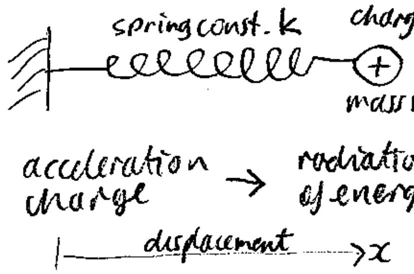
Use Correspondence Principle to recover Einstein's A & B coefficients for special case of linear harmonic oscillator

classical model



resonant frequency $\omega_0 = \sqrt{k/m}$

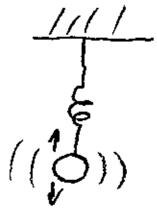
acceleration charge → radiation of energy → damping of motion

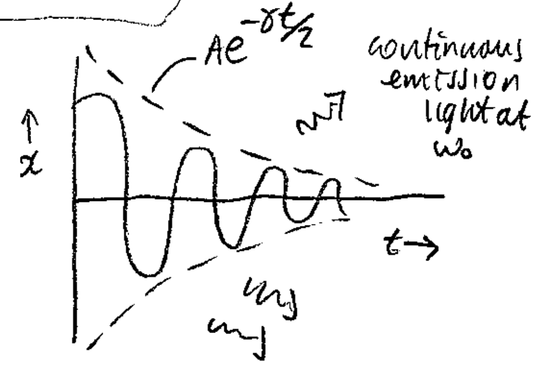
Radiative damping ⟹ Spontaneous emission ⟹ A coefficient

classical model

$$x = \underbrace{\left(Ae^{-\gamma t/2}\right)}_{\text{amplitude}} \underbrace{e^{-i\omega_0 t}}_{\text{only Fourier component in frequency } \omega_0}$$

For $\gamma = \frac{2e^2\omega_0^2}{3mc^2}$ SMALL

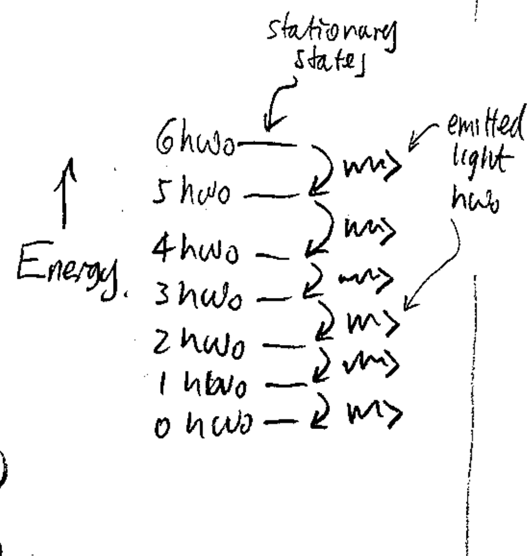




** [Corresponce principle] **

Quantum model

Only allowed emissions of frequency ω_0 → Energy $\hbar\omega_0$ with prob fixed by $\underbrace{\text{Amplitude } \left(Ae^{-\gamma t/2}\right)}_{\text{hence (see below)}}$



$$dW = A^n_{n+1}\, dt \quad \text{with} \quad A^n_{n+1} = n\gamma = \frac{2ne^2\omega_0^2}{3mc^2}$$

Prob. transition Energy $E = (n+1)\hbar\omega_0$ to $E = n\hbar\omega_0$

Details of recovery of $A_{n+1}^{n} = n\gamma$ from amplitude $Ae^{-\gamma t/2}$

For large n, A_{n+1}^{n} emissions must give classical emissions.

Classical model $x(t) = \underbrace{\left(Ae^{-\gamma t/2}\right)}_{\text{max. amplitude of oscillation}} e^{-i\omega_0 t}$

$\therefore$ Energy of oscillator $E = \frac{1}{2} k \begin{pmatrix} \text{max} \\ \text{amp} \end{pmatrix}^2 = \frac{1}{2} k \left(Ae^{-\frac{\gamma t}{2}}\right)^2$

$= \frac{1}{2} kA^2 e^{-\gamma t}$

Rate loss of energy $\frac{dE}{dt} = -\frac{1}{2} kA^2 e^{-\gamma t} \gamma = -E\gamma$

But $\underbrace{-\overline{dE}}_{\text{mean energy lost in time } dt} = \hbar\omega_0 \underbrace{dW}_{\text{probability of emission } \hbar\omega_0 \text{ in time } dt}$

$\therefore \quad \hbar\omega_0 \frac{dW}{dt} = E\gamma = \overbrace{(n\hbar\omega_0)}^{E}\gamma$

$\therefore \quad dW = \underbrace{n\gamma}_{\text{this must be } A_n^{n-1}} dt$

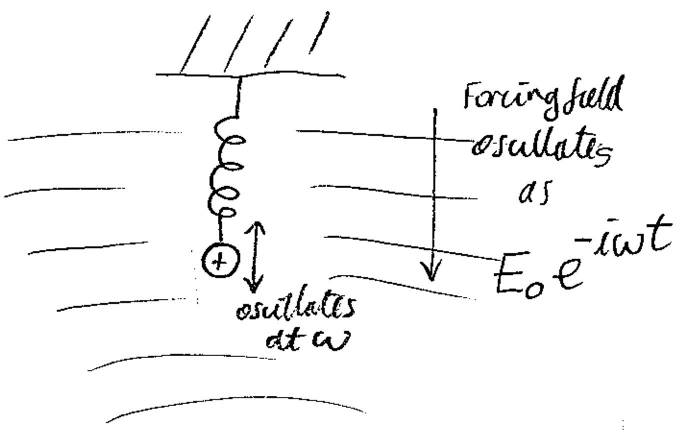
Electromagnetically forced oscillations of radiation damped linear harmonic oscillator $\Rightarrow$ stimulated emission and absorption $\xrightarrow[\text{(not shown here)}]{\text{eventually}}$ B coefficient

Classical model

Equilibrium oscillation

$$x(t) \doteq \underbrace{\left(\frac{e}{m}\cdot\frac{E_0}{\omega_0^2-\omega^2}\right)}_{\text{equilibrium amplitude}} \underbrace{e^{i(\omega t+\phi)}}_{\text{only Fourier component at forced frequency } \omega}$$

phase shift



** [correspondence principle] **

Quantum model

Only allowed emissions & absorptions of frequency ω $\rightarrow$ energy $\hbar\omega$

with prob. fixed by

Need to sustain amplitude of $x(t)$ at $\frac{e}{m}\cdot\frac{E_0}{\omega_0^2-\omega^2}$

Recover B coefficients from this condition.

Ask: which coefficients B yield classical equilibrium energy $E = \frac{1}{2}k\left(\frac{e}{m}\cdot\frac{E_0}{\omega_0^2-\omega^2}\right)^2$ for oscillator

I HAVEN'T TRIED THE CALCULATION!