	85	377-220	131	550-816	177	710-361	223	859-836
39	183-761	86	381-181	132	554-417	178	713-707	224	862-994
40	188-223	87	385-133	133	558-051	179	717-049	225	866-149
41	192-673	88	389-076	134	561-597	180	720-386	226	869-300
42	197-111	89	393-010	135	565-176	181	723-718	227	872-448
43	201-537	90	396-935	136	568-749	182	727-046	228	875-592
44	205-951	91	400-851	137	572-316	183	730-369	229	878-733
45	210-353	92	404-758	138	575-877	184	733-687	230	881-870
46	214-743	93	408-656	139	579-432	185	737-001	231	885-004
47	219-121								

36. The following tables show some numerical results which have been obtained in this way, with a few (contained in the lower part of the second table) calculated from values of  $\int_0^t \mu dt$  estimated for temperatures above 230°, roughly, according to the rate of variation of that function within the experimental limits.

37. Explanation of the Tables.

Column I. in each table shows the assumed ranges.

Column II. shows ranges deduced by means of Table II. of the former paper, so that the value of  $\int_T^S \mu dt$  for each may be the same as for the corresponding range shown in column I.

Column III. shows what would be the duty of a unit of heat if Carnot's theory required no modification (or the actual duty of a unit of heat with additions through the range, to compensate for the quantities converted into mechanical effect).

Column IV. shows the true duty of a unit of heat, and a comparison of the numbers in it with the corresponding numbers in column III. shows how much the true duty falls short of Carnot's theoretical duty in each case.

Column VI. is calculated by the formula

$$R = \epsilon^{-\frac{1}{1390} \int_T^S \mu dt},$$

where  $\epsilon = 2.71828$ , and for  $\int_T^S \mu dt$  the successive values shown in column III. are used.

Column IV. is calculated by the formula

$$W = 1390(1 - R)$$

from the values of  $1 - R$  shown in column V.

38. Table of the Motive Power of Heat.

Range of temperatures.				III.	IV.	V.	VI.
I.		II.		Duty of a unit of heat through the whole range.	Duty of a unit of heat supplied from the source.	Quantity of heat converted into mechanical effect.	Quantity of heat wasted.
S.	T.	S.	T.	$\int_T^S \mu dt.$	W.	1 - R.	R.
				ft.-lbs.	ft.-lbs.		
1	0	31.08	30	4.960	4.948	.00356	.99644
10	0	40.86	30	48.987	48.1	.0346	.9654
20	0	51.7	30	96.656	93.4	.067	.933
30	0	62.6	30	143.06	136	.098	.902
40	0	73.6	30	188.22	176	.127	.873
50	0	84.5	30	232.18	214	.154	.846
60	0	95.4	30	274.97	249	.179	.821
70	0	106.3	30	316.64	283	.204	.796
80	0	117.2	30	357.27	315	.227	.773
90	0	128.0	30	396.93	345	.248	.752
100	0	138.8	30	435.69	374	.269	.731
110	0	149.1	30	473.62	401	.289	.711
120	0	160.3	30	510.77	427	.308	.692
130	0	171.0	30	547.21	452	.325	.675
140	0	181.7	30	582.98	476	.343	.657
150	0	192.3	30	618.14	499	.359	.641
160	0	203.0	30	652.74	521	.375	.625
170	0	213.6	30	686.80	542	.390	.610
180	0	224.2	30	720.39	562	.404	.596
190	0	190	0	753.50	582	.418	.582
200	0	200	0	786.17	600	.432	.568
210	0	210	0	818.45	619	.445	.555
220	0	220	0	850.34	636	.457	.542
230	0	230	0	881.87	653	.470	.530

39. *Supplementary Table of the Motive Powers of Heat.*

Range of temperatures.				III.	IV.	V.	VI.
I.		II.		Duty of a unit of heat through the whole range.	Duty of a unit of heat supplied from the source.	Quantity of heat converted into mechanical effect.	Quantity of heat wasted.
S.	T.	S.	T.	$\int_T^S \mu dt.$ ft.-lbs.	W. ft.-lbs.	1-R.	R.
101·1	0	140	30	439·9	377	·271	·729
105·8	0	230	100	446·2	382	·275	·725
300	0	300	0	1099	757	·545	·455
400	0	400	0	1395	879	·632	·368
500	0	500	0	1690	979	·704	·296
600	0	600	0	1980	1059	·762	·238
$\infty$	0	$\infty$	0	$\infty$	1390	1·000	·000

40. Taking the range  $30^\circ$  to  $140^\circ$  as an example suitable to the circumstances of some of the best steam-engines that have yet been made (see Appendix to Account of Carnot's Theory, Sec. v.), we find in column III. of the supplementary table, 377 ft.-lbs. as the corresponding duty of a unit of heat instead of 440, shown in column III., which is Carnot's theoretical duty. We conclude that the recorded performance of the Fowey-Consols engine in 1845, instead of being only  $57\frac{1}{2}$  per cent. amounted really to 67 per cent., or  $\frac{2}{3}$  of the duty of a perfect engine with the same range of temperature; and this duty being ·271 (rather more than  $\frac{1}{4}$ ) of the whole equivalent of the heat used; we conclude further, that  $\frac{1}{5\cdot49}$ , or 18 per cent. of the whole heat supplied, was actually converted into mechanical effect by that steam-engine.

41. The numbers in the lower part of the supplementary table show the great advantage that may be anticipated from the perfecting of the air-engine, or any other kind of thermo-dynamic engine in which the range of the temperature can be increased much beyond the limits actually attainable in steam-engines. Thus an air-engine, with its hot part at  $600^\circ$ , and its cold part at  $0^\circ$  Cent., working with perfect œconomy, would convert 76 per cent. of the whole heat used into mechanical effect; or working with such œconomy as has been estimated for the Fowey-Consols engine, that is, producing 67 per cent. of the theoretical duty corresponding to its range of temperature, would convert 51 per cent. of all the heat used into mechanical effect.

42. It was suggested to me by Mr. Joule, in a letter dated December 9, 1848, that the true value of  $\mu$  might be "inversely

as the temperatures from zero\*," and values for various temperatures calculated by means of the formula,

$$\mu = J \frac{E}{1 + Et} \dots \dots \dots (11)$$

were given for comparison with those which I had calculated from data regarding steam. This formula is also adopted by Clausius, who uses it fundamentally in his mathematical investigations. If  $\mu$  were correctly expressed by it, we should have

$$\int_T^S \mu dt = J \log \frac{1 + ES}{1 + ET};$$

and therefore equations (1) and (2) would become

$$W = J \frac{S - T}{\frac{1}{E} + S} \dots \dots \dots (12)$$

$$R = \frac{\frac{1}{E} + T}{\frac{1}{E} + S} \dots \dots \dots (13)$$

43. The reasons upon which Mr. Joule's opinion is founded, that the preceding equation (11) may be the correct expression for Carnot's function, although the values calculated by means of it differ considerably from those shown in Table I. of my former paper, form the subject of a communication which I hope to have an opportunity of laying before the Royal Society previously to the close of the present session.

[To be continued.]

\* If we take  $\mu = k \frac{E}{1 + Et}$  where  $k$  may be any constant, we find

$$W = J \left( \frac{S - T}{\frac{1}{E} + S} \right)^k;$$

which is the formula I gave when this paper was communicated. I have since remarked, that Mr. Joule's hypothesis implies essentially that the coefficient  $k$  must be as it is taken in the text, the mechanical equivalent of a thermal unit. Mr. Rankine, in a letter dated March 27, 1851, informs me that he has deduced, from the principles laid down in his paper communicated last year to this Society, an approximate formula for the ratio of the maximum quantity of heat converted into mechanical effect to the whole quantity expended, in an expansive engine of any substance, which, on comparison, I find agrees exactly with the expression (12) given in the text as a consequence of the hypothesis suggested by Mr. Joule regarding the value of  $\mu$  at any temperature.—[April 4, 1851.]

7.34 grains substance gave 5.54 carbonic acid and 0.56 water.  
4.68 grains substance gave 2.29 sulphate of lead.

	Experiment.	Calculation.		
Carbon . . .	20.57	19.91	C <sup>10</sup>	60
Hydrogen . . .	00.85	00.99	H <sup>3</sup>	3
Oxygen . . .	...	16.19	O <sup>6</sup>	48
Bromine . . .	...	26.01	Br	78.26
Oxide of lead . . .	36.48	36.90	PbO	111.56
	<u>100.00</u>	<u>100.00</u>		<u>300.82</u>

The composition of this salt is therefore expressed by the formula  $\text{PbO}, \text{C}^{10} \text{H}^2 \text{Br O}^5 + \text{HO}$ .

I have also obtained a substitution product of iodine, by operating in a particular manner, which I shall describe in detail in a future paper. I shall conclude this communication with a list of the substances described in it along with their formulæ.

These experiments were performed in the laboratory of Dr. Anderson, to whom I am much indebted for assistance.

Pyromeconic acid . . .	$\text{C}^{10} \text{H}^3 \text{O}^5 + \text{HO}$ .
Pyromeconate of baryta . . .	$\text{BaO}, \text{C}^{10} \text{H}^3 \text{O}^5 + \text{HO}$ .
Pyromeconate of strontia . . .	$\text{SrO}, \text{C}^{10} \text{H}^3 \text{O}^5 + \text{HO}$ .
Pyromeconate of lime . . .	$\text{CaO}, \text{C}^{10} \text{H}^3 \text{O}^5 + \text{HO}$ .
Pyromeconate of magnesia . . .	$\text{MgO}, \text{C}^{10} \text{H}^3 \text{O}^5$ .
Pyromeconate of lead . . .	$\text{PbO}, \text{C}^{10} \text{H}^3 \text{O}^5$ .
Pyromeconate of copper . . .	$\text{CuO}, \text{C}^{10} \text{H}^3 \text{O}^5$ .
Pyromeconate of iron . . .	$\text{Fe}^2 \text{O}^3, 3\text{C}^{10} \text{H}^3 \text{O}^5$ .
Bromo-pyromeconic acid . . .	$\text{C}^{10} \text{H}^2 \text{Br O}^5 + \text{HO}$ .
Lead salt . . . . .	$\text{PbO}, \text{C}^{10} \text{H}^2 \text{Br O}^5 + \text{HO}$ .

Edinburgh, July 1, 1852.

XXV. *On the Dynamical Theory of Heat, with numerical results deduced from Mr. Joule's equivalent of a Thermal Unit, and M. Regnault's Observations on Steam.* By WILLIAM THOMSON, M.A., Fellow of St. Peter's College, Cambridge, and Professor of Natural Philosophy in the University of Glasgow.

[Concluded from p. 117.]

PART III.—*Applications of the Dynamical Theory to establish Relations between the Physical Properties of all Substances.*

44. **T**HE two fundamental equations of the dynamical theory of heat, investigated above, express relations between quantities of heat required to produce changes of volume and temperature in any material medium whatever, subjected to a uniform pressure in all directions, which lead to various remarkable conclu-

sions. Such of these as are independent of Joule's principle (expressed by equation (2) of § 20), being also independent of the truth or falseness of Carnot's contrary assumption regarding the permanence of heat, are common to his theory and to the dynamical theory; and some of the most important of them\* have been given by Carnot himself, and other writers who adopted his principles and mode of reasoning without modification. Other remarkable conclusions on the same subject might have been drawn from the equation  $\frac{dM}{dt} - \frac{dN}{dv} = 0$ , expressing

Carnot's assumption (of the truth of which experimental tests might have been thus suggested); but I am not aware that any conclusion deducible from it, not included in Carnot's expression for the motive power of heat through finite ranges of temperature, has yet been actually obtained and published.

45. The recent writings of Rankine and Clausius contain some of the consequences of the fundamental principle of the dynamical theory (expressed in the first fundamental proposition above) regarding physical properties of various substances; among which may be mentioned especially a very remarkable discovery regarding the specific heat of saturated steam (investigated also in this paper in § 58 below), made independently by the two authors, and a property of water at its freezing-point, deduced from the corresponding investigation regarding ice and water under pressure by Clausius; according to which he finds that, for each  $\frac{1}{10}^{\circ}$  Cent. that the solidifying point of water is lowered by pressure, its latent heat, which under atmospheric pressure is 79, is diminished by .081. The investigations of both these writers involve fundamentally various hypotheses which may be or may not be found by experiment to be approximately true; and which render it difficult to gather from their writings what part of their conclusions, especially with reference to air and gases, depend merely on the necessary principles of the dynamical theory.

46. In the remainder of this paper, the two fundamental propositions, expressed by the equations

$$\frac{dM}{dt} - \frac{dN}{dv} = \frac{1}{J} \frac{dp}{dt} \quad \dots \quad (2) \text{ of } \S 20,$$

and

$$M = \frac{1}{u} \cdot \frac{dp}{dt}, \quad \dots \quad (3) \text{ of } \S 21,$$

are applied to establish properties of the specific heats of any substance whatever; and then special conclusions are deduced for the case of a fluid following strictly the "gaseous laws" of

\* See above, § 22.

density, and for the case of a medium consisting of parts in different states at the same temperature, as water and saturated steam, or ice and water.

47. In the first place it may be remarked, that by the definition of M and N in § 20, N must be what is commonly called the "specific heat at constant volume" of the substance, provided the quantity of the medium be the standard quantity adopted for specific heats, which, in all that follows, I shall take as the unit of weight. Hence the fundamental equation of the dynamical theory, (2) of § 20, expresses a relation between this specific heat and the quantities for the particular substance denoted by M and  $p$ . If we eliminate M from this equation, by means of equation (3) of § 21, derived from the expression of the second fundamental principle of the theory of the motive power of heat, we find

$$\frac{dN}{dv} = \frac{d\left(\frac{1}{\mu} \frac{dp}{dt}\right)}{dt} - \frac{1}{J} \frac{dp}{dt} \quad \dots \quad (14),$$

which expresses a relation between the variation in the specific heat at constant volume, of any substance, produced by an alteration of its volume at a constant temperature, and the variation of its pressure with its temperature when the volume is constant; involving a function,  $\mu$ , of the temperature, which is the same for all substances.

48. Again, let K denote the specific heat of the substance under constant pressure. Then, if  $dv$  and  $dt$  be so related that the pressure of the medium, when its volume and temperature are  $v + dv$  and  $t + dt$  respectively, is the same as when they are  $v$  and  $t$ , that is, if

$$0 = \frac{dp}{dv} dv + \frac{dp}{dt} dt;$$

we have

$$K dt = M dv + N dt.$$

Hence we find

$$M = \frac{-\frac{dp}{dv}}{\frac{dp}{dt}} (K - N) \quad \dots \quad (15),$$

which merely shows the meaning in terms of the two specific heats, of what I have denoted by M. Using in this for M its value given by (3) of § 21, we find

$$K - N = \frac{\left(\frac{dp}{dt}\right)^2}{\mu \times -\frac{dp}{dv}} \quad \dots \quad (16),$$

an expression for the difference between the two specific heats, derived without hypothesis from the second fundamental principle of the theory of the motive power of heat.

49. These results may be put into forms more convenient for use, in applications to liquid and solid media, by introducing the notation :—

$$\left. \begin{aligned} \kappa &= v \times -\frac{dp}{dv} \\ e &= \frac{1}{\kappa} \frac{dp}{dt} \end{aligned} \right\} \dots \dots \dots (17),$$

where  $\kappa$  will be the reciprocal of the compressibility, and  $e$  the coefficient of expansion with heat.

Equations (14), (16) and (3), thus become

$$\frac{dN}{dv} = \frac{d\left(\frac{\kappa e}{\mu}\right)}{dt} - \frac{\kappa e}{J} \dots \dots \dots (18),$$

$$K - N = v \frac{\kappa e^2}{\mu} \dots \dots \dots (19),$$

$$M = \frac{1}{\mu} \cdot \kappa e \dots \dots \dots (20);$$

the third of these equations being annexed to show explicitly the quantity of heat developed by the compression of the substance kept at a constant temperature. Lastly, if  $\theta$  denote the rise in temperature produced by a compression from  $v + dv$  to  $v$  before any heat is emitted, we have

$$\theta = \frac{1}{N} \cdot \frac{\kappa e}{\mu} \cdot dv = \frac{\kappa e}{\mu K - v \kappa e^2} dv \dots \dots (21).$$

50. The first of these expressions for  $\theta$  shows that, when the substance contracts as its temperature rises (as is the case, for instance, with water between its freezing-point and its point of maximum density), its temperature would become lowered by a sudden compression. The second, which shows in terms of its compressibility and expansibility exactly how much the temperature of any substance is altered by an infinitely small alteration of its volume, leads to the approximate expression

$$\theta = \frac{\kappa e}{\mu K},$$

if, as is probably the case, for all known solids and liquids,  $e$  be so small that  $e \cdot v \kappa e$  is very small compared with  $\mu K$ .

51. If, now, we suppose the substance to be a gas, and introduce

the hypothesis that its density is strictly subject to the "gaseous laws," we should have, by Boyle and Mariotte's law of compression,

$$\frac{dp}{dv} = -\frac{p}{v} \dots \dots \dots (22);$$

and by Dalton and Gay-Lussac's law of expansion,

$$\frac{dv}{dt} = \frac{Ev}{1 + Et} \dots \dots \dots (23);$$

from which we deduce

$$\frac{dp}{dt} = \frac{Ep}{1 + Et}.$$

Equation (14) will consequently become

$$\frac{dN}{dv} = \frac{d\left\{ \frac{Ep}{\mu(1 + Et)} - \frac{p}{J} \right\}}{dt} \dots \dots (24),$$

a result peculiar to the dynamical theory and equation (16),

$$K - N = \frac{E^2pv}{\mu(1 + Et)^2} \dots \dots \dots (25),$$

which agrees with the result of § 53 of my former paper.

If V be taken to denote the volume of the gas at the temperature 0° under unity of pressure, (25) becomes

$$K - N = \frac{E^2V}{\mu(1 + Et)} \dots \dots \dots (26).$$

52. All the conclusions obtained by Clausius, with reference to air or gases, are obtained immediately from these equations by taking

$$\mu = J \frac{E}{1 + Et},$$

which will make  $\frac{dN}{dv} = 0$ , and by assuming, as he does, that N, thus found to be independent of the density of the gas, is also independent of its temperature.

53. As a last application of the two fundamental equations of the theory, let the medium with reference to which M and N are defined consist of a weight 1 - x of a certain substance in one state, and a weight x in another state at the same temperature, containing more latent heat. To avoid circumlocution and to fix the ideas, in what follows we may suppose the former state to be liquid and the latter gaseous; but the investigation, as will be seen, is equally applicable to the case of a solid in contact with the same substance in the liquid or gaseous form.

54. The volume and temperature of the whole medium being,

as before, denoted respectively by  $v$  and  $t$ , we shall have

$$\lambda(1-x) + \gamma x = v \quad \dots \dots \dots (27),$$

if  $\lambda$  and  $\gamma$  be the volumes of unity of weight of the substance in the liquid and the gaseous states respectively: and  $p$ , the pressure, may be considered as a function of  $t$ , depending solely on the nature of the substance. To express  $M$  and  $N$  for this mixed medium, let  $L$  denote the latent heat of a unit of weight of the vapour,  $c$  the specific heat of the liquid, and  $h$  the specific heat of the vapour when kept in a state of saturation. We shall have

$$Mdv = L \frac{dx}{dv} dv$$

$$Ndt = c(1-x)dt + hxd t + L \frac{dx}{dt} dt.$$

Now, by (27), we have

$$(\gamma - \lambda) \frac{dx}{dv} = 1 \quad \dots \dots \dots (28),$$

and

$$(\gamma - \lambda) \frac{dx}{dt} + (1-x) \frac{d\lambda}{dt} + x \frac{d\gamma}{dt} = 0 \quad \dots \dots (29).$$

Hence

$$M = \frac{L}{\gamma - \lambda} \quad \dots \dots \dots (30),$$

$$N = c(1-x) + hx - L \frac{(1-x) \frac{d\lambda}{dt} + x \frac{d\gamma}{dt}}{\gamma - \lambda} \quad (31).$$

55. The expression of the second fundamental proposition in this case becomes, consequently,

$$\mu = \frac{(\gamma - \lambda) \frac{dp}{dt}}{L} \quad \dots \dots \dots (32),$$

which agrees with Carnot's original result, and is the formula that has been used (referred to above in § 31) for determining  $\mu$  by means of Regnault's observations on steam.

56. To express the conclusion derivable from the first fundamental proposition, we have, by differentiating the preceding expressions for  $M$  and  $N$  with reference to  $t$  and  $v$  respectively,

$$\frac{dM}{dt} = \frac{1}{\gamma - \lambda} \cdot \frac{dL}{dt} - \frac{L}{(\gamma - \lambda)^2} \cdot \frac{d(\gamma - \lambda)}{dt}$$

$$\begin{aligned} \frac{dN}{dv} &= \left( h - c - L \frac{\frac{d\gamma}{dt} - \frac{d\lambda}{dt}}{\gamma - \lambda} \right) \frac{dx}{dv} \\ &= \left\{ \frac{h - c}{\gamma - \lambda} - \frac{L}{(\gamma - \lambda)^2} \right\} \frac{d(\gamma - \lambda)}{dt}. \end{aligned}$$

Hence equation (2) of § 20 becomes

$$\frac{dL}{dt} + c - h = \frac{1}{J} \frac{dp}{dt} \dots \dots \dots (33).$$

Combining this with the conclusion (32) derived from the second fundamental proposition, we obtain

$$\frac{dL}{dt} + c - h = \frac{L\mu}{J} \dots \dots \dots (34).$$

The former of these equations agrees precisely with one which was first given by Clausius, and the preceding investigation is substantially the same as the investigation by which he arrived at it. The second differs from another given by Clausius only in not implying any hypothesis as to the form of Carnot's function  $\mu$ .

57. If we suppose  $\mu$  and  $L$  to be known for any temperature, equation (32) enables us to determine the value of  $\frac{dp}{dt}$  for that temperature; and thence deducing a value of  $dt$ , we have

$$dt = \frac{\gamma - \lambda}{\mu L} dp \dots \dots \dots (35);$$

which shows the effect of pressure in altering the "boiling-point" if the mixed medium be a liquid and its vapour, or the melting-point if it be a solid in contact with the same substance in the liquid state. This agrees with the conclusion arrived at by my elder brother in his Theoretical Investigation of the Effect of Pressure in Lowering the Freezing-Point of Water\*. His result, obtained by taking as the value for  $\mu$  that derived from Table I. of my former paper for the temperature  $0^\circ$ , is that the freezing-point is lowered by  $\cdot 0075^\circ$  Cent. by an additional atmosphere of pressure. Clausius, with the other data the same, obtains  $\cdot 00733^\circ$  as the lowering of temperature by the same additional pressure, which differs from my brother's result only from having been calculated from a formula which implies the hypothetical expression  $J \frac{E}{1 + Et}$  for  $\mu$ . It was by applying equation (33) to determine  $\frac{dL}{dt}$  for the same case that Clausius arrived at the curious result regarding the latent heat of water under pressure mentioned above (§ 45).

58. Lastly, it may be remarked that every quantity which ap-

\* Transactions, vol. xvi. part 5. His paper was republished, with some slight modifications, in the Cambridge and Dublin Mathematical Journal, new series, vol. v.—Nov. 1850.

appears in equation (33), except  $h$ , is known with tolerable accuracy for saturated steam through a wide range of temperature; and we may therefore use this equation to find  $h$ , which has never yet been made an object of experimental research. Thus we have

$$-h = \frac{\gamma - \lambda}{J} \frac{dp}{dt} - \left( \frac{dL}{dt} + c \right).$$

For the value of  $\gamma$  the best data regarding the density of saturated steam that can be had must be taken. If for different temperatures we use the same values for the density of saturated steam (calculated according to the gaseous laws, and Regnault's observed pressure from  $\frac{1}{1693.5}$ , taken as the density at  $100^\circ$ ), the values obtained for the first term of the second member of the preceding equation are the same as if we take the form

$$-h = \frac{L\mu}{J} - \left( \frac{dL}{dt} + c \right)$$

derived from (34), and use the values of  $\mu$  shown in Table I. of my former paper. The values of  $-h$  in the second column in the following table have been so calculated, with, besides, the following data afforded by Regnault from his observations on the total heat of steam, and the specific heat of water

$$\frac{dL}{dt} + c = .305.$$

$$L = 606.5 + .305t - (.00002t^2 + .0000003t^3).$$

The values of  $-h$  shown in the third column are those derived by Clausius from an equation which is the same as what (34) would become if  $J \frac{E}{1 + Et}$  were substituted for  $\mu$ .

$t$ .	$-h$ according to Table I. of "Account of Carnot's Theory."	$-h$ according to Clausius.
0	1.863	1.916
50	1.479	1.465
100	1.174	1.133
150	0.951	0.879
200	0.780	0.676

59. From these results it appears, that through the whole range of temperatures at which observations have been made, the value of  $h$  is negative; and, therefore, if a quantity of saturated vapour be compressed in a vessel containing no liquid water, heat must be continuously abstracted from it in order

that it may remain saturated as its temperature rises; and conversely, if a quantity of saturated vapour be allowed to expand in a closed vessel, heat must be supplied to it to prevent any part of it from becoming condensed into the liquid form as the temperature of the whole sinks. This very remarkable conclusion was first announced by Mr. Rankine, in his paper communicated to this Society on the 4th of February last year. It was discovered independently by Clausius, and published in his paper in Poggendorff's *Annalen* in the months of April and May of the same year.

60. It might appear at first sight, that the well-known fact that steam rushing from a high-pressure boiler through a small orifice into the open air does not scald a hand exposed to it\*, is inconsistent with the proposition, that steam expanding from a state of saturation must have heat given to it to prevent any part from becoming condensed; since the steam would scald the hand unless it were dry, and consequently above the boiling-point in temperature. The explanation of this apparent difficulty, given in a letter which I wrote to Mr. Joule last October, and which has since been published in the *Philosophical Magazine*, is, that the steam in rushing through the orifice produces mechanical effect which is immediately wasted in fluid friction, and consequently reconverted into heat; so that the issuing steam at the atmospheric pressure would have to part with as much heat to convert it into water at the temperature  $100^{\circ}$  as it would have had to part with to have been condensed at the high pressure and then cooled down to  $100^{\circ}$ , which for a pound of steam initially saturated at the temperature  $t$  is, by Regnault's modification of Watt's law,  $\cdot 305(t - 100^{\circ})$  more heat than a pound of saturated steam at  $100^{\circ}$  would have to part with to be reduced to the same state; and the issuing steam must therefore be above  $100^{\circ}$  in temperature, and dry.

[\* *Note added June 26, 1852.*—At present I am inclined to believe that the rapidity of the current exercises a great influence on the sensation experienced in the circumstances, by causing the steam to mix with the surrounding air; for I have found that the hand suffers pain when exposed to the steam issuing from a common kettle, and dried by passing through a copper tube surrounded by red-hot coals or heated by lamps. But although there may be uncertainty regarding the causes of the different sensations in the different circumstances, I believe there is no reason for doubting either the fact of the dryness of the steam issuing from a high-pressure boiler (except when there is "priming" to a considerable extent), or the correctness of the explanation of this fact which I have given in the letter referred to.]

indicated successively  $-2^{\circ}$ ,  $-18^{\circ}$ ,  $-28^{\circ}$ ,  $-15^{\circ}$ . At 1<sup>h</sup> 45<sup>m</sup> the rain still fell in different directions, but not at Brussels; the sun shone at intervals, and the electrometer marked  $-6^{\circ}$ .

I ought to remark, that during the showers not a single peal of thunder was heard, and not the smallest flash of lightning was visible.

The example which I have just cited shows how, during the same shower, according to the instant at which an observation is made, we may obtain either positive or negative electricity; this electricity is very energetic during the showers. If the observation is made at the moment when the sign changes, it may appear to be nearly null; these inversions, it may be remarked, are always of short duration.

*XL. On the Mechanical Action of Radiant Heat or Light: On the Power of Animated Creatures over Matter: On the Sources available to Man for the production of Mechanical Effect. By Professor WILLIAM THOMSON.\**

*On the Mechanical Action of Radiant Heat or Light.*

**I**T is assumed in this communication that the undulatory theory of radiant heat and light, according to which light is merely radiant heat, of which the vibrations are performed in periods between certain limits of duration, is true. "The chemical rays," beyond the violet end of the spectrum, consist of undulations of which the full vibrations are executed in periods shorter than those of the extreme visible violet light, or than about the eight hundred million millionth of a second. The periods of the vibrations of visible light lie between this limit and another, about double as great, corresponding to the extreme visible red light. The vibrations of the obscure radiant heat beyond the red end are executed in longer periods than this; the longest which has yet been experimentally tested being about the eighty million millionth of a second.

The elevation of temperature produced in a body by the incidence of radiant heat upon it is a mechanical effect of the dynamical kind, since the communication of heat to a body is merely the excitation or the augmentation of certain motions among its particles. According to Pouillet's estimate of heat radiated from the sun in any time, and Joule's mechanical equivalent of a thermal unit, it appears that the mechanical value of the solar heat incident perpendicularly on a square foot above

\* From the Proceedings of the Royal Society of Edinburgh, February, 1852. Communicated by the Author.

the earth's atmosphere is about eighty-four foot-pounds per second.

Mechanical effect of the statical kind might be produced from the solar radiant heat, by using it as the source of heat in a thermo-dynamic engine. It is estimated that about 556 foot-pounds per second of ordinary mechanical effect, or about the work of "one horse power," might possibly be produced by such an engine exposing 1800 square feet to receive solar heat, during a warm summer day in this country; but the dimensions of the moveable parts of the engine would necessarily be so great as to occasion practical difficulties in the way of using it with œconomical advantage that might be insurmountable.

The *chemical* effects of light belong to the class of mechanical effects of the statical kind; and reasoning analogous to that introduced and experimentally verified in the case of electrolysis by Joule, leads to the conclusion that when such effects are produced there will be a loss of heating effect in the radiant heat or light which is absorbed by the body acted on, to an extent thermally equivalent to the mechanical value of the work done against forces of chemical affinity.

The deoxidation of carbon and hydrogen from carbonic acid and water, effected by the action of solar light on the green parts of plants, is (as the author recently found was pointed out by Helmholtz\* in 1847) a mechanical effect of radiant heat. In virtue of this action combustible substances are produced by plants; and its mechanical value is to be estimated by determining the heat evolved by burning them, and multiplying by the mechanical equivalent of the thermal unit. Taking, from Liebig's Agricultural Chemistry, the estimate 2600 pounds of dry fir-wood for the annual produce of one Hessian acre, or 26,910 square feet of forest land (which in mechanical value appears not to differ much from estimates given in the same treatise for produce of various kinds obtained from *cultivated* land), and assuming, as a very rough estimate, 4000 thermal units Centigrade as the heat of combustion of unity of mass of dry fir-wood, the author finds 550,000 foot-pounds (or the work of a horse-power, for 1000 seconds) as the mechanical value of the mean annual produce of a square foot of the land. Taking  $50^{\circ} 34'$  (that of Giessen) as the latitude of the locality, the author estimates the mechanical value of the solar heat which, were none of it absorbed by the atmosphere, would fall annually on each square foot of the land, at 530,000,000 foot-pounds; and infers that probably a good deal more,  $\frac{1}{1000}$  of the solar heat, which actually falls on growing plants, is converted into mechanical effect.

\* *Ueber die Erhaltung der Kraft*, von Dr. H. Helmholtz. Berlin, 1847. [A translation of this essay will appear in the First Part of the New Series of the Scientific Memoirs.—Ed.]

When the vibrations of light thus act during the growth of plants, to separate, against forces of chemical affinity, combustible materials from oxygen, they must lose *vis viva* to an extent equivalent to the statical mechanical effect thus produced; and therefore quantities of solar heat are actually put out of existence by the growth of plants, but an equivalent of statical mechanical effect is stored up in the organic products, and may be reproduced as heat, by burning them. All the heat of fires, obtained by burning wood grown from year to year, is in fact solar heat reproduced.

The actual convertibility of radiant heat into statical mechanical effect, by inanimate material agency, is considered in this paper as subject to Carnot's principle; and a possible connexion of this principle with the circumstances regarding the quality of the radiant heat (or the colour of the light), required to produce the growth of plants, is suggested.

#### *On the Power of Animated Creatures over Matter.*

The question, "Can animated creatures set matter in motion in virtue of an inherent power of producing mechanical effect?" must be answered in the negative, according to the well-established theory of animal heat and motion, which ascribes them to the chemical action (principally *oxidation*, or a combustion at low temperatures) experienced by the food. A principal object of the present communication is to point out the relation of this theory to the dynamical theory of heat. It is remarked, in the first place, that both animal heat and weights raised or resistance overcome, are *mechanical* effects of the chemical forces which act during the combination of food with oxygen. The former is a dynamical mechanical effect, being thermal motions excited; the latter is a mechanical effect of the statical kind. The whole mechanical value of these effects, which are produced by means of the animal mechanism in any time, must be equal to the mechanical value of the work done by the chemical forces. Hence, when an animal is going up-hill or working against resisting force, there is less heat generated than the amount due to the oxidation of the food, by the thermal equivalent of the mechanical effect produced. From an estimate made by Mr. Joule, it appears that from  $\frac{1}{4}$  to  $\frac{1}{6}$  of the mechanical equivalent of the complete oxidation of all the food consumed by a horse may be produced, from day to day, as weights raised. The oxidation of the whole food consumed being, in reality, far from complete, it follows that a less proportion than  $\frac{5}{6}$ , perhaps even less than  $\frac{3}{4}$ , of the heat due to the whole chemical action that actually goes on in the body of the animal, is given out as heat. An estimate, according to the same principle, upon very imperfect data, however, is made by the author, regarding the relation between the thermal and the non-thermal mechanical effects produced by a man at

work; by which it appears that probably as much as  $\frac{1}{6}$  of the whole work of the chemical forces arising from the oxidation of his food during the twenty-four hours, may be directed to raising his own weight, by a man walking up-hill for eight hours a day; and perhaps even as much as  $\frac{1}{4}$  of the work of the chemical forces may be directed to the overcoming of external resistances by a man exerting himself for six hours a day in such operations as pumping. In the former case there would not be more than  $\frac{5}{6}$ , and in the latter not more than  $\frac{3}{4}$  of the thermal equivalent of the chemical action emitted as animal heat, on the whole, during the twenty-four hours, and the quantities of heat emitted during the times of working would bear much smaller proportions respectively than these, to the thermal equivalents of the chemical forces actually operating during those times.

A curious inference is pointed out, that an animal would be sensibly less warm in going up-hill than in going down-hill, were the breathing not greater in the former case than in the latter.

The application of Carnot's principle, and of Joule's discoveries regarding the heat of electrolysis and the calorific effects of magneto-electricity, is pointed out; according to which it appears nearly certain that, when an animal works against resisting force, there is not a *conversion of heat into external mechanical effect*, but the full thermal equivalent of the chemical forces is *never produced*; in other words, that the animal body does not act as a *thermo-dynamic engine*; and very probable that the chemical forces produce the external mechanical effects through electrical means.

Certainty regarding the means in the animal body by which external mechanical effects are produced from chemical forces acting internally, cannot be arrived at without more experiment and observation than has yet been applied; but the relation of mechanical equivalence, between the work done by the chemical forces, and the final mechanical effects produced, whether solely heat, or partly heat and partly resistance overcome, may be asserted with confidence. Whatever be the nature of these means, consciousness teaches every individual that they are, to some extent, subject to the direction of his will. It appears, therefore, that animated creatures have the power of immediately applying, to certain moving particles of matter within their bodies, forces by which the motions of these particles are directed to produce desired mechanical effects.

#### *On the Sources available to Man for the production of Mechanical Effect.*

Men can obtain mechanical effect for their own purposes either by working mechanically themselves, and directing other animals to work for them, or by using natural heat, the gravitation of descending solid masses, the natural motions of water and air,

and the heat, or galvanic currents, or other mechanical effects produced by chemical combination, but in no other way at present known. Hence the stores from which mechanical effect may be drawn by man belong to one or other of the following classes:—

- I. The food of animals.
- II. Natural heat.
- III. Solid matter found in elevated positions.
- IV. The natural motions of water and air.
- V. Natural combustibles (as wood, coal, coal-gas, oils, marsh gas, diamond, native sulphur, native metals, meteoric iron).
- VI. Artificial combustibles (as smelted or electrolytically deposited metals, hydrogen, phosphorus).

In the present communication, known facts in natural history and physical science, with reference to the sources from which these stores have derived their mechanical energies, are adduced to establish the following general conclusions:—

1. *Heat radiated from the sun* (sunlight being included in this term) is the principal source of mechanical effect available to man\*. From it is derived the whole mechanical effect obtained by means of animals working, water-wheels worked by rivers, steam-engines, and galvanic engines, and part at least of the mechanical effect obtained by means of windmills and the sails of ships not driven by the trade-winds.

2. The motions of the earth, moon, and sun, and their mutual attractions, constitute an important source of available mechanical effect. From them all, but chiefly, no doubt, from the earth's motion of rotation, is derived the mechanical effect of water-wheels driven by the tides. The mechanical effect so largely used in the sailing of ships by the trade-winds is derived partly, perhaps principally, from the earth's motion of rotation, and partly from solar heat.

3. The other known sources of mechanical effect available to man are either terrestrial—that is, belonging to the earth, and available without the influence of any external body,—or meteoric,—that is, belonging to bodies deposited on the earth from external space. Terrestrial sources, including mountain quarries and mines, the heat of hot springs, and the combustion of native sulphur, perhaps also the combustion of all inorganic native combustibles, are actually used, but the mechanical effect obtained from them is very inconsiderable, compared with that which is obtained from sources belonging to the two classes mentioned above. Meteoric sources, including only the heat of newly-fallen meteoric bodies, and the combustion of meteoric iron, need not be reckoned among those available to man for practical purposes.

\* A general conclusion equivalent to this was published by Sir John Herschel in 1833. See his *Astronomy*, edit. 1849, § (399).

XLVII. *On a Universal Tendency in Nature to the Dissipation of Mechanical Energy.* By Prof. W. THOMSON\*.

THE object of the present communication is to call attention to the remarkable consequences which follow from Carnot's proposition, that there is an absolute waste of mechanical energy available to man when heat is allowed to pass from one body to another at a lower temperature, by any means not fulfilling his criterion of a "perfect thermo-dynamic engine," established, on a new foundation, in the dynamical theory of heat. As it is most certain that Creative Power alone can either call into existence or annihilate mechanical energy, the "waste" referred to cannot be annihilation, but must be some transformation of energy†. To explain the nature of this transformation, it is convenient, in the first place, to divide *stores* of mechanical energy into two classes—*statical* and *dynamical*. A quantity of weights at a height, ready to descend and do work when wanted, an electrified body, a quantity of fuel, contain stores of mechanical energy of the statical kind. Masses of matter in motion, a volume of space through which undulations of light or radiant heat are passing, a body having thermal motions among its particles (that is, not infinitely cold), contain stores of mechanical energy of the dynamical kind.

The following propositions are laid down regarding the *dissipation* of mechanical energy from a given store, and the *restoration* of it to its primitive condition. They are necessary consequences of the axiom, "*It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects.*" (Dynam. Th. of Heat, § 12.)

I. When heat is created by a reversible process (so that the mechanical energy thus spent may be *restored* to its primitive condition), there is also a transference from a cold body to a hot body of a quantity of heat bearing to the quantity created a definite proportion depending on the temperatures of the two bodies.

II. When heat is created by any unreversible process (such as friction), there is a *dissipation* of mechanical energy, and a full *restoration* of it to its primitive condition is impossible.

III. When heat is diffused by *conduction*, there is a *dissipation* of mechanical energy, and perfect *restoration* is impossible.

IV. When radiant heat or light is absorbed, otherwise than in

\* From the Proceedings of the Royal Society of Edinburgh for April 19, 1852. Communicated by the Author.

† See the Author's previous paper on the Dynamical Theory of Heat, § 22.

vegetation, or in chemical action, there is a *dissipation* of mechanical energy, and perfect *restoration* is impossible.

In connexion with the second proposition, the question, *How far is the loss of power experienced by steam in rushing through narrow steam-pipes compensated, as regards the economy of the engine, by the heat* (containing an exact equivalent of mechanical energy) *created by the friction?* is considered, and the following conclusion is arrived at:—

Let  $S$  denote the temperature of the steam (which is nearly the same in the boiler and steam-pipe, and in the cylinder till the expansion within it commences);  $T$  the temperature of the condenser;  $\mu$  the value of Carnot's function for any temperature  $t$ ; and  $R$  the value of

$$\frac{1}{\epsilon} \int_T^S \mu dt.$$

Then  $(1 - R)w$  expresses the greatest amount of mechanical effect that can be economized in the circumstances from a quantity  $\frac{1}{J}w$  of heat produced by the expenditure of a quantity  $w$  of

work in friction, whether of the steam in the pipes and entrance ports, or of any solids or fluids in motion in any part of the engine; and the remainder,  $Rw$ , is absolutely and irrecoverably wasted, unless some use is made of the heat discharged from the condenser. The value of  $1 - R$  has been shown to be not more than about  $\frac{1}{4}$  for the best steam-engines, and we may infer that in them at least three-fourths of the work spent in any kind of friction is utterly wasted.

In connexion with the third proposition, the quantity of work that could be got by equalizing the temperature of all parts of a solid body possessing initially a given non-uniform distribution of heat, if this could be done by means of perfect thermo-dynamic engines without any conduction of heat, is investigated. If  $t$  be the initial temperature (estimated according to any arbitrary system) at any point  $xyz$  of the solid,  $T$  the final uniform temperature, and  $c$  the thermal capacity of unity of volume of the solid, the required mechanical effect is of course equal to

$$J \iiint c(t - T) dx dy dz,$$

being simply the mechanical equivalent of the amount of heat put out of existence. Hence the problem becomes reduced to that of the determination of  $T$ . The following solution is obtained:—

$$T = \frac{\iiint \epsilon^{-\frac{1}{J} \int_0^t \mu dt} c t dx dy dz}{\iiint \epsilon^{-\frac{1}{J} \int_0^t \mu dt} c dx dy dz}.$$

If the system of thermometry adopted\* be such that  $\mu = \frac{J}{t + \alpha}$ , that is, if we agree to call  $\frac{J}{\mu} - \alpha$  the *temperature* of a body, for which  $\mu$  is the *value of Carnot's function* ( $\alpha$  and  $J$  being constants), the preceding expression becomes

$$T = \frac{\iiint c \, dx \, dy \, dz}{\iiint \frac{c}{t + \alpha} \, dx \, dy \, dz} - \alpha.$$

The following general conclusions are drawn from the propositions stated above, and known facts with reference to the mechanics of animal and vegetable bodies:—

1. There is at present in the material world a universal tendency to the dissipation of mechanical energy.

2. Any *restoration* of mechanical energy, without more than an equivalent of dissipation, is impossible in inanimate material processes, and is probably never effected by means of organized matter, either endowed with vegetable life or subjected to the will of an animated creature.

3. Within a finite period of time past the earth must have been, and within a finite period of time to come the earth must again be, unfit for the habitation of man as at present constituted, unless operations have been, or are to be performed, which are impossible under the laws to which the known operations going on at present in the material world are subject.

## XLVIII. *Proceedings of Learned Societies.*

ROYAL SOCIETY.

[Continued from p. 236.]

May 13, "REPORT of the general process adopted in Graduating and Comparing the Standard Meteorological Instruments for the Kew Observatory." By Mr. John Welsh. Communicated by Col. Sabine on the part of the Committee of Recommendations of the Government Grant.

In offering to the Committee a short statement of the progress made at the Kew Observatory in the construction and verification

\* According to "Mayer's hypothesis," this system coincides with that in which equal differences of temperature are defined as those with which the same mass of air under constant pressure has equal differences of volume, provided  $J$  be the mechanical equivalent of the thermal unit, and  $\frac{1}{\alpha}$  the coefficient of expansion of air.