Two Approaches to Time Reversal

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When I presented Frank Arntzenius and Hilary Greaves' (2007) ideas on time reversal in classical electrodynamics in our seminar earlier this term, I was still unsure precisely what their proposal was and how it works. In particular, they proposed a time reversal operation that does not flip the sign of velocities, so that the time reverse of a body moving from left to right is still a body moving from left to right. That seemed quite untenable. After an email exchange with Hilary and then Frank, I now feel that I have a pretty good picture of why they think this is admissible.

In brief, there are two ways of conceiving the operation of taking a time reverse under discussion. One, called "geometric" by Frank, is elaborated by David Malament (2004). It takes a time reverse by flipping the sign of a vector or co-vector field that represents the future direction of time and then propagates the ensuing changes through the quantities in a theory's models. The other, called "active" by Frank, seeks a symmetry in the theory's models that relates models with their time reversals directly. Loosely speaking, it leaves the time orientation unaffected and checks that each process has a time reversed version in the model set.

Frank and Hilary's paper employs the first; I was employing the second. That would not matter if we were dealing with the standard notion of time reversal; then both ways of conceiving the time reversal operation end up in the same transformation equations. However in Frank and Hilary's non-standard approach, they issue in different transformation equations. In particular, the first approach used by Frank and Hilary does not lead to velocity flipping sign under time reversal, whereas their ideas expressed in terms of the second do lead to velocity flipping signs under time reversal.

The details are below along with my reactions. In brief, I think that Frank and Hilary have found something important. Their "Feynman metaphysics" is appealing, in so far as it leads to a viable alternative to the standard rule of time reversal in electrodynamics; that is, in so far as it leads to a viable alternative that flips the sign of charge under time reversal. The viability of this alternative is important.

(Note added later: See Frank's remark below. In his view, Feynman metaphysics does not lead to an active transformation that differs from the standard, Malament transformation.)

Where I remain unconvinced is that the "geometric" approach to time reversal is a real alternative to the "active" approach. Rather I think it is only viable in so far as it presumes that time reversal invariance in the active sense has already been secured. When the geometric approach yields different transformation equations, they serve only to obscure the presence of active invariance, in which the real time reversibility of the theory resides.

1. Two Notions of Time Reversal

Flipping Time Orientation ("Geometric")

This approach depends upon dividing the content of the physical theory into a time orientation free core and everything else, that is, everything that is dependent on time orientation. The time orientation can be specified by a non-vanishing everywhere timelike vector field or by a corresponding co-vector. The operation of time reversal is the reversing of the temporal direction of the time orientation. That reversal is then propagated through all the quantities that are time orientation dependent, thereby creating the time reversal transformation.

For example, in most accounts, the worldline of an uncharged particle is independent of time orientation; the worldline just catalogs when the particle is at which point in spacetime. Defining a velocity requires selecting a particular path parameter from all possible and one is usually chosen that increases in the direction of the time orientation. As a result, flipping the time orientation flips the velocity vector.

A theory is time reversal invariant if models remain models after the time orientation is flipped.

Closure of Models Sets ("Active")

This approach depends upon inspecting the model set of a theory to see if it contains a symmetry with the properties of a time reversal transformation. The key property sought is that sequences of states in time transform into sequences of states in a time reversed order. Most simply, a worldline of particle moving from left to right must transform into one moving from right to left. The reversed states are allowed to be different from the original states. For example, velocities are switched in sign; or a magnetic field may be switched in sign; or a complex valued quantum wave function is replace by its complex conjugate.

My view is that these conditions need not pick out a unique time reversal operation. For a given theory, there may be more than one. (My plan is to write this up in a paper, if I can get the details sorted out!)

A theory is time reversal invariant if the model set of the theory contains the time reverse of every model.

2. The Implementations

The Standard Approach (Malament metaphysics)

In this approach the worldline of a charged particle is part of the time reversal independent core of electrodynamics. Its four-velocity is determined from the time orientation. Using the geometric approach and following David's arguments, we get the following as reversing an electrodynamic system in time:

Four force: $F^a \rightarrow F^a$
Charge: $q \rightarrow q$
Maxwell field tensor: $F^a{}_b \rightarrow -F^a{}_b$
Four-velocity: $V^a \rightarrow -V^a$

As a result, the Lorentz force law $F^a = q\, F^a{}_b\, V^a$ is invariant under time reversal.

These last transformations are the same as found using the active approach via the usual arguments.

Arntzenius-Greaves (Feynman metaphysics)

The major change is to declare that the four-velocity of a charged particle is part of the time orientation free structure. The idea is that charges just have charge; there is no intrinsic sense in the charge of positive and negative. That notion of positivity or negativity of charge comes from the sign on the four-velocity. A positive charge corresponds to a positive four-velocity. A negative charge corresponds to a negative four-velocity.

Here is the idea is slightly more detail. The Lorentz force law still obtains, but it is read differently. Charge enters as an absolute magnitude, written here as |q| to emphasize this fact:

$$F^a = |q|\, F^a{}_b\, V^a$$

where F^a is the four force on a charge |q| in a field with Maxwell tensor $F^a{}_b$ when the charge has four velocity V^a.

The four force will have a different sign for two particles of the same charge |q| but with four velocities differing in sign. This is reinterpreted in the standard accounts as a positive and negative charge when we revert to the standard practice of always conceiving of the four velocity as a positive vector. That is

• If we have a positive four velocity V^a, we compute the four force on it as
$F^a = |q|\, F^a{}_b\, (+V^a) = +q\, F^a{}_b\, V^a$

thereby treating the charge as a positive charge.

• A charge of the same magnitude but with a reversed four velocity in the same electromagnetic field, will have the same four force acting on it, but with a flipped sign. We will now write the four force as

$F^a = |q|\, F^a{}_b\, (-V^a) = -q\, F^a{}_b\, V^a$

We rewrite the four velocity as future directed, thereby requiring us to move the negative sign to the charge, with is now treated as a positive charge.

Question to Frank: Is this Correct?

Frank: yes, the point is just that flipping the 4-velocity but keeping q the same has the same effect as flipping the sign of q but keeping the 4-velocity fixed.

This "Feynman metaphysics" now leads to two different sets of transformation equations:

Geometric

Under geometric transformation, we flip the time orientation. Virtually every quantity is independent of time orientation. That includes each of F^a, $|q|$, $F^a{}_b$ and V^a. This is true standardly for F^a and $|q|$. It is the central posit of Feynman metaphysics that V^a is time orientation independent. Frank has told me in email that the Maxwell field tensor $F^a{}_b$ is independent of time orientation as a postulate. Hence, it follows immediately that the Lorentz force law is time reversal invariant and that the time reverse of any model containing these quantities will be a model containing these quantities with the same values.

Active

The time reversal operation is recovered in the usual way. All intrinsic properties are flipped. In particular, the four velocity of a charge is flipped. In order for the Lorentz force law to continue to hold, we must now flip the signs of one of q or $F^a{}_b$. Flipping the second gives the standard transformation equations. Leaving $F^a{}_b$ unchanged but flipping q gives a second set of transformation equations that conforms with Feynman metaphysics in so far as time reversals flip positive into negative charges.

Frank: I am not completely clear on how you are conceiving of an active time reversal here. But if upon active time reversal the 4-velocities of charged particles flip, then upon active time reversal then sign of $F^a{}_b$ should flip, not the sign of q. The sense in which according to the Feynman view time reversal changes the charge is only that if one flips the 4-velocity of a charged particle then it interacts with a given, fixed, em field as if it has the opposite charge.

3. My Concerns

About the Geometric Approach

While the geometric approach is initially attractive, I have grown uncomfortable with it and think that it really is not viable as a self-contained alternative to the active approach. These concerns are rooted in two worries:

(a) The geometric approach requires that we add metaphysics to the standard formulations of electrodynamics in order to yield a result. Maxwell's equations and the Lorentz force law by themselves are no longer deemed a complete account. More notions, invisible in the textbooks, have to be added. In one addition (Malament), we declare that worldlines but not velocities are time orientation independent. That leads to one set of transformations. In the other, we adopt Feynman metaphysics and get a different set. I don't see any non-question begging way to pick between the two. They employ the same theory (Maxwell's equations + Lorentz force law) and correspond in all observables.

Frank: we were worried about this too, which is why we wrote the bit about structuralism at the end.

(b) The focus of the theoretical analysis in the geometric approach is the time orientation vector field. It is treated as if it is a central physical quantity. That could be right if the theory is not time reversal invariant, for then it will be possible to use the theory's other structures to define a time orientation vector that picks out a physically preferred time direction. If, however, the theory is time reversal invariant, then a time orientation can be chosen only as a convention. In those circumstances, flipping the time orientation merely amounts to us thinking differently about which is the past and future. We then note that how we think about it makes no difference. We have to worry that this result reflects nothing physical. Here, contrary to its name, the geometric approach is really a subjective or passive approach and is reminiscent of coordinate based ways of introducing covariance principles. The active approach is actually closer to the geometric tradition of modern spacetime physics and philosophy.

These two worries combine to suggest the possibility of theories that formally satisfy the condition of time reversibility, but intuitively do not. For example, consider a theory of uncharged particles with just one model: a collection of uncharged particles in an expanding motion. We could declare that the four velocities of this collection of particles are time orientation independent quantities. Then flipping the direction of the time orientation will leave the four velocities and all motions unchanged. As a result the simple theory is judged time reversal invariant in the geometric approach, even though it is strongly asymmetric in time.

Frank: if the theory says that both expansion relative to the temporal orientation and collapse relative to the temporal orientation is allowed, then yes the theory is time reversal invariant. And that still seems right to me.

What has gone wrong with this bare (and rather dim-witted) application of the geometric approach is that the one model has its own natural sense of time orientation. The future is the direction of expansion. So the model is dependent upon the time orientation; it cannot be declared time orientation independent. A flip of the time orientation that leaves the motions unchanged ought no longer to produce a model of the theory.

The moral is that one cannot arbitrarily declare a quantity to be time orientation independent. A quantity can only be so declared if it resides in a structure sufficiently symmetric for the structure to have no intrinsic time direction. That is the case for worldlines of uncharged particles. They are definable without employing a time orientation. In the standard approach, it is not thought to be so for four velocities. We usually think that we have to pick one preferred time parameterization of a worldline to define a four velocity and that commits us to a direction in time. What enables escape from this time orientation dependence for the four velocities of charged particles is Feynman metaphysics. It identifies a symmetry in the models between positive charges with future pointing four velocities and negative charges with past pointing four velocities. This symmetry then grounds the declaration that the four velocity is time orientation independent.

The identification of symmetries like this just is the active approach. The symmetry described is the active transformation for charged particles. Hence proper application of the geometrical approach requires a prior, tacit application of the active approach. As a result, I do not believe that the geometric approach is a real alternative to the active approach. Rather it presupposes it.

Added May 4. I had a chance to talk to Hilary at the New Directions conference in Washington. That was helpful. She stresses that the above "one model" example is messy in a way I was aware of but was trying to ignore. The trouble is that there is no way to define just this one model without, in some way, already adding a time orientation to the spacetime structure. For example, if one wants to define the one model by equations, they must employ something like preferred coordinate systems, else the time reversed model will also be in the model set. The same thing would happen if one tries to specify a model set in words as I did. Then a correct implementation of the geometric method requires that the time orientation of the method must coincide with this tacitly introduced orientation. (I'm assuming the theory then comes out as not time reversal invariant. Flipping the time orientation then also flips the direction of expansion. So there is no model in which there is a collapse motion in the direction of the time orientation.)

Even with this complication, my objection still stands. If you are going to use the geometric method, you have to know already whether the system is tacitly adding a time orientation to the spacetime and then ensure that the time orientation of the geometric method coincides with it. That is, you already need to know the time irreversibility of the theory.

The same point is made more simply in the case of a time reversal invariant theory. In such a theory, to apply the geometric method, you begin by designating certain structures as unaffected by a flip of the time orientation. You can only do that if you know in advance that these are time reversal invariant; that is, that they treat past and future symmetrically. And that is just to say that a transformation that switches past and future leaves the theory's model set unchanged, which is the active definition of time reversal invariance.

For me, the interesting problem remains this: how do we identify which are the time reversal operations in a theory (if it admits any)? It may seem that this is only a problem for the active approach. The same problem, however, is expressed in the geometric approach in this form: how do we know which structures to designate as the time orientation free core that aren't to be flipped when the time orientation is flipped?

About the Active Approach

Frank, however, has worries about the active approach. They are:

(a) He claims the "geometric" approach is easier to transfer or even the only one of the two that can be transferred to the case of general relativity in which there are no global symmetries of the overall temporal structure. (This worries me too, but if it is right the upshot is that we have good notion of time reversal in general relativity since the geometric approach depends upon the active.)

(b) He is concerned about how we match up geometric structures across different models. e.g. if one model has a particle moving to the left and another the same particle moving to the right, how do we know that the "right" of the second model corresponds to the "right" of the first? (This worries me too, but not that much. If the worry is sustainable then it will bring down much more with it, such as the notion of active covariance principles.)

My thanks to Frank and Hilary for tolerating my incessant interrogation.

References

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