Presentation on Anderson's "Conditional Erasure and the Landauer Limit"

For HPS 2559: Thermodynamics and Statistical Mechanics, Spring 2020

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9 April, 2020

Outline

1. Anderson's Results on Erasure Bounds (§§2–3.2, 5)
2. Comparisons with Bennett (§3.3.1)
3. Comparisons with Norton (§4)

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Setup (pp. 67–8)

The setting is unitary non-relativistic quantum mechanics.

We start with copies of a system $\mathcal{S}$ coupled to an environment $\mathcal{E}$, each of which is in a state $\hat{\rho}_i^{\mathcal{S}} \otimes \hat{\rho}_{\text{th}}^{\mathcal{E}}$

- The $\hat{\rho}_i^{\mathcal{S}}$ have support on orthogonal subspaces (what Anderson means here by “distinguishable”).
- $\hat{\rho}_{\text{th}}^{\mathcal{E}}$ is a Gibbs state.
- The fraction of copies in state $\hat{\rho}_i^{\mathcal{S}}$ is denoted by p_i.

We then suppose that these copies undergo unitary evolution: $\hat{U}_i(\hat{\rho}_i^{\mathcal{S}} \otimes \hat{\rho}_{\text{th}}^{\mathcal{E}})\hat{U}_i^{\dagger} = \hat{\rho}_i^{\mathcal{S}\mathcal{E}}$, where

- $\text{Tr}_{\mathcal{E}}[\hat{\rho}_i^{\mathcal{S}\mathcal{E}}] = \hat{\rho}_{\text{reset}}^{\mathcal{S}}$, and $\hat{\rho}_{\text{reset}}^{\mathcal{S}}$ is some designated state.
- In general, the $\hat{U}_i$ can be distinct.

Notation

Quantities:

- $E_i^{\mathcal{E}}$ and T: the environment's energy (when $\mathcal{S}$ is initially in state $\hat{\rho}_i^{\mathcal{S}}$) and initial temperature.
- $S_i^{\mathcal{S}} = -\mathrm{Tr}_{\mathcal{S}}[\hat{\rho}_i^{\mathcal{S}} \log_2 \hat{\rho}_i^{\mathcal{S}}]$: von Neumann entropy *in bits*.
- $H(\{p_j\}) = -\sum_j p_j \log_2 p_j$: Shannon entropy *in bits*.
- k_B: Boltzmann's constant.

Operations:

- $\Delta\cdot$: change in $\cdot$, i.e., final value minus initial value.
- $\langle\cdot\rangle$: QM expectation value of $\cdot$.
- $\bar{\cdot}$: arithmetic average of $\cdot$ over the copies of the system.

Conditional Erasure (p. 70)

"Conditional" means no restrictions on the $\hat{U}_i$.

$$\Delta\langle E_i^{\mathcal{E}}\rangle \geq -k_B T \ln(2) \Delta S_i^{\mathcal{S}}$$

$$\overline{\Delta\langle E_i^{\mathcal{E}}\rangle} \geq -k_B T \ln(2) \sum_i p_i \Delta S_i^{\mathcal{S}}$$

Special case in which each $\Delta S_i^{\mathcal{S}} = 0$, these become the "Landauer-Bennett limits":

$$\Delta\langle E_i^{\mathcal{E}}\rangle \geq 0$$

$$\overline{\Delta\langle E_i^{\mathcal{E}}\rangle} \geq 0$$

Unconditional Erasure (p. 73)

"Unconditional" means that there's some $\hat{U}$ s.t. $\hat{U}_i = \hat{U}$ for all i.

$$\overline{\Delta\langle E_i^{\mathcal{E}}\rangle} \geq -k_B T \ln(2) \sum_i p_i \Delta S_i^{\mathcal{S}} + k_B T \ln(2) H(\{p_j\})$$

Special case 1: when the p_j are 0-1, this reduces to the conditional erasure case.
Special case 2 in which each $\Delta S_i^{\mathcal{S}} = 0$:

$$\overline{\Delta\langle E_i^{\mathcal{E}}\rangle} \geq k_B T \ln(2) H(\{p_j\})$$

Extra-special case in which $j \in \{1, 2\}$ and $p_1 = p_2 = 1/2$, this becomes the "Landauer limit":

$$\overline{\Delta\langle E_i^{\mathcal{E}}\rangle} \geq k_B T \ln(2) \approx 0.69 k_B T.$$

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Assumptions

- Totes just unitary NRQM, no classical thermo, no second law
- Environment starts in equilibrium = Gibbs state
- Elementary technical features of von Neumann entropy

Interpretation: Statistical Aspects

The results are *doubly* statistical, involving expectation values *and* averages over a collection.

Such averages are what are actually measured and reported in experiments for the "Landauer limit":

- Berut et al (2012): $\ln(2)k_B T$ to k_B

  silica bead in water with optical tweezers

- Jun et al (2014): $(0.71 \pm 0.03)k_B T$

  florescent particle in colloidal suspension with electrostatic force

- Hong et al (2012): $(1.45 \pm 0.35)k_B T$

  single domain nanomagnet with external fields

- Orlov et al (2012): $\sim 0.01 k_B T$ for "Landauer-Bennett limit"

  CR network

Interpretation: Is the "Landauer limit" the Landauer limit?

Usually Landauer's limit is

1. phrased in terms of thermodynamic quantities such as work, heat, or entropy,
2. and applies to *individual* erasure operations.

Each of these deserves further comment:

1. The approach taken here does not guarantee that the environments ends in an equilibrium state. Perhaps for sufficiently "large" systems it will be a good approximation?
2. The statistical nature seems unavoidable; it has a definitively more "Gibbsian" flavor. (The is in contrast to the derivation of a non-averaged Landauer-Bennett limit.)

Interpretation: Information and Computation

Landauer thought that his work showed that “information is physical” and that computation implicated thermodynamics.

- No substantive notions of information or computation appear in Anderson’s chapter.

Anderson does remark that “The Shannon entropy . . . is commonly taken as a measure of information encoded in $\mathcal{S}$ . . . information [that] is *lost* from $\mathcal{S}$ in erasure” (p. 73).

- However, the bound says very little about any changes for any one of the *individual* copies of $\mathcal{S}$ in the collection. Thus this interpretation does not seem to be supported by the technical result it expounds.

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Data and Protocol (Cf. p. 78)

Anderson			Bennett		
Data	Erasure Protocol	$\overline{\Delta\langle E^{\mathcal{E}}\rangle}$	Data	Erasure Protocol	$\overline{\Delta\langle E^{\mathcal{E}}\rangle}$
Known	Conditional	$\gtrapprox 0$	Known	Conditional	$\gtrapprox 0$
	Unconditional	> 0		Unconditional	> 0
Unknown	Conditional	$\gtrapprox 0$			
	Unconditional	> 0			
None	(Un)Conditional	$\gtrapprox 0$	Random	(Un)Conditional	$\gtrapprox 0$

Replacements

Cost: Reversible/Irreversible $\Rightarrow \overline{\Delta\langle E^{\mathcal{E}}\rangle} \gtrapprox 0/ > 0$.

Erase with(out) Copy $\Rightarrow$ (Un)conditional

Reset/Erase $\Rightarrow$ (Un)Conditional

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Data and Protocol: On Being “Random”

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Random?

Anderson takes Bennett’s “random” data to be a uniform mixture of the N data states: $\hat{\rho}^{\mathcal{S}}_{\text{ran}} = N^{-1}\sum_i \hat{\rho}^{\mathcal{S}}_i$.

But if it were interpreted as an *unknown* $\hat{\rho}^{\mathcal{S}}_i$, then the erasure protocol must be *unconditional*.

Data and Protocol: On Being "Data"

Anderson writes,

"we take a system to be encoding data if and only if it is prepared in *one* of the data states *and* if there exists a record or copy of the data instantiated in a physical system that is external both to the system $\mathcal{S}$ and to the observer-inaccessible environment" (pp. 75–6).

Data is "unknown" just when a record of it exists but is known at best "known statistically."

- Is the "record" requirement too vague? Trivial?
- Is it subject to counterexamples? Consider the calculation of new digits of π.

Data and Protocol: Can Andersonian Data be Unknown?

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How can there be a record of data that is unknown which facilitates a conditional erasure?
Is the “record” requirement ultimately superfluous?

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Points of Agreement

- Random data states are not necessarily equilibrium/thermalized states. (p. 84)
- Unconditional protocals are needed for resetting a state that is unknown. (p. 87)

Points of Disagreement

- Having knowledge of a system's state can matter to erasure costs because it enables conditional erasure, even if that knowledge does not bear on what the system's state *is*. (pp. 84–5. Cf. p. 188 of Norton 2011.)
- Reversible erasure requires a conditional process. (pp. 87–8. Cf. pp. 190, 198 of Norton 2011.)