Energy and Momentum of a Tachyon

The basic relativistic relations for the energy E and momentum p of a mass m moving at speed v are:

Energy = mass x c^2 $E = m\,c^2$ Momentum = mass x velocity $p = mv$

The mass m increases with speed v according to $m = m_0/\sqrt{1 - v^2/c^2}$, where m_0 is the rest mass of the body; that is, it is the mass of the body when it is at rest.

For normal masses moving at less than the speed of light, the energy and momentum are

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} \quad \text{and } p = mv = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

These formulae also apply to tachyons. However, to ensure that the energy and momentum are both real numbers, we need to assign an imaginary rest mass to the tachyon of $i\,m_0$, where $i = \sqrt{-1}$. This might seem odd. It works if we remember that the tachyon never comes to rest, so it never exhibits a bare imaginary mass.

For the tachyon, $v > c$. Hence, we rewrite the relativistic factor as

$$\sqrt{1 - v^2/c^2} = \sqrt{-1}\sqrt{\frac{v^2}{c^2} - 1} = i\sqrt{\frac{v^2}{c^2} - 1}$$

The tachyon energy is then

$$E = mc^2 = \frac{i\,m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{i\,m_0 c^2}{i\sqrt{\frac{v^2}{c^2} - 1}} = \frac{m_0 c^2}{\sqrt{\frac{v^2}{c^2} - 1}}$$

It follows that the energy E of the tachyon has limiting values:

$$E = \frac{m_0 c^2}{\sqrt{\frac{v^2}{c^2}-1}} = \frac{m_0 c^2}{\infty} = 0 \text{ when } v = \infty \quad \text{and} \quad E = \frac{m_0 c^2}{\sqrt{\frac{v^2}{c^2}-1}} \to \infty \;\; v \to c.$$

That is, the tachyon has least energy when it moves infinitely fast; and it slows arbitrarily close to the speed of light c when it gains energy.

The corresponding relations for momentum are

$$p = \frac{m_0 v}{\sqrt{\frac{v^2}{c^2}-1}} \sim \frac{m_0 v}{v/c} = m_0 c \text{ when } v \to \infty \quad \text{and} \quad p = \frac{m_0 v}{\sqrt{\frac{v^2}{c^2}-1}} \to \infty \text{ as } v \to c.$$