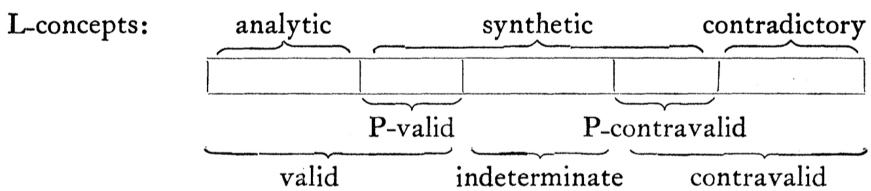
Formal System in Carnap's "Testability and Meaning"

Language-system, L
L rules
P rules
'S', 'S_1', 'S_2' etc. as designations of sentences.
L-consequence
P-consequence



"a," "b," etc. as names of space-time-points. Individual constants
"x," "y," etc. as corresponding variables. Individual variables
"P", "P1", "P2" etc., and "Q" as predicates.
Connectives, ~, v, . , ⊃, ≡
(x) for all
(∃x) there exists

Definition 1. …confirmation of S completely reducible to that of C …
... directly incompletely reducible …
… directly reducible …

Definition 2. … confirmation of S reducible to that of C …
… confirmation of S is incompletely reducible…

Definition 3 … confirmation of S is reducible …

Definitions 4 and 5. Confirmation of predicates.

Definition 6. Atomic form

Definition 7. Molecular form.

Definition 8. Generalized form. Essentially generalized form.

Theorem 1. If the confirmation both of S_1 and of S_2 is completely reducible to that of a class C of predicates, then the confirmation both of their disjunction and of their conjunction is also completely reducible to that of C.

Theorem 2. If S is a sentence of molecular form and the descriptive predicates occurring in S belong to C, the confirmation of S is completely reducible to that of C.

	two formulations	The confirmation of S is reducible					
		to that of C_1 ('P(a)' etc.)		to that of C_2 ('$\sim$P(a)' etc.)		to that of C (= C_1 + C_2)	
		compl.	incompl.	compl.	incompl.	compl.	incompl.
S_1	$(x)P(x); \sim (\exists x) \sim P(x)$	−	+	−	−	−	+
$\sim S_1$	$\sim (x)P(x); (\exists x)(\sim P(x))$	−	−	+	−	+	−
S_2	$(x) \sim P(x); \sim (\exists x)P(x)$	−	−	−	+	−	+
$\sim S_2$	$\sim (x) \sim P(x); (\exists x)P(x)$	+	−	−	−	+	−

Theorem 3. Let S be the universal sent (x)P(x). The confirmation of S is incompletely reducible to theat of the full sentences of 'P' and hence to that of 'P'. The confirmation of , S is completely reducible to that of the negation of any full sentence of 'P' and hence to that of 'P'.

Theorem 4. Let S be the existential sentence '(∃x)P(x)'. The confirmation of S is completely reducible to that of any full sen-tence of 'P' and hence to that of 'P'. The confirmation of - S is incompletely reducible to that of the negations of the full sentences of 'P' and hence to that of 'P'

Definition 9. Atomic or molecular forms for definitions of predicates.

Theorem 5. If 'P' is defined by a definition D based upon C, 'P' is reducible to C. If D has molecular form, “P” is completely reducible to C. If D has essentially generalized form, “P” is incompletely reducible to C.

Definition 10a. A universal sentence of the form

$$Q_1 \supset (Q_2 \supset Q_3)$$

is called a reduction sentence for Q_3 provided “~(Q_1.Q_2)” is not valid.

b. c. “reduction pair,” “bilateral reduction sentence.”

Theorem 6. If a reduction pair for 'Q' is valid, then 'Q' is completely reducible to the four (or two, respectively) other predicates occurring.

Definition 11. …introductive chain…

Definition 12. … is said to be introduced by this chain …

Definition 13. … introductive chain is said to have atomic (or molecular) form …

Theorem 7. If 'P' is introduced by an introductive chain based upon C, 'P' is reducible to C. If the chain has molecular form, 'P' is completely reducible to C; if the chain has essentially generalized form, 'P' is incompletely reducible to C.

Definition 14. … an introductive chain …, atomic (or molecular) predicate …

Definition 15. … atomic sentence…, … molecular sentence…

The representative sentence S		a reduction sentence of R (in L_2)	L_2
in L_1	in L_2		
1. analytic	analytic	analytic	consistent (if L_1 is consistent)
2. P-valid	P-valid	valid*	
3. indeterminate	P-valid	valid*	
4. P-contravalid	valid and P-contravalid	valid* and P-contravalid	inconsistent
5. contradictory	valid and contradictory	valid* and contradictory	inconsistent†

* analytic if fulfilling the general criterion (p. 451); otherwise P-valid.
† and moreover L-inconsistent if at least one sentence of R is analytic on the basis of the general criterion (p. 451).

Now the *complete criterion for 'analytic'* can be stated as follows:

Nature of S	Criterion for S being *analytic*
1. S does not contain any descriptive symbol.	S is valid.
2. All descriptive symbols of S are primitive.	Every sentence S′ which results from S when we replace any descriptive symbol at all places where it occurs in S by any symbol whatever of the same type—and hence S itself also—is valid.
3. S contains a defined descriptive symbol 'Q'.	The sentence S′ resulting from S by the elimination of 'Q' is valid.
4. S contains a descriptive symbol 'Q' introduced by a set R of reduction pairs; let L′ be the sub-language of L not containing 'Q', and S′ the representative sentence of R (comp. p. 451).	S′ is analytic in L′, and S is an L-consequence of R (e.g. one of the sentences of R); in other words, the implication sentence containing the conjunction of the sentences of R as first part and S as second part is analytic (i.e. every sentence resulting from this implication sentence where we replace 'Q' at all places by any symbol of the same type occurring in L′ is valid in L′).

Explanation 1. A predicate 'P' of a language L is called observable for an organism (e.g. a person) N, if …

Explanation 2. A predicate 'P' of a language L is called realizable by N, if …

Definition 16. A sentence S is called confirmable …

Definition 17. A sentence S is called bilaterally confirmable …

Definition 18. A predicate "P" is called confirmable …

Theorem 8. …"P" is completely confirmable…

Theorem 9. … [molecular] S is bilaterally confirmable …

Theorem 10. … S is bilaterally confirmable …

Definition 19. … test chain …

Definition 20. … testable …

Theorem 11. If a predicate is testable it is confirmable; if it is completely testable it is completely confirmable.

Definition 21. … S is testable …

Theorem 12, 13 and 14. Various connections between testable and bilaterally testable.