Inflationary Cosmology

Based on

G. Watson, "An Exposition of Inflationary Cosmology"
ASTRO-PH/0005000

Synopsis of Standard Cosmology

Robertson Walker spacetime

$$ds^2 = c^2 dt^2 - \underbrace{a^2(t)}_{\substack{\text{scale factor}\\ \text{(was } R(t))}} \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right]$$

k chooses spatial geometry
$k = +1$ spherical
$= 0$ flat
$= -1$ hyperbolic

[cosmic matter at rest in spatial coordinates r, θ, ϕ]

Hubble's Law
Hubble's constant

Distance between two galaxies $\quad \ell(t) = a(t)\,\ell(0)$

$\therefore$ velocity recession

$$v(t) = \frac{d}{dt}\ell(t) = \dot{a}(t)\,\ell(0) = \underbrace{\frac{\dot{a}(t)}{a(t)} \cdot \frac{\overbrace{a(t)\,\ell(0)}}{\ell(t)}}$$

compare Hubble's Law $v(t) = H(t)\,\ell(t)$

$$\boxed{H(t) = \frac{\dot{a}(t)}{a(t)}}$$

Einstein Equations $\boxed{G_{\mu\nu} + \frac{\Lambda g_{\mu\nu}}{c^2} = \frac{-8\pi G}{c^4} T_{\mu\nu}}$

$\Downarrow$ solve for dust with pressure

$$T_{\mu\nu} = (\underset{\substack{\uparrow \\ \text{density}}}{\rho} + \underset{\substack{\uparrow \\ \text{pressure}}}{p})\, u_\mu u_\nu - p g_{\mu\nu}$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \qquad \underbrace{\left(\frac{\dot{a}}{a}\right)^2}_{H^2} = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}}$$

Critical density at which $k=0$ space is flat case of $\Lambda=0$

$$H^2 = \frac{8\pi G \rho_c}{3} - \frac{0}{a^2} + \cancel{\frac{\Lambda}{3}}^{\,0} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

Flatness Problem

$$\boxed{\text{OBSERVE that present density } \rho \approx \rho_c}$$

i.e. $\Omega = \rho/\rho_c$ satisfies $|\Omega - 1| \approx 0$

BUT

$$\boxed{\text{Non-Inflationary DYNAMICS drives } \rho \text{ away from } \rho_c,\ |\Omega-1| \text{ away from } 0}$$

with $\Lambda = 0$ RW gives: $\underbrace{\left(\frac{\dot{a}}{a}\right)^2}_{H^2} = \frac{8\pi G \rho}{3} - \frac{k}{a^2}$ $\therefore$ $1 = \frac{\rho}{\underbrace{\left(\frac{3H^2}{8\pi G}\right)}_{\rho_c}} - \frac{k}{\underbrace{a^2 H^2}_{(\dot{a})^2}}$

$$\boxed{\frac{\rho}{\rho_c} - 1 = \Omega - 1 = \frac{k}{(\dot{a})^2}}$$

matter dominated
$a \sim t^{2/3}$
$\frac{1}{(\dot{a})^2} \sim t^{2/3}$

radiation dominated
$a \sim t^{1/2}$
$\frac{1}{(\dot{a})^2} \sim t$

$$\boxed{(\Omega - 1) \sim t^{2/3} \qquad (\Omega - 1) \sim t}$$

$\Omega - 1$ driven away from zero $\Longrightarrow$ would need $|\Omega - 1| \sim 10^{-16}$ at decoupling to get $(\Omega - 1)$ to present value

Horizon Problem

very hot. Radiation interacts extensively with particles

"recombination" electrons, nucleons form ordinary matter

cooler epoch cosmic background radiation propagates largely independently

0 ——— t_{rec} ——————————— t_0 now

distance light can travel

$$\underbrace{\int_0^{t_{rec}} \frac{c\,dt}{a(t)}}_{\text{Distance scale over which thermal equilibrium can be attained}} \ll \underbrace{\int_{t_{rec}}^{t_0} \frac{c\,dt}{a(t)}}_{\text{Distances from which we now see radiation}}$$

Formula for light propagation

$$0 = ds^2 = c^2dt^2 - a^2(t)\cdot\frac{dr^2}{1-kr^2}$$

$$\frac{dr}{\sqrt{1-kr^2}} = \frac{c\,dt}{a(t)}$$

Hence do not expect isotropy in cosmic background radiation

2 degrees separation in sky is enough to distinguish causally disconnected regions

OBSERVE ... ISOTROPY

Large Scale Structure Formation

Standard Big Bang cosmology assumes perfectly uniform initial matter distribution $\Rightarrow$ No inhomogeneities to seed structure (e.g. galaxies) formation

Introduce just the right amount of inhomogeneity to give present structures without ad hoc stipulation?

Monopole Problem

Various particle theories predict formation of anomalous structures (e.g. magnetic monopoles) in early, hot universe.

Not observed now.

Inflation

Key idea: Drive expansion of universe by form of matter that DOES NOT DILUTE as expansion proceeds.] in early "inflationary" phase

RW: $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

posit ρ as belonging to exotic matter

Drop, since inflation drives geometry close to Euclidean

OR reinterpret Λ as an exotic form of matter with $\rho_\Lambda = \frac{\Lambda}{8\pi G}$

so: $$\underbrace{H^2 = \left(\frac{\dot{a}}{a}\right)^2}_{} = \frac{8\pi G\rho_\Lambda}{3} = \text{CONSTANT}$$

$$\text{solve}\quad \underbrace{a(t) = a(0)\exp(Ht)}_{\boxed{\text{De Sitter spacetime}}}$$

Rapidly accelerating ("inflating") expansion

Flatness Problem Solved

$$(\Omega(t)-1) = \frac{k}{(\dot{a})^2} = \frac{k}{[a(0)\exp(Ht)]^2} \sim \exp(-2Ht)$$

!!!!

$(\Omega(t)-1)$ driven towards 0.
Allow inflation to proceed long enough so that $\Omega(t)-1$ gets so close to zero that it is still there after non-inflationary phase to present.

monopole problem solved

Inflationary expansion dilutes monopole density ← expect now to see none

Horizon Problem solved

$r(t)$ = co-moving coordinate of most distant particle visible at t } stabilizes to constant value during inflation.

Visible universe becomes small part of thermally equilibrated universe

For $k=0$, light follows $ds^2 = 0 = c^2dt^2 - a^2(t)\,dr^2$

$$dr = \frac{c\,dt}{a(t)}$$

Inflation from time: 0 to t_1

change in $r(t)$ $= \Delta r = \int_0^{t_1} \frac{c\,dt}{a(t)} = \frac{c}{a(0)} \int_0^{t} \frac{dt}{\exp(Ht)}$

$$= \frac{c}{a(0)} \cdot \frac{-1}{H} \left[\exp(-Ht)\right]_0^{t_1}$$

$$= \frac{c}{a(0)H} \left[1 - \exp(-Ht_1)\right] \longrightarrow \frac{c}{a(0)H}$$

$\boxed{\text{But during this phase inflation stretches the thermally equilibrated portion of spacetime to a much larger region}}$

Structure Formation in Inflationary Cosmology

- matter field driving inflation modelled by quantum "inflaton" field.

- Quantum perturbations in inflaton field frozen in by rapidity of inflation

$$\downarrow$$

later seed structure formation.