Overview: *On How to Approach the Approach to Equilibrium* by Joshua Luczak

I. Typicality accounts and their shortcomings

i. **Goal**: Account for why systems beginning in an initial macrostate M_p end up in the equilibrium macrostate M_{eq} (where the values of the system's macrovariables are spatially uniform), and why they stay there for incredibly long periods of time

ii. **Typicality**: Something is typical if it happens in the "vast majority" of cases.

iii. **Typicality measure**: A measure over microstates on a system's phase space energy hypersurface. Typicality measures represent the relative size of sets of states. Typical states show a certain property if the measure of the set that corresponds to this property is 1 or close to 1.

iv. Three categories:

1. *Dominance views*: thermodynamic behavior is accounted for by the fact that equilibrium microstates are typical, as revealed by the dominance of the equilibrium macrostate.

2. *Unspecified dynamical views*: Given any reasonable account of the system's dynamics, typical initial states are taken into the equilibrium macrostate and stay there for incredibly long periods.

3. *Ergodic views*: Relevant systems possess a dynamical property found toward the bottom of the ergodic hierarchy, and typical states of these systems approach equilibrium and remain in equilibrium for long periods of time

v. Limitations: Typicality views do not underpin facts about

1. *The rates at which systems approach equilibrium*- "What will the system do in the next 5 minutes?"

2. *The states systems pass through on their way to equilibrium*- "What states will it pass through?"

3. *Fluctuation phenomena*- "How likely is it that we will see the system fluctuate out of equilibrium in the next few minutes?"

In general, these views do not help us to form new expectations about the systems we study, or help us justify expectations we already have. This is because **these views do not incorporate enough dynamical information**. This shortcoming makes sense, because typicality views aim at recovering thermodynamic behavior.

II. Langevin approach to Brownian Motion (Phenomenal approach)

i. **Motivation**: What are the techniques physicists use to model systems that begin away from equilibrium, and why are they successful?

ii. **Theory of Brownian motion**: Investigates the irregular behavior of objects when they are placed into various kinds of fluid media. Can also be applied to many other kinds of phenomena.

iii. **Langevin equation**:

$$M\frac{dV}{dt} = -\zeta V + \delta F(t);\ \zeta = 6\pi\eta a$$

M: mass of Brownian particle	V: velocity of Brownian particle
a: radius of Brownian particle	η: viscosity of fluid medium
Frictional force: $-\zeta V$	Fluctuating force: $\delta F(t)$

Assumptions:

1. Expectation value of the fluctuating force is zero $\langle \delta F(t) \rangle = 0$
2. No correlation in collisions in distinct time intervals $\langle \delta F(t)\delta F(t') \rangle = 2B\delta(t-t')$

Solution:

$$V(t) = e^{-\frac{\zeta t}{M}}V(0) + \int_0^t dt'\, e^{-\frac{\zeta(t-t')}{M}}\delta F(t')/M$$

First term: exponential decay of the particle's initial velocity

Second term: extra velocity produced by the fluctuating force

Mean-squared velocity:

$$\langle V(t)^2 \rangle = e^{-\frac{2\zeta t}{M}}V(0)^2 + \frac{B}{\zeta M}(1 - e^{-\frac{2\zeta t}{M}})$$

This equation accomplishes many things (as does the mean-squared displacement):

i. Models the Brownian particle's approach to equilibrium, i.e. the time evolution of one of the system's macrovariables.

ii. Tells us the system will quickly approach equilibrium and remain there for long periods

iii. Helps us form expectations about the kinds of microstates the system will be in at various stages of its time-evolution

iv. Helps us form expectations about exactly how quickly the system will approach equilibrium

v. Taking the long-time limit gives us the Einstein-Smoluchowski relation: $B = \zeta k_B T$ which gives us quantitative information relating the strength of the fluctuating force to the magnitude of the friction.

III. Time-Reversal Noninvariance of the Langevin Equation

i. The Langevin equation is asymmetric under time reversal, but the system at hand has time-symmetric dynamics. How is this the case?

ii. The time-reversal noninvariance originates in the two force terms:

a) The frictional force term is time-asymmetric
b) The properties attributed to the fluctuation term are time-asymmetric (Assumptions 1 and 2 above)

Why do physicists make Assumptions 1 and 2? What would have to be true (or approximately true) at the microlevel to account for the success of this model despite the underlying time-symmetric dynamics?

IV. The **Collision Assumption** and **Microphysical Fact**

If we take a microphysical approach to the derivation of the Langevin equation, as opposed to the phenomenal approach detailed above, our two assumptions about the system are rephrased slightly.

Assumptions:

1. The incoming velocities of colliding molecules have an expectation value of 0: $\langle F_s(t) \rangle = 0$
2. The incoming velocities of colliding molecules at distinct times are uncorrelated:
$$\langle F_s(t)F_s(t') \rangle = 2B\delta(t - t')$$
i.e., no correlation exists between the forces acting on the Brownian particle at any two distinct times.

i. **Collision Assumption**: The velocity of any incoming colliding fluid particle is, at any time, **probabilistically independent** of the incoming velocity of the Brownian particle.

Temporal asymmetry is built into this assumption, which underlies Assumptions 1 and 2.

Velocity-reversing a collision process leads to correlations between the incoming velocities of the fluid molecules and the Brownian particle, and an increase in the Brownian particle's velocity after the collision.

ii. **Microphysical Fact**: At all times (except perhaps initially), the velocity of the incoming colliding fluid molecule and the incoming velocity of the Brownian particle are **approximately probabilistically independent**.

This is a weaker assumption. We cannot avoid the fact that these velocities are correlated immediately after the collision. So, we instead assume that correlations between fluctuating forces exponentially decay on incredibly short time scales.

V. Support for CA/MF:

i. *Very large number of particles in fluid*

Since the fluid is composed of a very large number of particles undergoing very many collisions, it is reasonable to think that **two particles that undergo a collision will undergo may collisions with other particles before colliding again**, if they ever do.

We can therefore reasonably expect that the forces acting on the Brownian particle to be **effectively** random, and the velocities of the incoming particles are effectively independent from that of the Brownian particle.

ii. *Bizarre behavior in temporally reversed system*

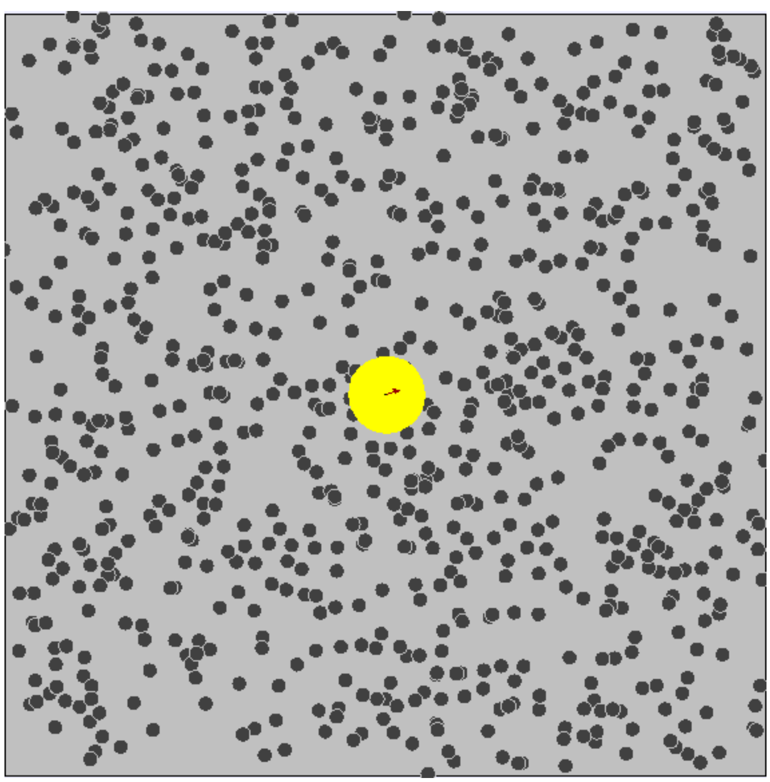
Time-forward system: Brownian particle slowed to equilibrium

Time-reversed system: "**Conspiratorial situation**" where the velocities of the fluid particles are already correlated with the Brownian particle, and are distributed such that their combined force increases the Brownian particle's velocity. The correlations between the fluid particles are explained by an event in the future, i.e. **effect before cause**.

We do not ever observe systems behaving like the time-reversed system.

iii. *Results from idealized hard sphere gas system*

Lanford's work: shows that for a system described by the Boltzmann equation, assuming an uncorrelated initial state, the system can **sustain a lack of correlation** as it approaches equilibrium (for a short time)



VI. Summary and questions

The main point:

Typicality accounts of how systems approach equilibrium are severely limited. They cannot tell us many important facts that we would like to know about a system that begins away from equilibrium, because they do not contain sufficient dynamical information.

In order to better understand how a system spontaneously approaches equilibrium and remains in equilibrium, we should look to the methods successfully used by physicists to model systems beginning away from equilibrium, such as a Brownian particle placed in a fluid medium.

Studying the Langevin approach to Brownian motion reveals the primary assumption made by physicists, about systems beginning away from equilibrium, namely, the MF. The MF is the assumption that the incoming velocities of the colliding fluid molecule and the Brownian particle are approximately uncorrelated.

The MF is justified by "scientifically informed reflection" as summarized above. If we take a Brownian particle slowing in a fluid and time-reverse the system, we encounter a bizarre, conspiratorial situation. Deviations from the MF lead us to expect systems to behave in ways we simply never witness.

Questions:

1. Has Luczak given us sufficient reason to abandon typicality accounts of the approach to equilibrium?

Let's refer back to one of our first readings of the semester, *Boltzman's Approach to Statistical Mechanics* by Sheldon Goldstein (2001):

"*For a nonequilibrium phase point X of energy E, the Hamiltonian dynamics governing the motion X_t arising from X would have to be ridiculously special to avoid reasonably quickly carrying X_t into Γ_{EQ} and keeping it there for an extremely long time*" (p. 5)

"What is required is that the initial state not be too contrived, that it be somehow reasonable, indeed that it be natural." (p. 13)

"*We say that a phenomenon has been explained if it holds for typical initial conditions, that is with rare exceptions as defined by a suitable "measure" μ of typicality.*

The phenomenon has been explained if the set E of exceptional initial conditions satisfies $\mu(E) \ll 1$. Of course it is essential that the measure of typicality be natural and not contrived […] For dynamical systems as we are discussing here, the measure of typicality should be naturally related to the dynamics" (p. 15)

In other words, is Luczak's argument for the Microphysical Fact at all stronger than Goldstein's argument for a "typicality" view?

2. Is the Collision Assumption the same thing as the ***Stoßzahlansatz***? How about the Microphysical Fact?

3. Does Luczak succeed in justifying the Microphysical Fact? Does his work constitute an advance with respect to Boltzmann's ***Stoßzahlansatz*** troubles?