Einstein-Ritz controversy (1908-1909)

"[E]xperience compels one to consider the representation by means of retarded potentials as the only one possible, if one is inclined to the view that the fact of the irreversibility of radiation must already find its expression in the fundamental equations. Ritz considers the restriction to the form of retarded potentials as one of the roots of the second law [of thermodynamics], while Einstein believes that irreversibility is exclusively due to reasons of probability." (Ritz and Einstein 1909: 324)

Einstein: thermo/entropic arrow $\Rightarrow$ EM arrow

Ritz: EM arrow $\Rightarrow$ thermo/entropic arrow

Retarded and Advanced Potentials

Sols. to Maxwell's equations with specified sources J^ν:

$$\frac{\partial^2 A^\nu}{\partial x^2} + \frac{\partial^2 A^\nu}{\partial y^2} + \frac{\partial^2 A^\nu}{\partial z^2} - \frac{\partial^2 A^\nu}{\partial t^2} = J^\nu$$

Kirchhoff's representation theorem for Minkowski spacetime:

$$\begin{aligned} A^\nu(x) &= \int_{\Omega^-} ret + \int_{\partial\Omega^-} ret \equiv {}_{ret}A^\nu(x) + {}_{in}A^\nu(x) \\ &= \int_{\Omega^+} adv + \int_{\partial\Omega^+} adv \equiv {}_{adv}A^\nu(x) + {}_{out}A^\nu(x) \end{aligned}$$

$$F_{\mu\nu} := \nabla_{[\nu}A_{\mu]}$$

$$\begin{aligned}
F^{\mu\nu}(x) &= {}_{ret}F^{\mu\nu}(x) + {}_{in}F^{\mu\nu}(x) \\
&= {}_{adv}F^{\mu\nu}(x) + {}_{out}F^{\mu\nu}(x) \\
&= \lambda[{}_{ret}F^{\mu\nu}(x) + {}_{in}F^{\mu\nu}(x)] + \\
&\quad (1-\lambda)[{}_{adv}F^{\mu\nu}(x) + {}_{out}F^{\mu\nu}(x)]
\end{aligned}$$

$$1 \leq \lambda \leq 0$$

1) There are NOT different solutions but just representations of the same solution.

2) The advanced (retarded) fields of a point charge are called the Liénard-Wiechert fields.

B.S. about causality/causation

Thus, Heald and Marion (1995) opine that

> This so-called advanced potential [the time component of the advanced four-potential] appears to have no physical significance because it corresponds to an anticipation of the charge distribution (and current distribution for the case of the vector potential [the space components of the four-potential]) at a future time. Such a potential does not satisfy the requirement that causality must be obeyed by a physical system.

Frisch (2000). EM fields associated with charges obey the retardation condition—each charge physically contributes a fully retarded component to the field. This is a law on a par with Maxwell's laws. It explains various EM asymmetries.

Frisch (2005) The retardation condition is a "causal constraint."

Charged particles interacting at a distance

The retarded theory T_{ret}. For each particle j calculate its (auxiliary) retarded Liénard-Wiechert field ${}_{ret}F^{\mu\nu}_{(j)}$. Then postulate that each particle k obeys the equation of motion

$$m_{(k)} a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} {}_{ret}F^{\mu\nu}_{(j)} u_{(k)\nu}$$

The advanced theory T_{adv}:

$$m_{(k)} a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} {}_{adv}F^{\mu\nu}_{(j)} u_{(k)\nu}$$

The symmetric theory T_{sym}:

$$m_{(k)} a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} \frac{1}{2}\left[{}_{ret}F^{\mu\nu}_{(j)} + {}_{adv}F^{\mu\nu}_{(j)} \right] u_{(k)\nu}$$

1) Ritz's retardation condition has a clear meaning in this context—*without* any need to appeal to causal notions.

2) There are important differences between these theories, e.g. T_{sym} is time reversal invariant while T_{ret} and T_{ret} are not, and they admit different solution sets.

Challenge: Without using the words "cause" and "causal" (or cognates, e.g. "produces" or "contributes") explain what the retardation means in orthodox Maxwellian EM.

$\hat{T}_{ret}$: Maxwell + the posit that for a system of N charged particles

$$F^{\mu\nu}_{ret}(x) = \sum_{j=1}^{N} {}_{ret}F^{\mu\nu}_{(j)}(x) + {}_{\mathrm{hom}}F^{\mu\nu}_{1}(x)$$

$\hat{T}_{adv}$:

$$F^{\mu\nu}_{adv}(x) = \sum_{j=1}^{N} {}_{adv}F^{\mu\nu}_{(j)}(x) + {}_{\mathrm{hom}}F^{\mu\nu}_{2}(x)$$

$\hat{T}_{sym}$:

$$F^{\mu\nu}_{sym}(x) = \frac{1}{2}\sum_{j=1}^{N}[\, {}_{ret}F^{\mu\nu}_{(j)}(x) + {}_{adv}F^{\mu\nu}_{(j)}(x)] + {}_{\mathrm{hom}}F^{\mu\nu}_{3}(x)$$

But are these different theories or just the same theory written in different ways? We can always choose the homogeneous solutions so that the total fields $F^{\mu\nu}_{ret}(x) = F^{\mu\nu}_{adv}(x) = F^{\mu\nu}_{sym}(x)$.

In some circumstances one of the representations may seem more natural than the others–see Fig. 1

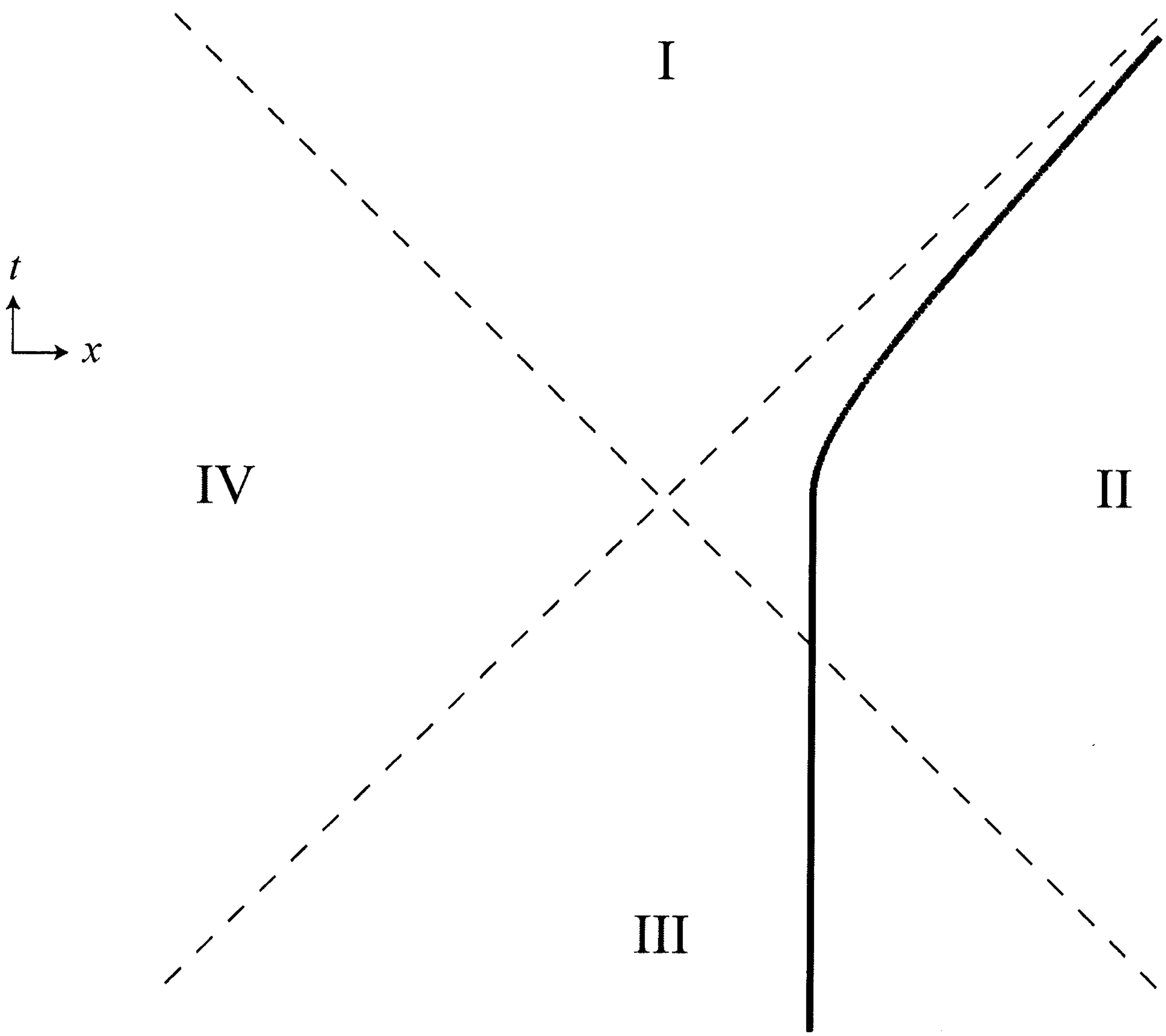

Fig. 1

Some EM arrows: real and alleged

A1) We often observe spherical EM waves diverging from a localized source but rarely observe the time reverse.

a) What has this to do with advanced and retarded field/potentials? Nothing.

b) Nothing peculiar here to EM—similar asymmetries for water and sound waves, even in Newtonian regimes where advanced and retarded fields do not come into play.

c) The most plausible explanation is statistical—just as Einstein asserted.

A2) Zeh (2001):

Why does the condition ${}_{in}F^{\mu\nu} = 0$ (in contrast to ${}_{out}F^{\mu\nu} = 0$) approximately apply in most situations?

"Situations" = Ω finite. But

(i) if what Zeh claims were true, how could we see objects outside our local neighborhood?

(ii) The ubiquity of the cosmic background radiation.

A3) Frisch (2005):

> (RADASYM) There are many situations in which the total field can be represented as being approximately equal to the sum of the retarded fields associated with a small number of charges (but not as the sum of the advanced fields associated with the charges), and there are almost no situations in which the total field can be represented as being approximately equal to the sum of the advanced fields associated with a small number of charges.

Frisch also intends this to apply to local situations. And he thinks that "the brunt of the explanatory work is done by the retardation condition—the assumption that the field physically contributed by a charge is fully retarded."

But is there any need to appeal to the ill-defined notion of "physically contributed"?

A4) Suppose that the limit exists as $\Omega \to \infty$.

${}^{\infty}_{in}F^{\mu\nu}$ is the (truly) source-free radiation, and ${}^{\infty}_{in}F^{\mu\nu} = 0$ is called the *Summerfeld radiation condition*.

${}^{\infty}_{out}F^{\mu\nu}$ is the unabsorbed radiation that escapes to spatial infinity, and ${}^{\infty}_{out}F^{\mu\nu} = 0$ is called the *absorber condition*.

Having ${}^{\infty}_{in}F^{\mu\nu} = 0$ but ${}^{\infty}_{out}F^{\mu\nu} \neq 0$ (or vice versa) would give a global EM asymmetry, possibly linked to the cosmological arrow.

a) Kirchhoff's representation theorem may not be well-defined in some cosmological models. In the non-conformally flat case, the retarded (respectively, advanced) representation contains an additional "tail" term consisting of an integral over the interior as well as the surface of the past (respectively, future) light cone of the field point.

b) The limit $\Omega \to \infty$ may exist in the retarded but not the advanced representation or vice versa.

c) ${}^{\infty}_{in}F^{\mu\nu} = 0$ cannot hold in big bang models with particle horizons.

A5) Radiation and radiation damping.

The equation of motion for a charged particle is not

$$ma^{\mu} = q({}_{ext}F^{\mu\nu}u_{\nu}) + {}_{nem}F^{\mu}$$

An accelerated charged particle radiates and feels a damping force. The time reverse is not observed.

The motion is described by the Lorentz-Dirac equation

$$\begin{aligned} ma^{\mu} &= q({}_{ext}F^{\mu\nu}u_{\nu}) + {}_{nem}F^{\mu} + \frac{2}{3}q^2(\dot{a}^{\mu} - a^2u^{\mu}) \\ &= q({}_{ext}F^{\mu\nu}u_{\nu}) + {}_{nem}F^{\mu} + \frac{2}{3}q^2(\eta^{\mu\nu} + u^{\mu}u^{\nu})\dot{a}_{\nu} \end{aligned}$$

- Pathologies—run away solutions, preacceleration, ...

- Limit to intended domain of application—a value for the self-interaction term that is small in comparison with the external force. In this domain can replace the L-D equation with the reduced-order equation, which is free of the above pathologies:

$$ma^{\mu} = q({}_{ext}F^{\mu\nu}u_{\nu}) + \frac{2}{3}\frac{q^3}{m}\left[u^{\beta}\nabla_{\beta}({}_{ext}F^{\mu\nu})u_{\nu} + \frac{q}{m}\,{}_{ext}F^{\mu\nu}\,{}_{ext}F_{\nu\beta}u^{\beta} + \frac{q}{m}u^{\mu}\,{}_{ext}F^{\alpha\beta}\,{}_{ext}F_{\beta\gamma}u_{\alpha}u^{\gamma}\right]$$

Both the L-D equation and the reduced-order equation fail to be time reversal invariant.

(A1) *Neo-Ritzian idea*: One of these equations, or some variant, is a law of nature. Then we finally have a clean EM arrow!

E.g. Rohrlich who takes a version of the reduced order equation to be the exact, true equation of motion governing radiation reaction. The asymmetry is said to be due to the need to use retarded fields.

Bad idea: The correct theory is QED, and this theory is time reversal invariant.

(A2) *Einstein redux*: The asymmetry is explained by probabilistic/statistical considerations.

Dirac's derivation of the damping force

Postulate that the radiation reaction force Γ^{μ} experienced by a point particle with charge q is given by

$$\Gamma^{\mu} := \frac{q}{2}\left[{}_{ret}F^{\mu\nu} - {}_{adv}F^{\mu\nu}\right]u_{\nu} \qquad (D1)$$

where ${}_{ret}F^{\mu\nu}$ and ${}_{adv}F^{\mu\nu}$ are the retarded and advanced Liénard-Wiechert fields of the charge itself.

Do a power series expansion, drop some terms, and out pops the radiation reaction force.

Full equation of motion (also called the L-D equation)

$$\begin{aligned} ma^{\mu}_{(k)} &= q_k \sum_{j\neq k}^{N} {}_{ret}F^{\mu\nu}_{(j)} u_{(k)\nu} \qquad (D2) \\ &\quad + q_k\left({}_{in}F^{\mu\nu} u_{(k)\nu}\right) \\ &\qquad + \frac{q_k}{2}\left[{}_{ret}F^{\mu\nu}_{(k)} - {}_{adv}F^{\mu\nu}_{(k)}\right]u_{(k)\nu} \end{aligned}$$

This equation is TRI—so can't explain the asymmetry.

Another way to see that $(D2)$ cannot explain the asymmetry. Rewrite $(D2)$ using Kirchhoff theorem:

$$ma^{\mu}_{(k)} = q_k \sum_{j \neq k}^{N} {}_{adv}F^{\mu\nu}_{(j)} u_{(k)\nu} \qquad (D2')$$

$$+ q_k \left({}_{out}F^{\mu\nu} u_{(k)\nu} \right)$$

$$+ \frac{q_k}{2} \left[{}_{adv}F^{\mu\nu}_{(k)} - {}_{ret}F^{\mu\nu}_{(k)} \right] u_{(k)\nu}$$

Inject an asymmetry: Suppose that the Sommerfeld radiation condition ${}^{\infty}_{in}F^{\mu\nu} = 0$ obtains. Then

$${}^{\infty}_{out}F^{\mu\nu} = \sum_{all\, j}^{N} \left[{}_{ret}F^{\mu\nu}_{(j)} - {}_{adv}F^{\mu\nu}_{(j)} \right] \qquad (D3)$$

Substitute $(D3)$ into $(D2')$ and the latter reduces to $(D2)$.

This does not explain why we observe radiation damping rather than anti-damping. Substitute $(D3)$ into $(D2)$ and the latter reduces to $(D2')$.

Attempt to explain radiation reaction in the Wheeler-Feynman action-at-a-distance theory

Th equation of motion is the one from T_{sym}:

$$m_{(k)} a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} \frac{1}{2} [{}_{ret}F^{\mu\nu}_{(j)} + {}_{adv}F^{\mu\nu}_{(j)}] u_{(k)\nu}$$

Contorted arguments plus a dubious Absorber Condition used to show that this equation reduces to $(D2)$.

But we have seen that $(D2)$ does not explain the asymmetry of radiation reaction.

QED

Classical model: a point particle of charge q is linearly accelerated by an external potential V.

z_o: the position the particle would have at $t = 0$ if there there were no radiation reaction force.

z: the actual position at $t = 0$ when the radiation reaction force—per the Lorentz-Dirac equation—is acting.

$\delta z_C := z - z_o$: the classical position shift due to radiation reaction

The goal is to compare the classical position shift δz_C to the position shift δz_{QED}.

QED model: charged scalar field $\hat{\varphi}$ coupled to the electromagnetic field and subject to the same external potential V as in the classical model.

Step 1. Calculate $< \hat{z} >^{off}$ of position at $t = 0$ with the electromagnetic field turned off using an initial state $|i\rangle$ in which the momentum of the particle is strongly peaked about a value pointing in the positive z-direction.

Step 2. Using perturbation theory, calculate to lowest order in q the expectation value of position $< \hat{z} >^{on}$ at $t = 0$ with the electromagnetic field turned on.

To this order, time dependent perturbation theory give a final state $|f\rangle$ of of the form

$$\begin{aligned} |f\rangle &= |1\varphi, 0\gamma\rangle + |1\varphi, 1\gamma\rangle \\ &= |i\rangle + |s\rangle \end{aligned}$$

where $|1\varphi, 0\gamma\rangle$ state with one scalar particle and no photon, $|1\varphi, 1\gamma\rangle$ state with one scalar particle and one photon, and $|s\rangle$ arises from the the forward scattering of the wave packet.

Step 3. Compute $\delta z_{QED} := <\hat{z}>^{on} - <\hat{z}>^{(s)} - <\hat{z}>^{off}$

Upshot: Higuchi and Martin (2004, 2005):

in the $\hbar \to 0$ limit, $\delta z_{QED} = \delta z_C$.

QED is TRI—so if $|I\rangle \rightsquigarrow |F\rangle$ is allowed so is ${}^R|F\rangle \rightsquigarrow {}^R|I\rangle$.

The observed asymmetry of radiation and radiation reaction is to be traced to the fact that, in the circumstances we find ourselves, it is overwhelmingly more "probable" that $|I\rangle$ type states will be realized than it is that ${}^R|F\rangle$ type states will be realized.

If circumstances were different and $|I\rangle$ type and ${}^R|F\rangle$ type states are equally "probable" then the observed asymmetry would disappear.

Here “probable” not something computed from QED itself but from external statistical considerations—the ball is kicked back to stat bach but now QSM.