Black Hole Thermodynamics: More Than an Analogy - John Dougherty and Craig Callender

"the laws of black hole thermodynamics ... simply *are* the ordinary laws of thermodynamics applied to a black hole" - Wald in *QFT in Curved Spacetime and BHT* (1994, pg. 174)

1. **The Laws of BHT, Quickly Given**
   (a) **Zeroth Law of BHT**: The surface gravity κ of the event horizon is constant everywhere on the horizon.
   (b) **First Law of BHT**: Changes in the mass M of a neutral ($Q = 0$) black hole are related to changes in the horizon area A and angular momentum J as
   $$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega\delta J + \phi\delta Q \tag{1}$$
   where Ω is the angular velocity and ϕ is the electromagnetic potential at the horizon.
   (c) **Second Law of BHT**: The area of the event horizon A will not decrease in any physical process
   $$\delta A \geq 0 \tag{2}$$
   where a physical process is any process which satisfies the null energy condition ($T_{\mu\nu}n^\mu n^\nu > 0$ for null n^μ) and the cosmic censorship hypothesis (no naked singularities).
   (d) **Third Law of BHT**: No finite series of operations may reduce the surface gravity of a black hole to zero.
   (e) We see the formal analogy when we allow the surface gravity to stand in as thermodynamic temperature, and the horizon area to stand in as thermodynamic entropy.
2. **Beyond Mathematical Coincidence**
   Two reasons why the analogy is typically taken to be more than formal:
   (a) **Generalized Second Law**: The sum of the thermodynamic entropy of the exterior region and black hole entropy never decreases.
   (b) **Hawking Radiation**: Black holes radiate with temperature $\frac{\kappa}{2\pi}$.
3. **Pale Shadows of Thermodynamic Laws - Is the Analogy All that Strong?**
   (a) The Zeroth Law is commonly taken to state thermal equilibrium is transitive. Which assumes
      i. the thermal equilibrium state exists,
      ii. systems spontaneously approach this state,
      iii. 'equilibrium with' relation exists, and
      iv. that the relation is transitive.
   (b) **Disanalogy 1**: The Zeroth Law of BHT can be properly analogized with only a consequence of the Zeroth Law, and neglects much of the rich structure.
   Perhaps, we can recover i. and ii. above be appealing to black hole stationarity as an equilibrium state and assuming that a collapsed body will "settle down" to such a state. However,
   (c) **Disanalogy 2**: In thermodynamics, isolated systems in equilibrium minimize their internal energy. The BHT counterpart of internal energy is the mass. It is not clear what it means when one says that stationary black holes minimize their mass.
   (d) **Disanalogy 3**: There is no 'equilibrium with' relation in BHT (iii. and iv. cannot be fulfilled).

(e) **Disanalogy 4**: In thermodynamics total energy is distinct from internal energy, in BHT internal energy is total energy.

(f) **Disanalogy 5**: Two black holes, with the same temperature κ coalesce into one. The black hole entropy increases (since the area increases), contrary to thermodynamics.

(g) **Disanalogy 6**: Volume is an important variable in thermodynamics (e.g. ideal gas law), but BH entropy is proportional to area. Do we substitute BH entropy wherever volume appears in thermodynamic laws, or do we say that thermodynamic volume is not the counterpart of volume in BHT?

(h) **Disanalogy 7**: BHT is non-extensive. Consider a Schwarzschild BH. Double it. Entropy doesn't double (as it would in thermodynamics), and temperature is halved (as it would not in thermodynamics).

4. **Entropy of What?**

(a) If the laws of BHT are thermodynamical laws, then they must describe the behavior of thermodynamic systems. Up till now we have identified these systems in BHT to be the black hole event horizon. However, there are problems with this...

i. Event horizons are globally defined. Identifying the black hole region requires knowing what happens at infinity, at least if we don't restrict ourself to special cases.

ii. We are ignorant of the location of the event horizon. There is no relationship between the location of the event horizon and curvature, so we may not know where we are in spatial relation to them.

iii. Event horizons are"importantly ignorant." Anything within the event horizon is invisible to an observer at infinity. e.g. a star falls into a large black hole and then collapses, but an event horizon does not form since there is already one and they are the *outermost* surfaces of the region of no escape. Thus, BHT doesn't apply to collapsed objects behind an event horizon.

(b) Given these issues, recourse has been made to locally-defined replacements for event horizons. There are some problems...

i. Many are foliation-dependent, "the cardinal relativistic sin."

ii. In many cases event horizons can be present and not locally-defined horizons (e.g. some foliations allow for event horizons but not trapped horizons)

iii. Should Hawking radiation be associated with locally-defined horizons or black hole event horizons, given that Hawking radiation is a local phenomenon? (this in tension with the Generalized Second Law, as defined by Bekenstein, as applied to the globally defined black hole event horizon)

(c) In short, there is a tension. Use event horizons, inherit the problems of nonlocality, or use locally-defined horizons and inherent their individual problems.

5. **Entropy and Epistemicism**

(a) "The key assumption throughout BHT is that the entropy of a body that has fallen into a black hole is lost. Why accept that?"(the initial worry which motivated the Generalized Second Law was that when matter falls into a black hole the total entropy decreases, violating the thermodynamic second law)

(b) The key assumption above is accepted because BHT is committed to an information-theoretic understanding of thermodynamic entropy. The inital Bekenstein paper (1973) is explicitly committed to this.

(c) However, this is a "massively controversial assumption, one worthy of scrutiny. The foundations of stat mech roughly divide along objective/epistemic lines.

i. objective: found in the work of Boltzmann/Gibbs - the justification of thermodynamic behavior is ultimately in the detailed dynamics of the microphysics

ii. Entropy (Shannon) is understood by how much information is conveyed in a signal. Its maximized with maximized uncertainty and vanishes when the uncertainty disappears. Probability distributions interpreted as rational degrees of belief, etc.... Entropy becomes a feature of one's epistemic state.

(d) Confusion arises because Shannon entropy and Gibbs fine-grained entropy are formally identical. Though they are conceptually distinct, and can have different values.

(e) Shannon entropy should not be identified with thermodynamic entropy because thermodynamic entropy is directly connected to efficiency and the amount of work a system can do, which are perfectly objective facts about boxes of gas, etc...

(f) "If this is correct, then there is no reason to believe that a body slipping past an event horizon would lose its entropy." If a steam engine falls into a black hole its entropy remains unchanged, and one could follow it in and check. "Its a mistake to believe that the entropy of the body changes in any way." So no compensation, in terms of the Generalized second law, is necessary.

6. **All Together Now**

The analogy may not be more than formal.

(a) The analogy is not that tight.

(b) There is a tension in the analogy itself (which horizon type to ascribe thermodynamic properties)

(c) A major motivation is based on contentious reasoning about the relationship between entropy and knowledge.