The Gibbs Framework: Pro and Anti

	Wallace	Goldstein *et. al*
Boltzmann equilibrium	"A system is in **Boltzmann equilibrium** if it lies in the largest of the macrostates (called the equilibrium macrostate) given the system's energy." p.3 "The approach to Boltzmann equilibrium is essentially a consequence of phase-space geometry combined with some reasonable assumptions about the dynamics […] **This conception of equilibrium makes the approach to equilibrium a statistical or probabilistic matter**" p. 3	"In every energy shell there is usually one macro set $\Gamma_\nu = \Gamma_{eq}$ that corresponds to **thermal equilibrium** and takes up by far most (say, more than 99.99%) of the volume" p.4 "Now increase of **Boltzmann entropy** means that the phase point $X(t)$ moves to bigger and bigger macro sets Γ_ν." p.19
Gibbs equilibrium	"A system is at **Gibbs equilibrium** if ρ is time-invariant under the system's dynamics […] if the system is ergodic, the equilibrium distribution must be uniform on each energy hypersurface" p.4	"In the view we call the ensemblist view, **a system is in thermal equilibrium if and only if its phase point *X* is random with the appropriate distribution**[…] In the individualist view, in contrast, a system is in thermal equilibrium (at a given energy) if and only if its phase point X lies in a certain subset of phase space." p.34
Time-independence of Gibbs entropy and coarse-graining	"There is an immediate problem with this first-pass version of the Gibbsian approach**: it seems to have the corollary that real systems do not increase in entropy or approach equilibrium**." p.4-5 "in statistical mechanics (as distinct from thermodynamics) the entropy is ultimately no more than a book-keeping device" p.17 "There is no dynamical principle according to which the coarse-grained entropy is a constant of the motion; indeed, **it is mathematically possible for the coarse-grained entropy to increase to a maximum value and then remain there indefinitely**." p.6	"The **time independence of $S_G(\rho)$ conflicts with the formulation of the second law given by Clausis**[…] Clausius's statement is actually correct for the Boltzmann entropy." p.6-7 "Some authors (e.g., Wallace, 2019; Tolman, 1938, § 51) have considered a partition (of an energy shell in) phase space[…] **It is plausible that [the coarse-grained entropy] indeed tends to increase**[…] the argument for the increase of [entropy] given by Tolman (1938) is without merit." p.38

Arbitrariness when carving up phase space into macostates	"Yes, formally speaking the Boltzmann entropy depends only on a system's macrostate, but it relies for its definition on a partition of the energy hypersurface into macrostates, and **that partition is modal in nature-** most obviously because the energy hypersurface itself depends on the dynamics." p.18 "…the macrostate partition at the heart of Boltzmannian statistical mechanics is just as vulnerable to these [subjectivity] criticisms as is the Gibbsian coarse-graining- indeed, it is **a special case of coarse-graining**" p.19	"…this description **still leaves quite some freedom of choice and thus arbitrariness in the partition**[…] Wallace (2019) complained that this element makes the Boltzmann entropy "subjective" as well, but that complaint does not seem valid: rather, S_B and its increase provide an objective answer to a question that is of interest from the human perspective. Moreover […] **this anthropomorphic element becomes less relevant for larger N. It is usually not problematical and not subject to the same problems as the subjective entropy**." p.18 "…we usually never go through the trouble of actually selecting Γ_{eq} and the other Γ_v: it often suffices to imagine that they *could* be selected. Specifically, for thermal equilibrium, it often suffices to specify the distribution […] with the understanding that **Γ_{eq} should contain, in a reasonable case, the typical points relative to that distribution**." p.34
Status of the probability density/measure ρ	"**What we want to explain in non-equilibrium statistical mechanics is itself something modal**: not that systems *invariably* go to equilibrium but that they do so *almost certainly* […] A probabilistic property of a system is poorly suited to explain why the system deterministically behaves in such-and-such a way, but it is well suited to explain why it very probably behaves in that way." p.16 "For if we want to explain why a deterministic system will with high probability do X, **probabilistic statements about its current state are pretty much all we can expect** as explanada." p.20	"While every classical system has a definite phase point X (even if we observers do not know it), **a system does not "have a ρ"**; that is, it is not clear which distribution ρ to use.[…] In general, several possibilities for ρ come to mind: (a) ignorance[…] (b) preparation procedure[…] (c) coarse graining[…]" p.4 Three options for ρ in the "individualist" approach to Gibbs entropy (p.31): (i) frequency in repeated preparation (**genuine probability**), (ii) degree of belonging, (iii) typicality

Reinterpreting probabilities in terms of typicality	"…it is always open to the Boltzmannian to insist that apparently 'probabilistic' predictions should be reinterpreted as, say, claims about what is **typical** when an experiment is repeatedly performed on a very large number of copies of the system. But this is just a claim about the general foundations of probability in statistical mechanics (specifically, that it should be understood on frequentist lines). **It in no way eliminates probability from the actual statement and use of statistical mechanics**." p.10	"A feature of behavior is said to be typical in a set S if it occurs for most (i.e. the overwhelming majority of) elements of S." p.31 [Example: digits of π] "when considering a random experiment in probability theory, we usually imagine that we can repeat the experiment, with relative frequencies in agreement with distribution ρ […] what we are getting at is **that Gibbs's ensembles are best understood as measures of typicality, not of genuine probability** […] There is room for different choices of ρ, and this fits well with the fact that Γ_{ν_o} has boundaries with some degree of arbitrariness." p.32-33
Subjectivity/role of observer's knowledge	"It will be objected […] we are saying something objective about the world, not something about my beliefs. I agree, as it happens; that just tells us that **the probabilities in statistical mechanics cannot be interpreted epistemically. And then, of course, it is a mystery how they can be interpreted**, given that the underlying dynamics is deterministic […] In the **thermodynamics context**, by contrast, it is far less clear to me why my knowledge of a system's state cannot play an explanatory role." p.20	"Wallace, an ensemblist, feels the force of arguments against subjective entropy but **thinks there is no alternative**." p. 30 "The basic problem with the ensemblist definition of thermal equilibrium is the same as with the Gibbs entropy: a system has an X but not a ρ. Is it subjective? **But whether or not a system is in thermal equilibrium is not subjective**." p.34
Recurrence and the Gibbs and Boltzmann frameworks	"once it is recognized that in Gibbsian statistical mechanics 'equilibrium' is a statement about the probability distribution of a system, **there is no contradiction between the (classical) recurrence theorem and the claim that entropy is non-decreasing.** For the former tells us that any given system has some timescale at which it has returned to its initial state, and the latter (for Boltzmann-apt systems) tells us that at any time after the equilibration timescale the system is overwhelmingly likely to be in the equilibrium macrostate, and these statements are compatible." p.17	"Contrary to von Neumann's statement[…] **the second law as formulated in (34) is not refuted by either [the time reversal or recurrence] objection**: after all, the second law applies to *most*, not *all*, phase points $X(0)$, and it does not claim that $X(t)$ will stay in thermal equilibrium *forever*, but only for a very, very long time." p.21

Recovering thermodynamics from statistical mechanics	p.11: requirements for recovering thermodynamics from an underlying mechanical theory “we have a (sketch of a) satisfactory derivation of thermodynamics from Gibbsian statistical mechanics […] The situation is parallel to the statistical-mechanical case. For the Gibbsian, there is no factive difference between the two approaches: the validity of the Gibbsian approach entails that of the Boltzmannian approach, and the two strategies differ only semantically.” p.13 “modern physics is extensively applying, and testing, thermodynamics in the microscopic regime, where the Boltzmann-aptness assumption completely fails and predictions are explicitly probabilistic.” p.15	“Section 4.1: Cases of Wrong Values” example “Consider for example the phenomenon that by thermal contact, heat always flows from the hotter to the cooler body, not the other way around. The usual explanation of this phenomenon is that entropy decreases when hear flows to the hotter body, and the second law excludes that. Now that explanation would not get off the ground if entropy meant subjective entropy: in the absence of observers, does heat flow from the cooler to the hotter? In distant stars, does heat flow from the cooler to the hotter? In the days before humans existed, did heat flow from the cooler to the hotter? After the human race becomes extinct…” p.13
Thermal coefficients: Two-time correlation function	“Since $C(t)$ is an explicitly probabilistic quantity, it is not even defined on the Boltzmannian approach.[…] So: even for Boltzmann-apt systems, **there are important cases where probabilistic methods seem necessary and do not reduce to Boltzmannian methods in any simple way**.” p.8-9	“Actually, that is not correct. The individualist will be happy as soon as it is shown that for most phase points in $\mathcal{H}_{mc}$, the rate of heat conduction is practically constant and can be computed from $C(t)$in the way considered.” Goldstein p.37
Spontaneous symmetry breaking in a ferromagnet	“Here, ‘the’ equilibrium microstate below a certain temperature has a non-zero expectation value of magnetic spin […] it then follows from the rotational symmetry of the underlying dynamics that there must be another microstate obtained by applying a rotation to each microstate in the first, of equal volume to ‘the’ equilibrium microstate.” p.9 “**We cannot, for instance, say ‘typical states are equally likely to end up in each equilibrium macrostate’** since ‘being equally likely to end up in equilibrium macrostate’ is not a property that any given microstate can have in a deterministic theory.” p.10	“a broad ρ will lead to approximately equal probabilities for ν_1 and ν_2. This leads to the question why practical procedures lead to broad ρs, and that comes from **typicality** as expressed in the development conjecture. Put differently, for a large number of identical ferromagnets, **it is typical that about half of them are in ν_1 and about half of them in ν_2.**” Goldstein p.37

Quantum statistical mechanics	"a hypothetical 'Gibbsian' quantum statistical mechanics works **with density operators *understood as probability distributions over mixed states*; a 'Boltzmannian' statistical mechanics instead works with density operators *understood as individual mixed states***. But nothing at the level of the mathematics will distinguish the two approaches[…] And I have been arguing that the *machinery* of the Gibbsian approach, not a hypothetical interpretation of that machinery, is compatible with the Boltzmannian conception […] **at the level of machinery, there *is* no difference between the two approaches**." p.21-22	"…**the density matrix $\hat{\rho}$ plays the role analogous to the classical distribution density ρ**, and again, the question arises as to what exactly $\hat{\rho}$ refers to: an observer's ignorance or what? Our discussion of options (a)-(c) above for the Gibbs entropy will apply equally to the von Neumann entropy[…] The closest quantum analog of the Boltzmann entropy is the following. **A macro state ν should correspond to, instead of a subset Γ_ν of phase space, a subspace $\mathcal{H}_\nu$ of Hilbert space $\mathcal{H}$, called a macro space** […] It seems convincing that $S_{qB}(\nu)$ yields the correct value of thermodynamic entropy." p.8
The grounding framework	"fluctuations around the Boltzmann equilibrium values can be described either as fluctuations *within* Gibbs equilibrium, or as fluctuations *into and out of* Boltzmann equilibrium, but this is simply a semantic difference […] p.7 "In those systems to which the latter is applicable**, the Gibbsian framework can be seen as grounding the Boltzmannian one**. The situation is not symmetric, for obvious conceptual reasons. The Boltzmannian framework *per se* contains no explicit notion of probability, and so does not permit us even to define the Gibbsian probability distribution." p.7	"By a fuzzy macro set we mean using functions $\gamma_\nu(x) \geq 0$ instead of sets Γ_ν as expressions of a macro state ν: some phase points x look a lot like ν, others less so, and $\gamma_\nu(x)$ quantifies how much. The point here is to get rid of sharp boundaries between the sets as the boundaries are artificial and somewhat arbitrary anyway. So **what would be the appropriate generalization of the Boltzmann entropy to a fuzzy macro state**? It should be k times the log of the volume over which is effectively distributed- in other words, **the Gibbs entropy**…" p.29

Questions for discussion:

I. Does carving up phase space into macrostates truly make the Boltzmannian approach probabilistic or subjective? Is Goldstein's defense against this accusation enough to dismiss it?
II. Gibbs entropy vs. "subjective entropy": is Goldstein arguing against a straw-man in Section 4: "Subjective Entropy is Not Enough"?
   a. To what extent are Gibbs entropy and "subjective entropy" the same thing?
III. Is Goldstein's definition of typicality satisfactory? (We can compare this definition to our readings from last week.)
IV. Do Gibbsian ensembles measure typicality or genuine probability?
V. The time reversal problem for the Boltzmann entropy of the entire universe
   a. For Goldstein, Past Hypothesis + Lanford's theorem ➔ Development Conjecture (DC). (Goldstein p. 27)
   b. Goldstein uses DC to explain away the fact that according to the Boltzmann equation, entropy increases in both time directions. Does DC truly solve the time reversal problem?
VI. A potential point of agreement: subjectivity and the irrelevance of the observer's knowledge of a system
   a. Do the two authors agree when discarding subjectivity, or are they referring to different ideas when using term?
   b. Wallace allows subjectivity to play an explanatory role in thermodynamics, while Goldstein recoils at the idea. Should we have the same visceral reaction against assigning any role in macroscopic science to subjectivity?
VII. Whose account of the "grounding" framework is more convincing- Wallace or Goldstein? Is the Boltzmann entropy a special case of the Gibbs entropy (Wallace) or is the Gibbs entropy a "fuzzy" Boltzmann entropy (Goldstein)?
VIII. Does Wallace have a response to Goldstein's assertion that a Boltzmann interpretation is necessary for the accuracy of macroscopic hydrodynamic equations? Is this a "naturalist" counter-example to Wallace's thermal coefficients, ferromagnet and Brownian motion? (see Goldstein p.5)
IX. There is a lot missing from this list, e.g. ergodicity & mixing, empirical vs. marginal distributions, Boltzmann's H-theorem. Many topics discussed in Goldstein's paper have been excluded. What other questions or remarks do you have?

Sources:

1. D. Wallace, "The Necessity of Gibbsian Statistical Mechanics", forthcoming; http://philsci-archive.pitt.edu/15290/
2. S. Goldstein, J. Lebowitz, R. Tumulka, N. Zanghi, "Gibbs and Boltzmann Entropy in classical and quantum mechanics", forthcoming; https://arxiv.org/abs/1903.11870