POPULAR
SCIENTIFIC LECTURES

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THE ECONOMICAL NATURE OF PHYSICAL INQUIRY.*

WHEN the human mind, with its limited powers, attempts to mirror in itself the rich life of the world, of which it is itself only a small part, and which it can never hope to exhaust, it has every reason for proceeding economically. Hence that tendency, expressed in the philosophy of all times, to compass by a few organic thoughts the fundamental features of reality. "Life understands not death, nor death life." So spake an old Chinese philosopher. Yet in his unceasing desire to diminish the boundaries of the incomprehensible, man has always been engaged in attempts to understand death by life and life by death.

Among the ancient civilised peoples, nature was filled with demons and spirits having the feelings and desires of men. In all essential features, this animistic view of nature, as Tylor† has aptly termed it, is shared in common by the fetish-worshipper of modern Africa

* An address delivered before the anniversary meeting of the Imperial Academy of Sciences, at Vienna, May 25, 1882.

† *Primitive Culture.*

and the most advanced nations of antiquity. As a theory of the world it has never completely disappeared. The monotheism of the Christians never fully overcame it, no more than did that of the Jews. In the belief in witchcraft and in the superstitions of the sixteenth and seventeenth centuries, the centuries of the rise of natural science, it assumed frightful pathological dimensions. Whilst Stevinus, Kepler, and Galileo were slowly rearing the fabric of modern physical science, a cruel and relentless war was waged with firebrand and rack against the devils that glowered from every corner. To-day even, apart from all survivals of that period, apart from the traces of fetishism which still inhere in our physical concepts,* those very ideas still covertly lurk in the practices of modern spiritualism.

By the side of this animistic conception of the world, we meet from time to time, in different forms, from Democritus to the present day, another view, which likewise claims exclusive competency to comprehend the universe. This view may be characterised as the *physico-mechanical* view of the world. To-day, that view holds, indisputably, the first place in the thoughts of men, and determines the ideals and the character of our times. The coming of the mind of man into the full consciousness of its powers, in the eighteenth century, was a period of genuine disillusionment. It produced the splendid precedent of a life

* Tylor, *loc. cit.*

really worthy of man, competent to overcome the old barbarism in the practical fields of life; it created the *Critique of Pure Reason*, which banished into the realm of shadows the sham-ideas of the old metaphysics; it pressed into the hands of the mechanical philosophy the reins which it now holds.

The oft-quoted words of the great Laplace,* which I will now give, have the ring of a jubilant toast to the scientific achievements of the eighteenth century: "A mind to which were given for a single instant all the forces of nature and the mutual positions of all its masses, if it were otherwise powerful enough to subject these problems to analysis, could grasp, with a single formula, the motions of the largest masses as well as of the smallest atoms; nothing would be uncertain for it; the future and the past would lie revealed before its eyes." In writing these words, Laplace, as we know, had also in mind the atoms of the brain. That idea has been expressed more forcibly still by some of his followers, and it is not too much to say that Laplace's ideal is substantially that of the great majority of modern scientists.

Gladly do we accord to the creator of the *Mécanique céleste* the sense of lofty pleasure awakened in him by the great success of the Enlightenment, to which we too owe our intellectual freedom. But to-day, with minds undisturbed and before *new* tasks, it

* *Essai philosophique sur les probabilités*. 6th Ed. Paris, 1840, p. 4. The necessary consideration of the initial velocities is lacking in this formulation.

becomes physical science to secure itself against self-deception by a careful study of its character, so that it can pursue with greater sureness its true objects. If I step, therefore, beyond the narrow precincts of my specialty in this discussion, to trespass on friendly neighboring domains, I may plead in my excuse that the subject-matter of knowledge is common to all domains of research, and that fixed, sharp lines of demarcation cannot be drawn.

The belief in occult magic powers of nature has gradually died away, but in its place a new belief has arisen, the belief in the magical power of science. Science throws her treasures, not like a capricious fairy into the laps of a favored few, but into the laps of all humanity, with a lavish extravagance that no legend ever dreamt of! Not without apparent justice, therefore, do her distant admirers impute to her the power of opening up unfathomable abysses of nature, to which the senses cannot penetrate. Yet she who came to bring light into the world, can well dispense with the darkness of mystery, and with pompous show, which she needs neither for the justification of her aims nor for the adornment of her plain achievements.

The homely beginnings of science will best reveal to us its simple, unchangeable character. Man acquires his first knowledge of nature half-consciously and automatically, from an instinctive habit of mimicking and forecasting facts in thought, of supplementing sluggish experience with the swift wings of thought, at first only for his material welfare. When he hears a noise in the underbrush he constructs there, just as the animal does, the enemy which he fears ; when he sees a certain rind he forms mentally the image of the fruit which he is in search of ; just as we mentally associate a certain kind of matter with a certain line in the spectrum or an electric spark with the friction of a piece of glass. A knowledge of causality in this form certainly reaches far below the level of Schopenhauer's pet dog, to whom it was ascribed. It probably exists in the whole animal world, and confirms that great thinker's statement regarding the will which created the intellect for its purposes. These primitive psychical functions are rooted in the economy of our organism not less firmly than are motion and digestion. Who would deny that we feel in them, too, the elemental power of a long practised logical and physiological activity, bequeathed to us as an heirloom from our forefathers?

Such primitive acts of knowledge constitute to-day the solidest foundation of scientific thought. Our instinctive knowledge, as we shall briefly call it, by virtue of the conviction that we have consciously and intentionally contributed nothing to its formation, confronts us with an authority and logical power which consciously acquired knowledge even from familiar sources and of easily tested fallibility can never possess. All so-called axioms are such instinctive knowledge. Not consciously gained knowledge alone, but powerful intellectual instinct, joined with vast conceptive powers, constitute the great inquirer. The greatest advances of science have always consisted in some successful formulation, in clear, abstract, and communicable terms, of what was instinctively known long before, and of thus making it the permanent property of humanity. By Newton's principle of the equality of pressure and counterpressure, whose truth all before him had felt, but which no predecessor had abstractly formulated, mechanics was placed by a single stroke on a higher level. Our statement might also be historically justified by examples from the scientific labors of Stevinus, S. Carnot, Faraday, J. R. Mayer, and others.

All this, however, is merely the soil from which science starts. The first real beginnings of science appear in society, particularly in the manual arts, where the necessity for the communication of experience arises. Here, where some new discovery is to be described and related, the compulsion is first felt of clearly defining in consciousness the important and essential features of that discovery, as many writers can testify. The aim of instruction is simply the saving of experience; the labor of one man is made to take the place of that of another.

The most wonderful economy of communication is found in language. Words are comparable to type, which spare the repetition of written signs and thus serve a multitude of purposes; or to the few sounds of which our numberless different words are composed.

Language, with its helpmate, conceptual thought, by fixing the essential and rejecting the unessential, constructs its rigid pictures of the fluid world on the plan of a mosaic, at a sacrifice of exactness and fidelity but with a saving of tools and labor. Like a piano-player with previously prepared sounds, a speaker excites in his listener thoughts previously prepared, but fitting many cases, which respond to the speaker's summons with alacrity and little effort.

The principles which a prominent political economist, E. Hermann,* has formulated for the economy of the industrial arts, are also applicable to the ideas of common life and of science. The economy of language is augmented, of course, in the terminology of science. With respect to the economy of written intercourse there is scarcely a doubt that science itself will realise that grand old dream of the philosophers of a Universal Real Character. That time is not far distant. Our numeral characters, the symbols of mathematical analysis, chemical symbols, and musical notes, which might easily be supplemented by a system of color-signs, together with some phonetic alphabets now in use, are all beginnings in this direction. The logical extension of what we have, joined with a use of the ideas which the Chinese ideography furnishes us, will render the special invention and promulgation of a Universal Character wholly superfluous.

The communication of scientific knowledge always

* *Principien der Wirthschaftslehre*, Vienna, 1873.

involves description, that is, a mimetic reproduction of facts in thought, the object of which is to replace and save the trouble of new experience. Again, to save the labor of instruction and of acquisition, concise, abridged description is sought. This is really all that natural laws are. Knowing the value of the acceleration of gravity, and Galileo's laws of descent, we possess simple and compendious directions for reproducing in thought all possible motions of falling bodies. A formula of this kind is a complete substitute for a full table of motions of descent, because by means of the formula the data of such a table can be easily constructed at a moment's notice without the least burdening of the memory.

No human mind could comprehend all the individual cases of refraction. But knowing the index of refraction for the two media presented, and the familiar law of the sines, we can easily reproduce or fill out in thought every conceivable case of refraction. The advantage here consists in the disburdening of the memory; an end immensely furthered by the written preservation of the natural constants. More than this comprehensive and condensed report about facts is not contained in a natural law of this sort. In reality, the law always contains less than the fact itself, because it does not reproduce the fact as a whole but only in that aspect of it which is important for us, the rest being either intentionally or from necessity omitted. Natural laws may be likened to intellectual type of a higher order, partly movable, partly stereotyped, which last on new editions of experience may become downright impediments.

When we look over a province of facts for the first time, it appears to us diversified, irregular, confused, full of contradictions. We first succeed in grasping only single facts, unrelated with the others. The province, as we are wont to say, is not *clear*. By and by we discover the simple, permanent elements of the mosaic, out of which we can mentally construct the whole province. When we have reached a point where we can discover everywhere the same facts, we no longer feel lost in this province; we comprehend it without effort; it is *explained* for us.

Let me illustrate this by an example. As soon as we have grasped the fact of the rectilinear propagation of light, the regular course of our thoughts stumbles at the phenomena of refraction and diffraction. As soon as we have cleared matters up by our index of refraction we discover that a special index is necessary for each color. Soon after we have accustomed ourselves to the fact that light added to light increases its intensity, we suddenly come across a case of total darkness produced by this cause. Ultimately, however, we see everywhere in the overwhelming multifariousness of optical phenomena the fact of the spatial and temporal periodicity of light, with its velocity of propagation dependent on the medium and the period. This tendency of obtaining a survey of a given province with the least expenditure of thought, and of representing all its facts by some one single mental process, may be justly termed an economical one.

The greatest perfection of mental economy is attained in that science which has reached the highest formal development, and which is widely employed in physical inquiry, namely, in mathematics. Strange as it may sound, the power of mathematics rests upon its evasion of all unnecessary thought and on its wonderful saving of mental operations. Even those arrangement-signs which we call numbers are a system of marvellous simplicity and economy. When we employ the multiplication-table in multiplying numbers of several places, and so use the results of old operations of counting instead of performing the whole of each operation anew; when we consult our table of logarithms, replacing and saving thus new calculations by old ones already performed; when we employ determinants instead of always beginning afresh the solution of a system of equations; when we resolve new integral expressions into familiar old integrals; we see in this simply a feeble reflexion of the intellectual activity of a Lagrange or a Cauchy, who, with the keen discernment of a great military commander, substituted for new operations whole hosts of old ones. No one will dispute me when I say that the most elementary as well as the highest mathematics are economically-ordered experiences of counting, put in forms ready for use.

In algebra we perform, as far as possible, all numerical operations which are identical in form once for all, so that only a remnant of work is left for the individual case. The use of the signs of algebra and analysis, which are merely symbols of operations to be performed, is due to the observation that we can materially disburden the mind in this way and spare its powers for more important and more difficult duties, by imposing all mechanical operations upon the hand. One result of this method, which attests its economical character, is the construction of calculating machines. The mathematician Babbage, the inventor of the difference-engine, was probably the first who clearly perceived this fact, and he touched upon it, although only cursorily, in his work, *The Economy of Manufactures and Machinery.*

The student of mathematics often finds it hard to throw off the uncomfortable feeling that his science, in the person of his pencil, surpasses him in intelligence, —an impression which the great Euler confessed he often could not get rid of. This feeling finds a sort of justification when we reflect that the majority of the ideas we deal with were conceived by others, often centuries ago. In great measure it is really the intelligence of other people that confronts us in science. The moment we look at matters in this light, the uncanniness and magical character of our impressions cease, especially when we remember that we can think over again at will any one of those alien thoughts.

Physics is experience, arranged in economical order. By this order not only is a broad and comprehensive view of what we have rendered possible, but also the defects and the needful alterations are made manifest, exactly as in a well-kept household. Physics shares with mathematics the advantages of succinct description and of brief, compendious definition, which precludes confusion, even in ideas where, with no apparent burdening of the brain, hosts of others are contained. Of these ideas the rich contents can be produced at any moment and displayed in their full perceptual light. Think of the swarm of well-ordered notions pent up in the idea of the potential. Is it wonderful that ideas containing so much finished labor should be easy to work with?

Our first knowledge, thus, is a product of the economy of self-preservation. By communication, the experience of *many* persons, individually acquired at first, is collected in *one*. The communication of knowledge and the necessity which every one feels of managing his stock of experience with the least expenditure of thought, compel us to put our knowledge in economical forms. But here we have a clue which strips science of all its mystery, and shows us what its power really is. With respect to specific results it yields us nothing that we could not reach in a sufficiently long time without methods. There is no problem in all mathematics that cannot be solved by direct counting. But with the present implements of mathematics many operations of counting can be performed in a few minutes which without mathematical methods would take a lifetime. Just as a single human being, restricted wholly to the fruits of his own labor, could never amass a fortune, but on the contrary the accumulation of the labor of many men in the hands of one is the foundation of wealth and power, so, also, no knowledge worthy of the name can be gathered up in a single human mind limited to the span of a human life and gifted only with finite powers, except by the most exquisite economy of thought and by the careful amassment of the economically ordered experience of thousands of co-workers. What strikes us here as the fruits of sorcery are simply the rewards of excellent housekeeping, as are the like results in civil life. But the business of science has this advantage over every other enterprise, that from *its* amassment of wealth no one suffers the least loss. This, too, is its blessing, its freeing and saving power.

The recognition of the economical character of science will now help us, perhaps, to understand better certain physical notions.

Those elements of an event which we call "cause and effect" are certain salient features of it, which are important for its mental reproduction. Their importance wanes and the attention is transferred to fresh characters the moment the event or experience in question becomes familiar. If the connexion of such features strikes us as a necessary one, it is simply because the interpolation of certain intermediate links with which we are very familiar, and which possess, therefore, higher authority for us, is often attended with success in our explanations. That *ready* experience fixed in the mosaic of the mind with which we meet new events, Kant calls an innate concept of the understanding (*Verstandesbegriff*).

The grandest principles of physics, resolved into their elements, differ in no wise from the descriptive principles of the natural historian. The question, "Why?" which is always appropriate where the explanation of a contradiction is concerned, like all proper habitudes of thought, can overreach itself and be asked where nothing remains to be understood. Suppose we were to attribute to nature the property of producing like effects in like circumstances; just these like circumstances we should not know how to find. Nature exists once only. Our schematic mental imitation alone produces like events. Only in the mind, therefore, does the mutual dependence of certain features exist.

All our efforts to mirror the world in thought would be futile if we found nothing permanent in the varied changes of things. It is this that impels us to form the notion of substance, the source of which is not different from that of the modern ideas relative to the conservation of energy. The history of physics furnishes numerous examples of this impulse in almost all fields, and pretty examples of it may be traced back to the nursery. "Where does the light go to when it is put out?" asks the child. The sudden shrivelling up of a hydrogen balloon is inexplicable to a child; it looks everywhere for the large body which was just there but is now gone.

Where does heat come from? Where does heat go to? Such childish questions in the mouths of mature men shape the character of a century.

In mentally separating a body from the changeable environment in which it moves, what we really do is to extricate a group of sensations on which our thoughts are fastened and which is of relatively greater stability than the others, from the stream of all our sensations. Absolutely unalterable this group is not. Now this, now that member of it appears and disappears, or is altered. In its full identity it never recurs. Yet the sum of its constant elements as compared with the sum of its changeable ones, especially if we consider the continuous character of the transition, is always so great that for the purpose in hand the former usually appear sufficient to determine the body's identity. But because we can separate from the group every single member without the body's ceasing to be for us the same, we are easily led to believe that after abstracting all the members something additional would remain. It thus comes to pass that we form the notion of a substance distinct from its attributes, of a thing-in-itself, whilst our sensations are regarded merely as symbols or indications of the properties of this thing-in-itself. But it would be much better to say that bodies or things are compendious mental symbols for groups of sensations—symbols that do not exist outside of thought. Thus, the merchant regards the labels of his boxes merely as indexes of their contents, and not the contrary. He invests their contents, not their labels, with real value. The same economy which induces us to analyse a group and to establish special signs for its component parts, parts which also go to make up other groups, may likewise induce us to mark out by some single symbol a whole group.

On the old Egyptian monuments we see objects represented which do not reproduce a single visual impression, but are composed of various impressions. The heads and the legs of the figures appear in profile, the head-dress and the breast are seen from the front, and so on. We have here, so to speak, a mean view of the objects, in forming which the sculptor has retained what he deemed essential, and neglected what he thought indifferent. We have living exemplifications of the processes put into stone on the walls of these old temples, in the drawings of our children, and we also observe a faithful analogue of them in the formation of ideas in our own minds. Only in virtue of some such facility of view as that indicated, are we allowed to speak of *a* body. When we speak of a cube with trimmed corners—a figure which is not a cube—we do so from a natural instinct of economy, which prefers to add to an old familiar conception a correction instead of forming an entirely new one. This is the process of all judgment.

The crude notion of "body" can no more stand the test of analysis than can the art of the Egyptians or that of our little children. The physicist who sees a body flexed, stretched, melted, and vaporised, cuts up this body into smaller permanent parts; the chemist splits it up into elements. Yet even an element is not unalterable. Take sodium. When warmed, the white, silvery mass becomes a liquid, which, when the heat is increased and the air shut out, is transformed into a violet vapor, and on the heat being still more increased glows with a yellow light. If the name sodium is still retained, it is because of the continuous character of the transitions and from a necessary instinct of economy. By condensing the vapor, the white metal may be made to reappear. Indeed, even after the metal is thrown into water and has passed into sodium hydroxide, the vanished properties may by skilful treatment still be made to appear; just as a moving body which has passed behind a column and is lost to view for a moment may make its appearance after a time. It is unquestionably very convenient always to have ready the name and thought for a group of properties wherever that group by any possibility can appear. But more than a compendious economical symbol for these phenomena, that name and thought is not. It would be a mere empty word for one in whom it did not awaken a large group of well-ordered sense-impressions. And the same is true of the molecules and atoms into which the chemical element is still further analysed.

True, it is customary to regard the conservation of weight, or, more precisely, the conservation of mass, as a direct proof of the constancy of matter. But this proof is dissolved, when we go to the bottom of it, into such a multitude of instrumental and intellectual operations, that in a sense it will be found to constitute simply an equation which our ideas in imitating facts have to satisfy. That obscure, mysterious lump which we involuntarily add in thought, we seek for in vain outside the mind.

It is always, thus, the crude notion of substance that is slipping unnoticed into science, proving itself constantly insufficient, and ever under the necessity of being reduced to smaller and smaller world-particles. Here, as elsewhere, the lower stage is not rendered indispensable by the higher which is built upon it, no more than the simplest mode of locomotion, walking, is rendered superfluous by the most elaborate means of transportation. Body, as a compound of light and touch sensations, knit together by sensations of space, must be as familiar to the physicist who seeks it, as to the animal who hunts its prey. But the student of the theory of knowledge, like the geologist and the astronomer, must be permitted to reason back from the forms which are created before his eyes to others which he finds ready made for him.

All physical ideas and principles are succinct directions, frequently involving subordinate directions, for the employment of economically classified experiences, ready for use. Their conciseness, as also the fact that their contents are rarely exhibited in full, often invests them with the semblance of independent existence. Poetical myths regarding such ideas,—for example, that of Time, the producer and devourer of all things,—do not concern us here. We need only remind the reader that even Newton speaks of an *absolute* time independent of all phenomena, and of an absolute space—views which even Kant did not shake off, and which are often seriously entertained to-day. For the natural inquirer, determinations of time are merely abbreviated statements of the dependence of one event upon another, and nothing more. When we say the acceleration of a freely falling body is 9·810 metres per second, we mean the velocity of the body with respect to the centre of the earth is 9·810 metres greater when the earth has performed an additional 86400th part of its rotation—a fact which itself can be determined only by the earth's relation to other heavenly bodies. Again, in velocity is contained simply a relation of the position of a body to the position of the earth.* Instead of referring events to the earth we may refer them to a clock, or even to our internal sensation of time. Now, because all are connected,

* It is clear from this that all so-called elementary (differential) laws involve a relation to the Whole.

and each may be made the measure of the rest, the illusion easily arises that time has significance independently of all.*

The aim of research is the discovery of the equations which subsist between the elements of phenomena. The equation of an ellipse expresses the universal *conceivable* relation between its co-ordinates, of which only the real values have *geometrical* significance. Similarly, the equations between the elements of *phenomena* express a universal, mathematically conceivable relation. Here, however, for many values only certain directions of change are *physically* admissible. As in the ellipse only certain *values* satisfying the equation are realised, so in the physical world only certain *changes* of value occur. Bodies are always accelerated towards the earth. Differences of temperature, left to themselves, always grow less; and so on. Similarly, with respect to space, mathematical and physiological researches have shown that the space of experience is simply an *actual* case of many conceivable cases, about whose peculiar properties experience alone can instruct us. The elucidation which this idea diffuses cannot be questioned, despite the absurd uses to which it has been put.

Let us endeavor now to summarise the results of

* If it be objected, that in the case of perturbations of the velocity of rotation of the earth, we could be sensible of such perturbations, and being obliged to have some measure of time, we should resort to the period of vibration of the waves of sodium light,—all that this would show is that for practical reasons we should select that event which best served us as the *simplest* common measure of the others.

our survey. In the economical schematism of science lie both its strength and its weakness. Facts are always represented at a sacrifice of completeness and never with greater precision than fits the needs of the moment. The incongruence between thought and experience, therefore, will continue to subsist as long as the two pursue their course side by side; but it will be continually diminished.

In reality, the point involved is always the completion of some partial experience; the derivation of one portion of a phenomenon from some other. In this act our ideas must be based directly upon sensations. We call this measuring.* The condition of science, both in its origin and in its application, is a *great relative stability* of our environment. What it teaches us is interdependence. Absolute forecasts, consequently, have no significance in science. With great changes in celestial space we should lose our co-ordinate systems of space and time.

When a geometer wishes to understand the form of a curve, he first resolves it into small rectilinear elements. In doing this, however, he is fully aware that these elements are only provisional and arbitrary devices for comprehending in parts what he cannot comprehend as a whole. When the law of the curve is found he no longer thinks of the elements. Similarly, it would not become physical science to see in its self-

* Measurement, in fact, is the definition of one phenomenon by another (standard) phenomenon.

created, changeable, economical tools, molecules and atoms, realities behind phenomena, forgetful of the lately acquired sapience of her older sister, philosophy, in substituting a mechanical mythology for the old animistic or metaphysical scheme, and thus creating no end of suppositious problems. The atom must remain a tool for representing phenomena, like the functions of mathematics. Gradually, however, as the intellect, by contact with its subject-matter, grows in discipline, physical science will give up its mosaic play with stones and will seek out the boundaries and forms of the bed in which the living stream of phenomena flows. The goal which it has set itself is the *simplest* and *most economical* abstract expression of facts.

* * *

The question now remains, whether the same method of research which till now we have tacitly restricted to physics, is also applicable in the psychical domain. This question will appear superfluous to the physical inquirer. Our physical and psychical views spring in exactly the same manner from instinctive knowledge. We read the thoughts of men in their acts and facial expressions without knowing how. Just as we predict the behavior of a magnetic needle placed near a current by imagining Ampère's swimmer in the current, similarly we predict in thought the acts and behavior of men by assuming sensations, feelings, and wills similar to our own connected with their bodies. What we here instinctively perform would appear to us as one of the subtlest achievements of science, far outstripping in significance and ingenuity Ampère's rule of the swimmer, were it not that every child unconsciously accomplished it. The question simply is, therefore, to grasp scientifically, that is, by conceptional thought, what we are already familiar with from other sources. And here much is to be accomplished. A long sequence of facts is to be disclosed between the physics of expression and movement and feeling and thought.

We hear the question, "But how is it possible to explain feeling by the motions of the atoms of the brain?" Certainly this will never be done, no more than light or heat will ever be deduced from the law of refraction. We need not deplore, therefore, the lack of ingenious solutions of this question. The problem is not a problem. A child looking over the walls of a city or of a fort into the moat below sees with astonishment living people in it, and not knowing of the portal which connects the wall with the moat, cannot understand how they could have got down from the high ramparts. So it is with the notions of physics. We cannot climb up into the province of psychology by the ladder of our abstractions, but we can climb down into it.

Let us look at the matter without bias. The world consists of colors, sounds, temperatures, pressures, spaces, times, and so forth, which now we shall not call sensations, nor phenomena, because in either term an arbitrary, one-sided theory is embodied, but simply *elements*. The fixing of the flux of these elements, whether mediately or immediately, is the real object of physical research. As long as, neglecting our own body, we employ ourselves with the interdependence of those groups of elements which, including men and animals, make up *foreign* bodies, we are physicists. For example, we investigate the change of the red color of a body as produced by a change of illumination. But the moment we consider the special influence on the red of the elements constituting our body, outlined by the well-known perspective with head invisible, we are at work in the domain of physiological psychology. We close our eyes, and the red together with the whole visible world disappears. There exists, thus, in the perspective field of every sense a portion which exercises on all the rest a different and more powerful influence than the rest upon one another. With this, however, all is said. In the light of this remark, we call *all* elements, in so far as we regard them as dependent on this special part (our body), *sensations*. That the world is our sensation, in this sense, cannot be questioned. But to make a system of conduct out of this provisional conception, and to abide its slaves, is as unnecessary for us as would be a similar course for a mathematician who, in varying a series of variables of a function which were previously assumed to be constant, or in interchanging the independent variables, finds his method to be the source of some very surprising ideas for him.*

If we look at the matter in this unbiassed light it will appear indubitable that the method of physiological psychology is none other than that of physics; what is more, that this science is a part of physics. Its subject-matter is not different from that of physics. It will unquestionably determine the relations the sensations bear to the physics of our body. We have already learned from a member of this academy (Hering) that in all probability a sixfold manifoldness of the chemical processes of the visual substance corresponds to the sixfold manifoldness of color-sensation, and a threefold manifoldness of the physiological processes to the threefold manifoldness of space-sensations. The paths of reflex actions and of the will are followed up and disclosed; it is ascertained what region of the brain subserves the function of speech, what region the function of locomotion, etc. That which still clings to our body, namely, our thoughts, will, when those investigations are finished, present no difficulties new in principle. When experience has once clearly exhibited these facts and science has

*I have represented the point of view here taken for more than thirty years and developed it in various writings (*Erhaltung der Arbeit*, 1872, parts of which are published in the article on *The Conservation of Energy* in this collection; *The Forms of Liquids*, 1872, also published in this collection; and the *Bewegungsempfindungen*, 1875). The idea, though known to philosophers, is unfamiliar to the majority of physicists. It is a matter of deep regret to me, therefore, that the title and author of a small tract which accorded with my views in numerous details and which I remember having caught a glance of in a very busy period (1879–1880), have so completely disappeared from my memory that all efforts to obtain a clue to them have hitherto been fruitless.

marshalled them in economic and perspicuous order, there is no doubt that we shall *understand* them. For other "understanding" than a mental mastery of facts never existed. Science does not create facts from facts, but simply *orders* known facts.

Let us look, now, a little more closely into the modes of research of physiological psychology. We have a very clear idea of how a body moves in the space encompassing it. With our optical field of sight we are very familiar. But we are unable to state, as a rule, how we have come by an idea, from what corner of our intellectual field of sight it has entered, or by what region the impulse to a motion is sent forth. Moreover, we shall never get acquainted with this mental field of view from self-observation alone. Self-observation, in conjunction with physiological research, which seeks out physical connexions, can put this field of vision in a clear light before us, and will thus first really reveal to us our inner man.

Primarily, natural science, or physics, in its widest sense, makes us acquainted with only the firmest connexions of groups of elements. Provisorily, we may not bestow too much attention on the single constituents of those groups, if we are desirous of retaining a comprehensible whole. Instead of equations between the primitive variables, physics gives us, as much the easiest course, equations between *functions* of those variables. Physiological psychology teaches us how to separate the visible, the tangible, and the audible from bodies—a labor which is subsequently richly requited, as the division of the subjects of physics well shows. Physiology further analyses the visible into light and space sensations; the first into colors, the last also into their component parts; it resolves noises into sounds, these into tones, and so on. Unquestionably this analysis can be carried much further than it has been. It will be possible in the end to exhibit the common elements at the basis of very abstract but definite logical acts of like form,—elements which the acute jurist and mathematician, as it were, *feels* out, with absolute certainty, where the uninitiated hears only empty words. Physiology, in a word, will reveal to us the true real elements of the world. Physiological psychology bears to physics in its widest sense a relation similar to that which chemistry bears to physics in its narrowest sense. But far greater than the mutual support of physics and chemistry will be that which natural science and psychology will render each other. And the results that shall spring from this union will, in all likelihood, far outstrip those of the modern mechanical physics.

What those ideas are with which we shall comprehend the world when the closed circuit of physical and psychological facts shall lie complete before us, (that circuit of which we now see only two disjoined parts,) cannot be foreseen at the outset of the work. The men will be found who will see what is right and will have the courage, instead of wandering in the intricate paths of logical and historical accident, to enter on the straight ways to the heights from which the mighty stream of facts can be surveyed. Whether the notion which we now call matter will continue to have a scientific significance beyond the crude purposes of common life, we do not know. But we certainly shall wonder how colors and tones which were such innermost parts of us could suddenly get lost in our physical world of atoms; how we could be suddenly surprised that something which outside us simply clicked and beat, in our heads should make light and music; and how we could ask whether matter can feel, that is to say, whether a mental symbol for a group of sensations can feel?

We cannot mark out in hard and fast lines the science of the future, but we can foresee that the rigid walls which now divide man from the world will gradually disappear; that human beings will not only confront each other, but also the entire organic and so-called lifeless world, with less selfishness and with livelier sympathy. Just such a presentiment as this perhaps possessed the great Chinese philosopher Licius some two thousand years ago when, pointing to a heap of mouldering human bones, he said to his scholars in the rigid, lapidary style of his tongue: "These and I alone have the knowledge that we neither live nor are dead."