

Hempel's Satisfaction Criterion of Confirmation

Inductive Generalization : Instance $\xrightarrow{\text{confirms}}$ Generalization

Enumerative Induction : some A's are B. \longrightarrow All A's are B.

Restricted to propositions of syllogistic logic

A: All A's are B.

I: Some A's are B.

E: No A's are B.

O: Some A's are not-B.

Extend to cases like

$$(2 = 1 + 1)$$

$$4 = 1 + 3$$

$$6 = 1 + 5$$

$$8 = 3 + 5$$

$$10 = 3 + 7$$

⋮

instances

$\xrightarrow{\text{confirms}}$

Göddbach's conjecture.

All even numbers are the sum of two primes

generalization

... Extend to predicate logic.

First pass : Examples of instances in predicate logic

$$P(a) \& P(b) \& P(c) \xrightarrow{\text{In domain } \{a, b, c\}} (\forall x) P(x) \quad \text{Rule A}$$

$$P(a) \vee P(b) \vee P(c) \xrightarrow{\text{In domain } \{a, b, c\}} (\exists x) P(x) \quad \text{Rule E}$$

Instance

Generalization

↑
 what generalization H would say if {a, b, c} were only individuals.

Development of H with respect to class {a, b, c}

≈



Hempel's technical innovation = Give a general definition for development.

General Recursive Definition of Development

1. Go to deepest level of nesting & apply Rule A and Rule E

2. Repeat at all higher levels

Example

$$H: (x) \left[\underbrace{(y) \text{ Knows}(x,y)}_{\text{Knows}(x,a) \ \& \ \text{Knows}(x,b)} \supset \underbrace{(\exists z) \text{ Likes}(x,z)}_{\text{Likes}(x,a) \ \vee \ \text{Likes}(x,b)} \right]$$

Develop for domain $\{a,b\}$

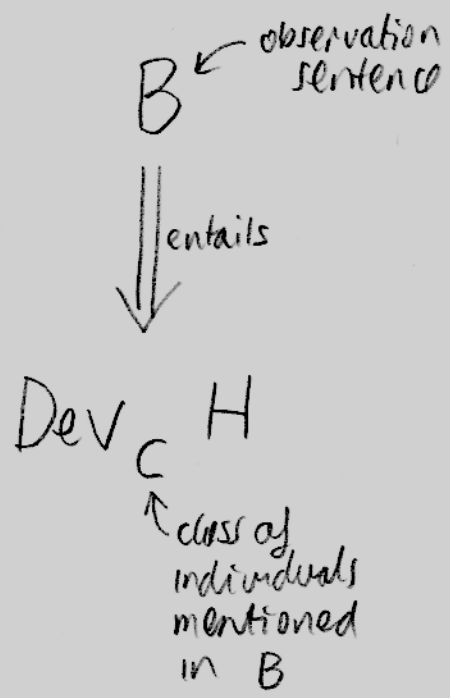
$$\left[\begin{array}{l} \left[\text{knows}(a,a) \ \& \ \text{knows}(a,b) \right] \supset \left[\text{Likes}(a,a) \ \vee \ \text{Likes}(a,b) \right] \\ \& \left[\text{knows}(b,a) \ \& \ \text{knows}(b,b) \right] \supset \left[\text{Likes}(b,a) \ \vee \ \text{Likes}(b,b) \right] \end{array} \right]$$

"Dev_{a,b} H"



The Satisfaction Criterion

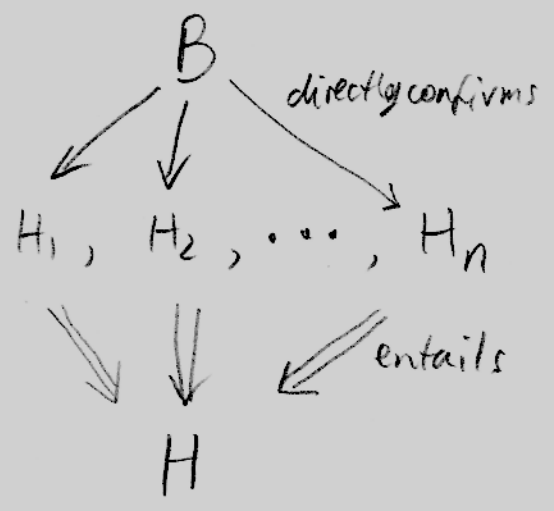
"Directly confirms"



The obvious definition

$$\begin{aligned}
 & \text{Raven}(a) \ \& \ \text{Black}(a) \\
 & \Downarrow \\
 & \text{Raven}(a) \supset \text{Black}(a) \\
 & = \text{Dev}_{\{a\}}(x) (\text{Raven}(x) \supset \text{Black}(x))
 \end{aligned}$$

"confirms"



Needed for obvious extension

$$\begin{aligned}
 & \text{Raven}(a) \ \& \ \text{Black}(a) \\
 & \Downarrow \\
 & (x) (\text{Raven}(x) \supset \text{Black}(x)) \\
 & \Downarrow \\
 & \text{Raven}(b) \supset \text{Black}(b)
 \end{aligned}$$

... Plus more clauses needed
to block obvious perversions

Tautology $\xrightarrow{\text{confirms}}$ $(x) P(x)$
 $P(a) \vee \sim P(a)$ $\vee (x) (\sim P(x))$

$= \text{Ded}_{\{a\}} (x) P(x) \vee (x) (\sim P(x))$

etc.

More serious problem:
(Evidence in finite domain) cannot confirm (Hypothesis that entails infinitely many individuals)

Finite Evidence

Hypothesis

$2 \rightarrow 3$

$(x) (\text{Prime}(x) \supset$

$3 \rightarrow 5$

$\exists y \text{ Prime}(y) \ \& \ \text{Greater}(y, x))$

$5 \rightarrow 7$

$7 \rightarrow 11$

$11 \rightarrow 13$

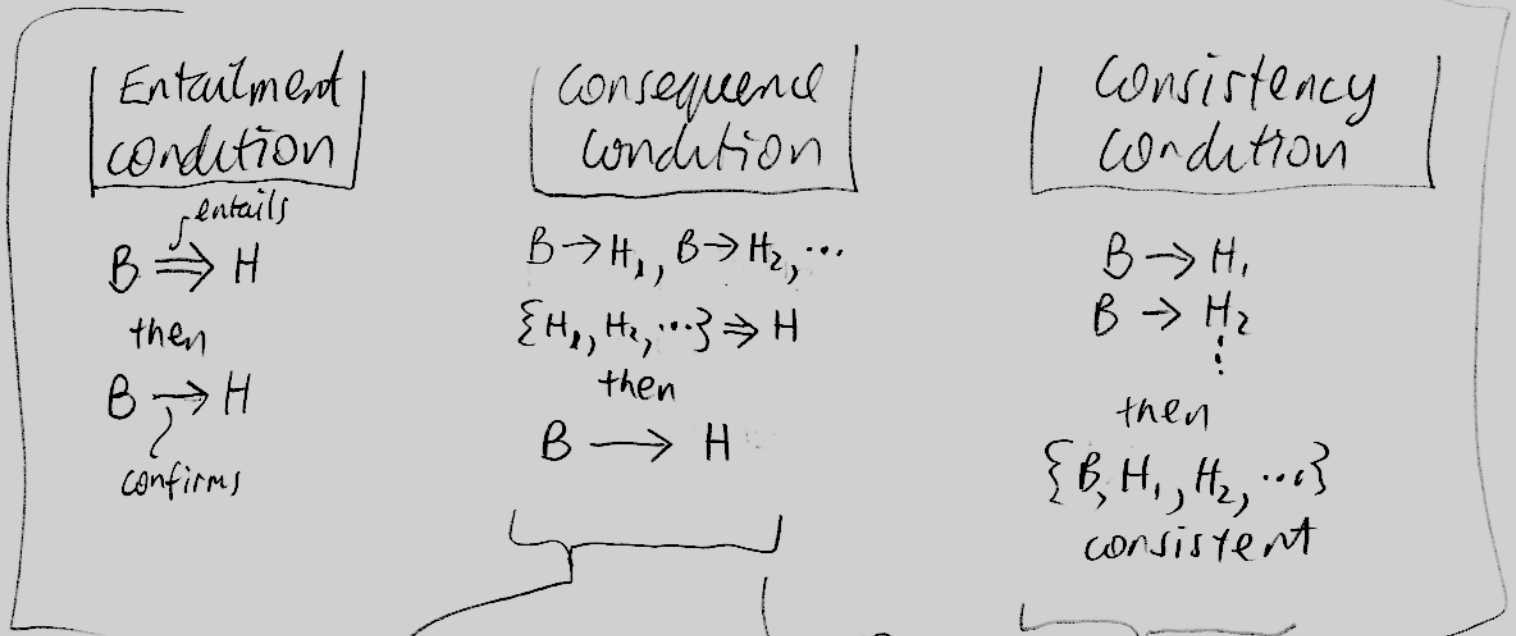
\vdots

$101 \rightarrow 103$

Hempel's conditions of Adequacy

responded to

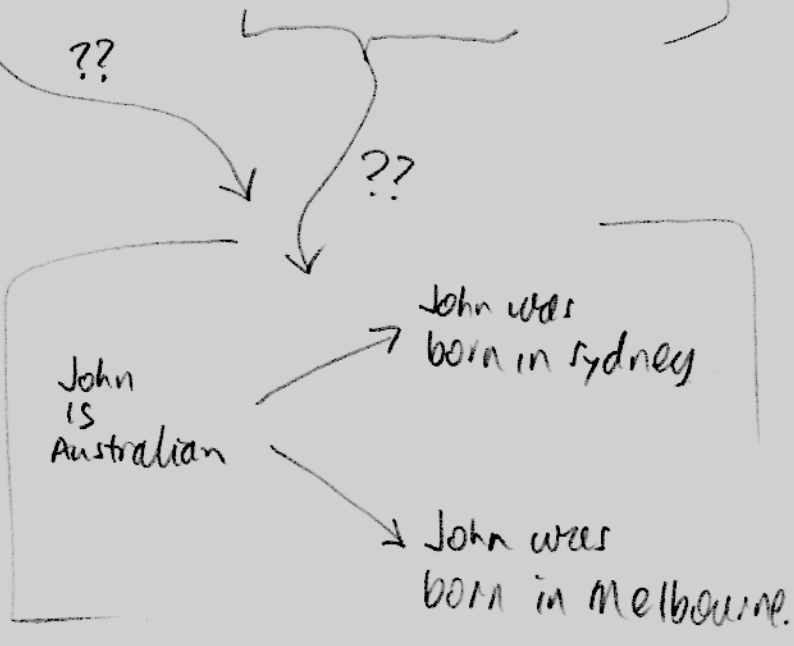
"Nirad's Criterion" $P(a) \& Q(a)$ confirms $(x) P(x) \supset Q(x)$
 (... but not logically equivalent)
 $(x) (\sim Q(x) \supset \sim P(x))$



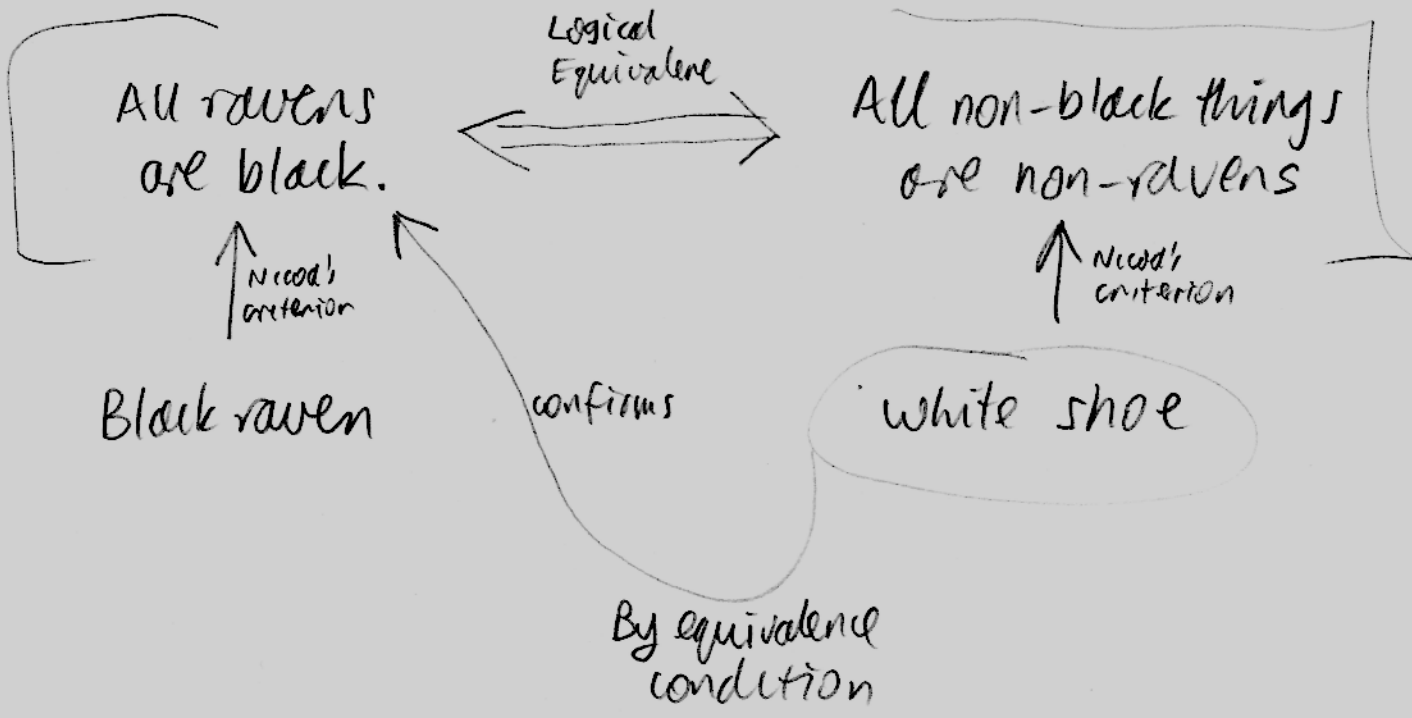
Equivalence condition

$$\begin{array}{l}
 B \rightarrow H_1 \\
 H_1 \iff H_2 \\
 \hline
 B \rightarrow H_2
 \end{array}$$

Paradox of the ravens



Paradox of the Ravens



Mool:

- Hempel's original goal:
Graft a probabilistic theory of confirmation onto the satisfaction criterion.
- Goodman's grue was taken to show that a purely syntactic relation of confirmation must fail