Harrigan & Spekkens, "Einstein, incompleteness and the epistemic view of quantum states"

2007

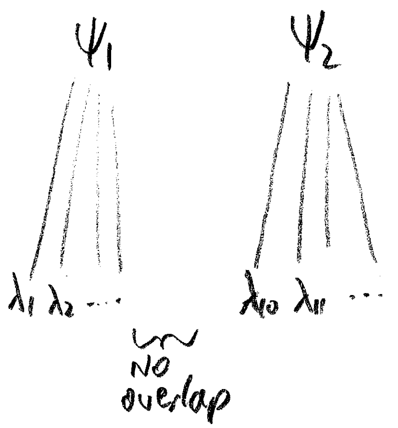
Standard view so far: $|\psi\rangle$ is a COMPLETE representation of the system's state OR $|\psi\rangle$ is an INCOMPLETE representation of the system's state

⇓

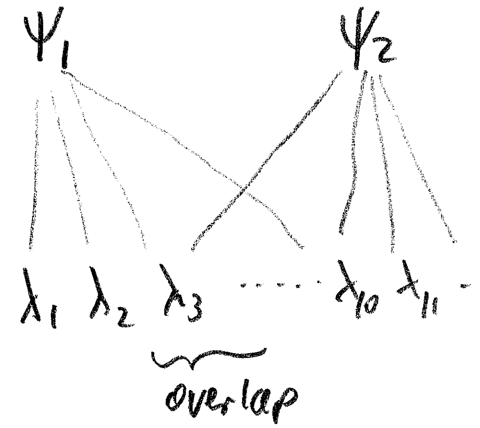
SPLIT

Novelty

"ψ-ontic" or "ψ-epistemic"



← Full specification of state →



"Ontic" since

Fix λ $\Longrightarrow$ Fix ψ

PBR "ψ is a property of λ"

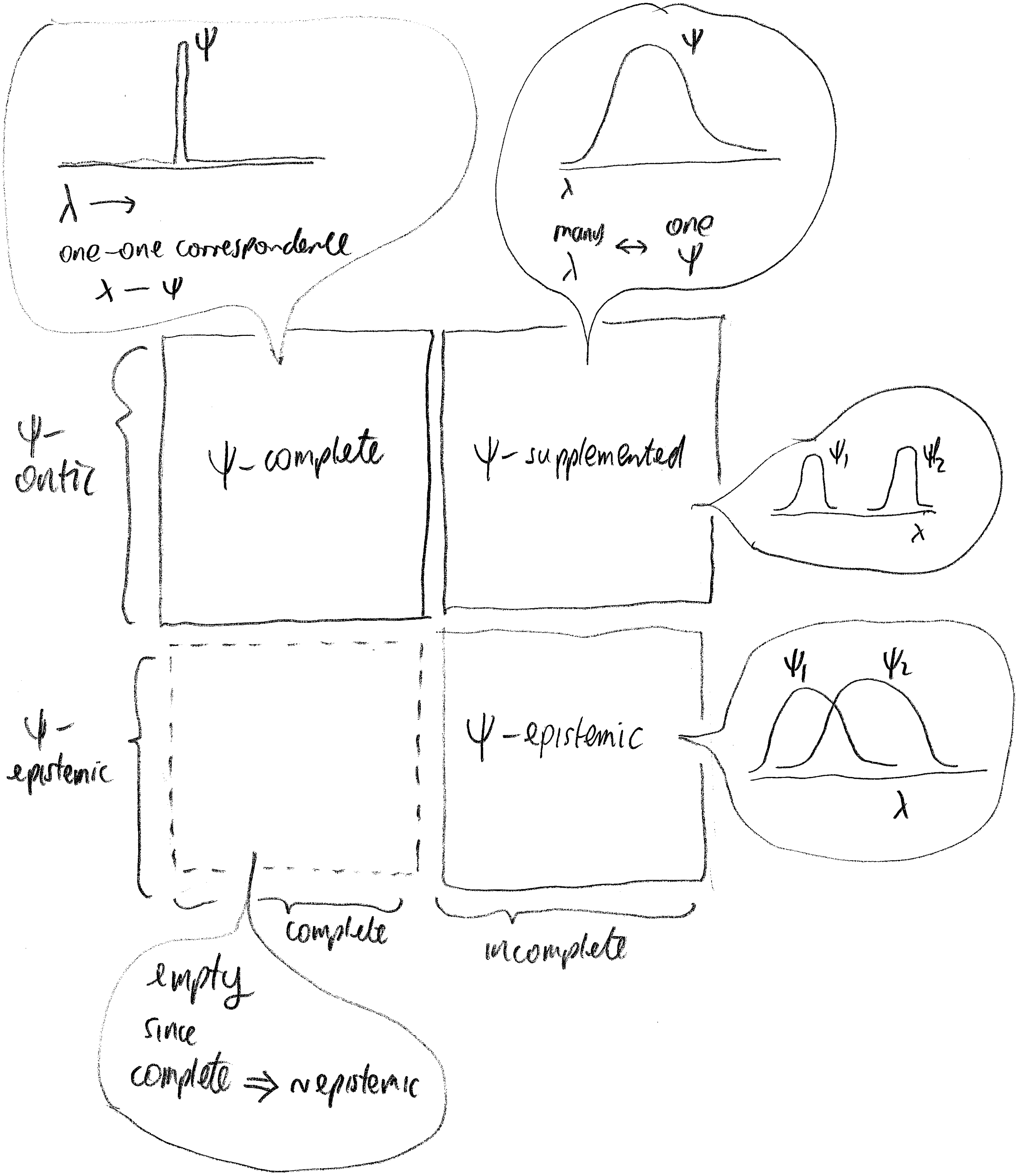
"Epistemic" since

Fix λ $\Longrightarrow$ ψ left uncertain

JDN: BUT

Fix ψ $\not\Longrightarrow$ Fix λ $\therefore$ still epistemic!

Poor terminology?


ψ
$\lambda \rightarrow$
one-one correspondence
$\lambda - \psi$
ψ
λ
many λ $\leftrightarrow$ one ψ
ψ-ontic
ψ-complete
ψ-supplemented
ψ_1 ψ_2
λ
ψ-epistemic
ψ-epistemic
ψ_1 ψ_2
λ
complete
incomplete
empty
since
complete $\Rightarrow$ $\sim$epistemic

An ontological model is

Local L

if and only if it is

Separable

For (disjoint?) regions of space $R_1, R_2, \ldots$

$$\Lambda_R = \Lambda_{R_1} \times \Lambda_{R_2} \times \cdots$$

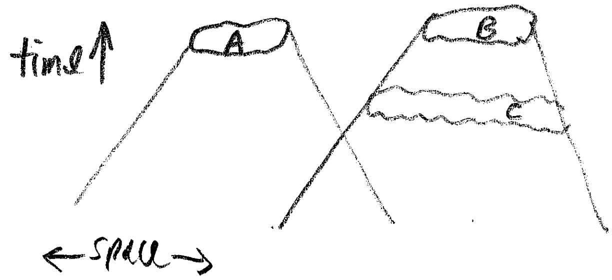
Λ_R: set of all states λ in combined region

!! $\boxed{\text{Cartesian product}}$!!

NOT Tensor product that would allow entanglement

and

Locally Causal



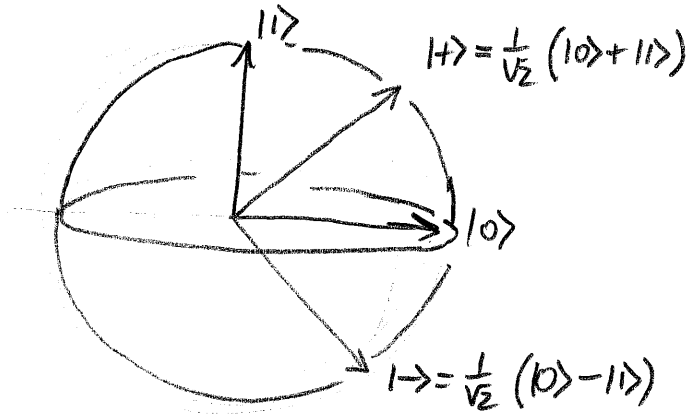
Events in C screen off events in B from events in A

$$p(B|A,\lambda_c) = p(B|\lambda_c)$$

λ_c: ontic state of region c

Alice steers Bob

Each holds one qubit in maximally entangled state



$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\Big(\underset{\text{Alice}}{|0\rangle}\,\underset{\text{Bob}}{|1\rangle} + \underset{\text{Alice}}{|1\rangle}\,\underset{\text{Bob}}{|0\rangle}\Big)$$

NB: NOT a singlet state!

"M_{01}" Alice measures in basis $|0\rangle, |1\rangle$ $\xrightarrow[\text{state collapses}]{\text{Bob's}}$ "P_{01}" Bob's qubit prepared in either $|1\rangle, |0\rangle$

- it is $|0\rangle$ → "P_0"
- it is $|1\rangle$ → "P_1"

"$M_\pm$" Alice measures in basis $|+\rangle, |-\rangle$ $\xrightarrow[\text{state collapses}]{\text{Bob's}}$ "$P_\pm$" Bob's qubit prepared in either $|+\rangle, |-\rangle$

- it is $|+\rangle$ → "P_+"
- it is $|-\rangle$ → "P_-"

Background sums for steering.

$|\psi^+\rangle$ in terms of $|+\rangle, |-\rangle$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + |1\rangle|0\rangle\right)$$

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$|0\rangle|1\rangle = \frac{1}{2}\left(|+\rangle|+\rangle + |-\rangle|+\rangle - |+\rangle|-\rangle - |-\rangle|-\rangle\right)$$

$$|1\rangle|0\rangle = \frac{1}{2}\left(|+\rangle|+\rangle - |-\rangle|+\rangle + |+\rangle|-\rangle - |-\rangle|-\rangle\right)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|+\rangle - |-\rangle|-\rangle\right)$$

$M_\pm$ Alice measures in $|+\rangle, |-\rangle$ basis

- Alice finds $|+\rangle$ $\longrightarrow$ Bob finds $|+\rangle$
- Alice finds $|-\rangle$ $\longrightarrow$ Bob finds $|-\rangle$

The big result: State is ψ-ontic ① & Quantum statistics respected ② $\Longrightarrow$ $\sim$ Locality ③

Proposal: This is Einstein's argument

Proof by contradiction.

Assume all ①,②,③ hold in case of Alice steering Bob.

BOB's true state only

$$P(\lambda \mid P_{01}) = \tfrac{1}{2} p(\lambda \mid P_1) + \tfrac{1}{2} p(\lambda \mid P_2)$$

Alice prepares Bob to be
$|0\rangle$ or $|1\rangle$ / $|1\rangle$ / $|0\rangle$

$$P(\lambda \mid P_\pm) = \tfrac{1}{2} p(\lambda \mid P_+) + \tfrac{1}{2} p(\lambda \mid P_-)$$

for computation, see ⑧

Locality $\Rightarrow$ $P(\lambda \mid P_{01}) = P(\lambda \mid P_\pm)$

since Alice's distant preparation cannot alter Bob's true state

The contradiction

$$4\Big[\overbrace{p(\lambda|P_{01})}^{\text{must be non-zero since } \underline{\text{some}} \text{ states are compatible with } |\psi+\rangle}\Big]^2 = 2p(\lambda|P_{01})\cdot 2p(\lambda|P_{\pm})$$

$$= \big[p(\lambda|P_0) + p(\lambda|P_1)\big]\big[p(\lambda|P_+) + p(\lambda|P_-)\big]$$

$$= \underbrace{\underbrace{p(\lambda|P_0)\,p(\lambda|P_+)}_{\text{Non-zero only if both } |0\rangle \text{ and } |+\rangle \text{ can be prepared at same time}}}_{\text{contradicts } \psi\text{-ontic}} + \underbrace{\underbrace{p(\lambda|P_0)\,p(\lambda|P_-)}_{|0\rangle \text{ and } |-\rangle}}_{} + \underbrace{\underbrace{p(\lambda|P_1)\,p(\lambda|P_+)}_{|1\rangle \text{ and } |+\rangle}}_{} + \underbrace{\underbrace{p(\lambda|P_1)\,p(\lambda|P_-)}_{|1\rangle \text{ and } |-\rangle}}_{}$$

$$= 0 \quad \text{if } \psi\text{-ontic obtains}$$

compute $p(\lambda | P_{01})$

$$p(\lambda | P_{01}) = \frac{p(P_{01} | \lambda)\, p(\lambda)}{p(P_{01})}$$ via Bayes.

$$= \frac{\{p(P_0|\lambda) + p(P_1|\lambda)\}\, p(\lambda)}{p(P_{01})}$$ since P_0, P_1 mutually exclusive

$$= \frac{p(P_0 \& \lambda) + p(P_1 \& \lambda)}{p(P_{01})}$$

$$= p(\lambda|P_0) \frac{p(P_0)}{\underbrace{p(P_{01})}_{1/2}} + p(\lambda|P_1) \frac{p(P_1)}{\underbrace{p(P_{01})}_{\frac{1}{2}}}$$

$$= \frac{1}{2} p(\lambda|P_0) + \frac{1}{2} p(\lambda|P_1)$$

similarly $p(\lambda | P_\pm) = \frac{1}{2} p(\lambda|P_+) + \frac{1}{2} p(\lambda|P_-)$