Thomson, "On a dynamical theory of heat ...." 1852

"Carnot's function" (Eqn 3, 4, pp 19–20)

$$\left(\frac{\partial p}{\partial t}\right)_v = \mu M$$

Heat added per unit volume during an isothermal expansion

p pressure
t temperature
v volume

Thomson shows by means of an infinitely small Carnot cycle that μ is the same for all substances

I think this proof works for both
- conversion theory
- Carnot's caloric theory of heat

$\boxed{\text{Determine } \mu \text{ for an ideal gas}}$

since μ is the same for all substances, this fixes μ universally

$$p = \frac{nRt}{v}$$

$$\therefore \left(\frac{\partial p}{\partial t}\right)_v = \frac{nR}{v}$$

During an isothermal expansion by dv

$$\text{work done} = \text{heat gained} = dq = p\,dv = nRt\,\frac{dv}{v}$$

This step assumes heat & work are interconvertible
Hence the μ recovered will not be suitable for the Carnot caloric version

$$M = \left(\frac{dq}{dv}\right)_{\substack{\text{this}\\ \text{process}}} = \frac{nRt}{v}$$

combine

$$\mu = \left(\frac{\partial p}{\partial t}\right)_v \Big/ M = 1/T$$

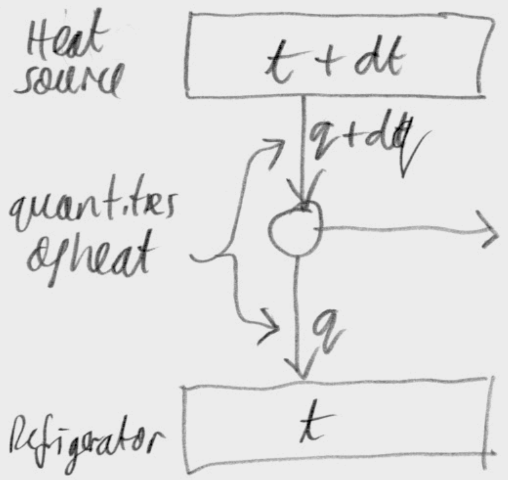
NB Thomson leaves μ undetermined until very late

$$\text{p.172} \quad \mu = J\,\frac{E}{1+Et} = J\,\frac{1}{1/E + t}$$

converts t to absolute temperature scale

mechanical equivalent of heat
= 1 in my calculation

Relation of μ to mechanical effect recoverable from reversible heat engines



work recovered

$$dW = \underbrace{Jdq}_{\text{from first law}} = \underbrace{\mu q \, dt}_{\text{seems to be the defenition of } \mu \text{ ??}}$$

Modern identification

$$\frac{q+dq}{q} = \frac{t+dt}{t}$$

$$1 + \frac{dq}{q} = 1 + \frac{dt}{t}$$

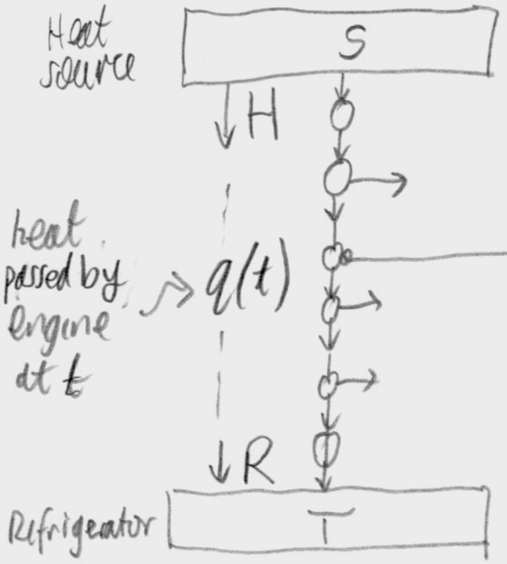
$$dq = q\left(\tfrac{1}{t}\right) dt$$

$$\therefore\ dW = J dq = q\left(\tfrac{J}{t}\right) dt$$

$$\boxed{\mu = J/t}$$ ← absolute temp

$$= 1/t$$ ← if we set $J = 1$ by using same units for heat & work

Integration of μ for finite temperature differences



work recovered from engine at t

$$\underbrace{dW = J\,dq = \mu q\,dt}_{\text{solve for relation between } H, R}$$

$$\frac{dq}{dt} = \frac{1}{J}\mu q$$

$$\therefore \int_R^H \frac{dq}{q} = \frac{1}{J}\int_T^S \mu\,dt$$

many infinitely small temperature difference heat engines

$$\boxed{\ln H/R = \frac{1}{J}\int_T^S \mu\,dt}$$

compare modern result. $\mu = J/T$

$$\ln H/R = \frac{1}{\cancel{J}}\int_T^S \frac{\cancel{J}}{T}\,dT = \ln \frac{S}{T}$$

$$H/R = \frac{S}{T} \qquad \boxed{\frac{H}{S} = \frac{R}{T}}$$

Why does Thomson leave μ an undetermined function?

① He repeatedly insists that it must be found experimentally

② He can compare his results with those from Carnot' caloric based theory.

Work recovered from heat H in moving from heat source at S to heat source at T in Carnot's theory

$$dW = \mu H \, dt$$

since heat is not consumed, H is a constant

Integrating:

$$\boxed{W = H \int_T^S \mu \, dt}$$

Thomson & Carnot agree when S is very close to T.
"J" drops out of Thomson's formulae

Compare with Thomson:

$$\ln H/R = \frac{1}{J} \int_T^S \mu \, dt$$

$$\therefore R/H = \exp\left(-\frac{1}{J}\int_T^S \mu \, dt\right)$$

$$\boxed{\therefore W = J(H-R) = JH\left[1 - \exp\left(-\frac{1}{J}\int_T^S \mu \, dt\right)\right]}$$

when $S \approx T$, $\int_T^S \mu dt$ is small. Express as power series

$$\boxed{W = JH\left[1 - \left(1 + \frac{1}{J}\int_T^S \mu \, dt\right) + \frac{1}{2!} \cdots\right] \approx H \int_T^S \mu \, dt}$$