Hempel's Satisfaction Criterion of Confirmation

Inductive Generalization: Instance $\xrightarrow{\text{confirms}}$ Generalization

Enumerative Induction: Some As are B. $\longrightarrow$ All A's are B.

Restricted to propositions of syllogistic logic

A: All A's are B.
I: Some A's are B.
E No A's are B
O: Some A's are not-B

Extend to cases like

(2 = 1+1)
4 = 1+3
6 = 1+5
8 = 3+5
10 = 3+7
⋮

instances

confirms →

Goldbach's conjecture.
All even numbers are the sum of two primes

generalization

... Extend to predicate logic.

First pass : Examples of instances in predicate logic

$$P(a) \,\&\, P(b) \,\&\, P(c) \xrightarrow{\text{In domain } \{a,b,c\}} (x)\,P(x) \qquad \text{Rule A}$$

$$\underbrace{P(a) \vee P(b) \vee P(c)}_{\text{Instance}} \longrightarrow \underbrace{(\exists x)\,P(x)}_{\text{Generalization}} \qquad \text{Rule E}$$

what generalization H would say if $\{a,b,c\}$ were only individuals. $\approx$ Development of H with respect to class $\{a,b,c\}$

Hempel's technical innovation = Give a general definition for development.

General Recursive Definition of Development	1. Go to deepest level of nesting & apply Rule A and Rule E	2. Repeat at all higher levels

Example

$$H:\ (x)\left[\underbrace{(y)\,\mathrm{Knows}(x,y)}_{\substack{\mathrm{Knows}(x,a)\\ \&\ \mathrm{Knows}(x,b)}} \supset \underbrace{(\exists z)\,\mathrm{Likes}(x,z)}_{\substack{\mathrm{Likes}(x,a)\\ \vee\ \mathrm{Likes}(x,b)}}\right]$$

Develop for domain $\{a,b\}$

$$\left[\begin{array}{l} \left[\begin{array}{l}\mathrm{Knows}(a,a)\\ \&\ \mathrm{Knows}(a,b)\end{array} \supset \begin{array}{l}\mathrm{Likes}(a,a)\\ \vee\ \mathrm{Likes}(a,b)\end{array}\right] \\[1em] \&\ \left[\begin{array}{l}\mathrm{Knows}(b,a)\\ \&\ \mathrm{Knows}(b,b)\end{array} \supset \begin{array}{l}\mathrm{Likes}(b,a)\\ \vee\ \mathrm{Likes}(b,b)\end{array}\right] \end{array}\right]$$

"$\mathrm{Dev}_{\{a,b\}}\, H$"

The Satisfaction Criterion

"Directly confirms"

$$\begin{tikzcd} B \arrow[d, Rightarrow, "\text{entails}"] \\ \mathrm{Dev}_C\, H \end{tikzcd}$$

B ← observation sentence

C ← class of individuals mentioned in B

The obvious definition

$$\begin{tikzcd} \mathrm{Raven}(a) \,\&\, \mathrm{Black}(a) \arrow[d, Rightarrow] \\ \mathrm{Raven}(a) \supset \mathrm{Black}(a) \end{tikzcd}$$

$$= \mathrm{Dev}_{\{a\}}\, (x)(\mathrm{Raven}(x) \supset \mathrm{Black}(x))$$

"Confirms"

$$\begin{tikzcd}
& B \arrow[dl] \arrow[d] \arrow[dr, "\text{directly confirms}"] & \\
H_1, \arrow[dr, Rightarrow] & H_2, \dots, \arrow[d, Rightarrow] & H_n \arrow[dl, Rightarrow, "\text{entails}"'] \\
& H &
\end{tikzcd}$$

Needed for obvious extension

$$\begin{tikzcd} \mathrm{Raven}(a) \,\&\, \mathrm{Black}(a) \arrow[d] \\ (x)(\mathrm{Raven}(x) \supset \mathrm{Black}(x)) \arrow[d, Rightarrow] \\ \mathrm{Raven}(b) \supset \mathrm{Black}(b) \end{tikzcd}$$

... Plus more clauses needed to block obvious perversions

Tautology $\xrightarrow{\text{confirms}}$ $(x)\,P(x)$

$P(a) \vee \sim P(a)$ $\qquad\qquad \vee\, (x)(\sim P(x))$

$= Dev_{\{a\}}\ (x)\,P(x) \vee (x)(\sim P(x))$

etc.

More serious problem:

(Evidence in finite domain) cannot confirm (Hypothesis that entails infinitely many individuals)

Finite Evidence	Hypothesis
$2 \rightarrow 3$	
$3 \rightarrow 5$	$(x)\,(\mathrm{Prime}(x) \supset$
$5 \rightarrow 7$	$\exists y\ \mathrm{Prime}(y)\ \&\ \mathrm{Greater}(y,x))$
$7 \rightarrow 11$	
$11 \rightarrow 13$	
$\vdots$	
$101 \rightarrow 103$	

Hempel's Conditions of Adequacy

responded to

"Nicod's Criterion"

$P(a) \,\&\, Q(a)$ confirms $(x)\, P(x) \supset Q(x)$

(... but not logically equivalent $(x)\,(\sim Q(x) \supset \sim P(x))$)

Entailment condition

$B \overset{\text{entails}}{\Longrightarrow} H$

then

$B \underset{\text{confirms}}{\longrightarrow} H$

Consequence condition

$B \to H_1,\ B \to H_2, \dots$

$\{H_1, H_2, \dots\} \Rightarrow H$

then

$B \longrightarrow H$

Consistency condition

$B \to H_1$

$B \to H_2$

$\vdots$

then

$\{B, H_1, H_2, \dots\}$ consistent

??

??

Equivalence condition

$$\frac{B \longrightarrow H_1 \qquad H_1 \Longleftrightarrow H_2}{B \longrightarrow H_2}$$

Paradox of the ravens

John is Australian

John was born in Sydney

John was born in Melbourne.

Paradox of the Ravens

All ravens are black. $\xleftrightarrow{\text{Logical Equivalence}}$ All non-black things are non-ravens

Nicod's criterion

Black raven

Nicod's criterion

white shoe

confirms

By equivalence condition

More:

- Hempel's original goal:
  Graft a probabilistic theory of confirmation onto the satisfaction criterion.

- Goodman's grue was taken to show that a purely syntactic relation of confirmation must fail