Einstein's A&B coefficient derivation of the Blackbody radiation law

I Derivation based on a non-classical, probabilistic treatment of energy transfers between matter & radiation — To be treated here

II Demonstrates that emission/absorption of radiant energy $h\nu$ by matter is accompanied by a momentum gain/loss of $h\nu/c$

Molecules — interacts with — Radiation

States

Energy ε_m

Energy ε_n

Probability

W_n

$= p_n \exp\left(-\frac{\varepsilon_n}{kT}\right)$

"weight of the state"

Energy density at ν

$\rho(\nu)$

Processes

Probability in time

spontaneous emission — ε_m → ε_n — energy $\varepsilon_m - \varepsilon_n$

$$dW = A_m^n\, dt$$

stimulated emission — ε_m → ε_n — energy $\varepsilon_m - \varepsilon_n$

$$dW = B_m^n\, \rho\, dt$$

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Absorption — ε_n → ε_m — energy $\varepsilon_m - \varepsilon_n$

$$dW = B_n^m\, \rho\, dt$$

Notation: Since $\varepsilon_m > \varepsilon_n$, B_m^n and B_n^m represent very different processes Beware!

Condition for Equilibrium

Rate transitions $n \to m$ in time dt	=	Rate transitions $m \to n$ in time dt
$W_n \cdot B_n^m \rho$		$W_m (B_m^n \rho + A_m^n)$

$$\boxed{\therefore\ p_n \left(\exp\left(-\frac{\varepsilon_n}{kT}\right)\right) B_n^m \rho = p_m \exp\left(-\frac{\varepsilon_m}{kT}\right)\left(B_m^n \rho + A_m^n\right)}$$

a little manipulation

At high T, $\rho \to \infty$, $\exp\left(-\frac{\varepsilon_m}{kT}\right) \to 1$

$\therefore$ Equality requires $p_n B_n^m = p_m B_m^n$

$$\rho = \frac{A_m^n / B_m^n}{\exp\left[\underbrace{\frac{\varepsilon_m - \varepsilon_n}{kT}}\right] - 1}$$

$\alpha \nu^3$ some constant α

$\varepsilon_m - \varepsilon_n = h\nu$ some constant h

Wien displacement law

$$\rho_\nu(T) = \nu^3 f\left(\frac{\nu}{T}\right)$$