

# Inflationary Cosmology

Based on

G. Watson, "An Exposition of  
Inflationary Cosmology"

ASTRO-PH/0005000

# Synopsis of Standard Cosmology

Robertson  
Waller  
spacetime

$$ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right]$$

↑  
scale  
factor  
(was  $R(t)$ )

↑  
chooses spatial geometry

- $k = +1$  spherical
- $= 0$  flat
- $= -1$  hyperbolic

↑  
[cosmic matter at rest  
in spatial coordinates  
 $r, \theta, \phi$ ]

Hubble's Law  
Hubble's constant

Distance  
between  
two galaxies

$$l(t) = a(t) l(0)$$

∴ velocity  
recession

$$v(t) = \frac{d}{dt} l(t) = \dot{a}(t) l(0) = \frac{\dot{a}(t)}{a(t)} \cdot \underbrace{a(t) l(0)}_{l(t)}$$

compose Hubble's Law  $v(t) = H(t) l(t)$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

Einstein Equations

$$G_{\mu\nu} + \frac{\Lambda g_{\mu\nu}}{c^2} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

↓ solve for dust with pressure

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$$

↑ density    ↑ pressure

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3} \quad \underbrace{\left(\frac{\dot{a}}{a}\right)^2}_{H^2} = \frac{8\pi G \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Critical density at which  $k=0$  space is flat case of  $\Lambda=0$

$$H^2 = \frac{8\pi G \rho_c}{3} - \frac{0}{a^2} + \frac{\Lambda}{3} \quad \rho_c = \frac{3H^2}{8\pi G}$$

# Flatness Problem

OBSERVE that present density  $\rho \approx \rho_c$

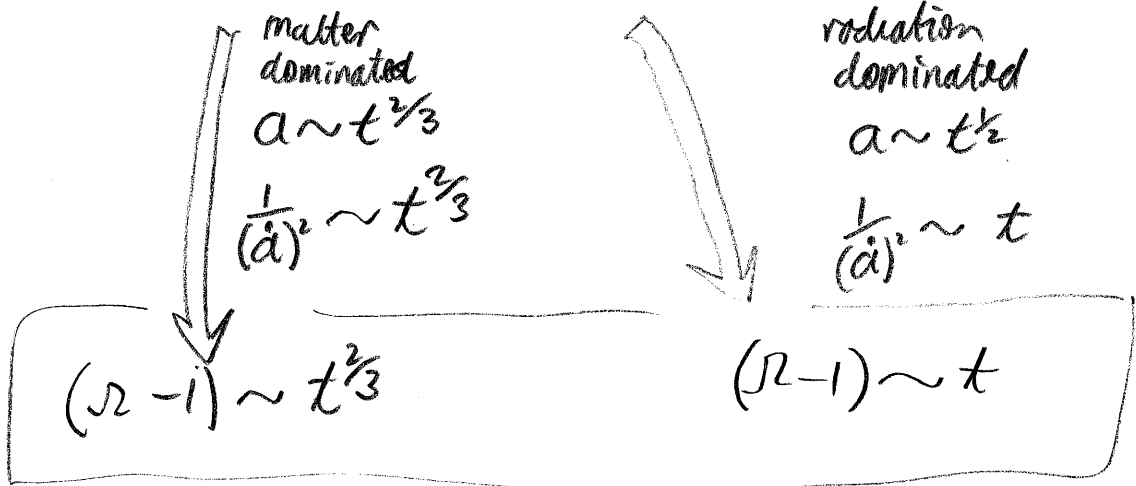
i.e.  $\Omega = \rho/\rho_c$  satisfies  $|\Omega - 1| \approx 0$

BUT

Non-inflationary DYNAMICS drives  $\rho$  away from  $\rho_c$   
 $|\Omega - 1|$  away from 0

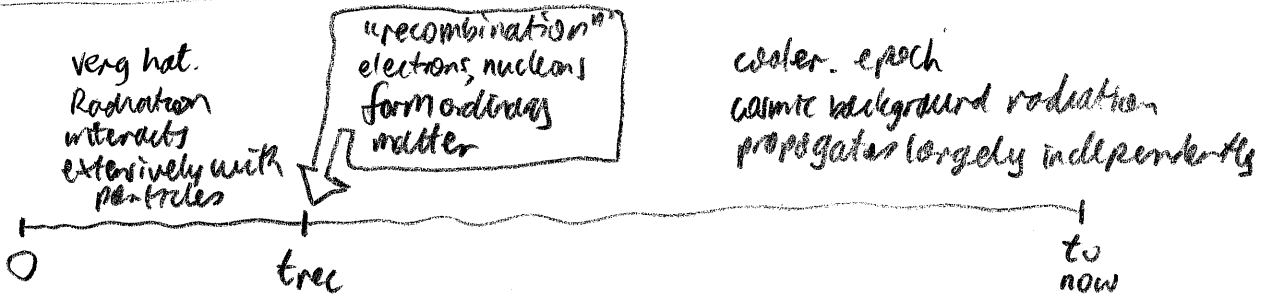
with  $\Lambda=0$  RW gives:  $\underbrace{\left(\frac{\dot{a}}{a}\right)^2}_{H^2} = \frac{8\pi G \rho}{3} - \frac{k}{a^2}$   $\therefore 1 = \underbrace{\frac{\rho}{\left(\frac{3H^2}{8\pi G}\right)}}_{\rho_c} - \underbrace{\frac{k}{a^2 H^2}}_{(\dot{a})^2}$

$$\frac{\rho}{\rho_c} - 1 = \Omega - 1 = \frac{k}{(\dot{a})^2}$$



$\Omega - 1$  driven away from zero  $\Rightarrow$  would need  $|\Omega - 1| \sim 10^{-16}$  at decoupling to get  $(\Omega - 1)$  to present value

# Horizon Problem



distance light can travel

$$\int_0^{tree} \frac{cdt}{d(t)}$$

Distance scale over which thermal equilibrium can be attained

$$\ll \int_{tree}^{to} \frac{cdt}{d(t)}$$

Distances from which we now see radiation

Formula for light propagation

$$0 = ds^2 = c^2 dt^2 - d^2(t) \cdot \frac{dr^2}{1-kr^2}$$

$$\frac{dr}{\sqrt{1-kr^2}} = \frac{cdt}{d(t)}$$

Hence do not expect isotropes in cosmic background radiation

2 degrees separation in sky is enough to distinguish causally disconnected regions

OBSERVE ... ISOTROPY

# Large scale Structure Formation

Standard  
Big Bang  
cosmology  
assumes perfectly  
uniform initial  
matter distribution



No inhomogeneities  
to feed  
structure (e.g. galaxies)  
formation

Introduce just the right amount of  
inhomogeneity to give present structures  
without ad hoc stipulation?

# monopole problem

Various particle theories predict  
formation of anomalous structures  
(e.g. magnetic monopoles)  
in early, hot universe.

Not observed now.

# Inflation

Key idea: Drive expansion of universe by form of matter that DOES NOT DILUTE as expansion proceeds. in early "inflationary" phase

RW:  $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$

posit  $\rho$  as belonging to static matter

in drop since inflation drives geometry close to Euclidean

OR reinterpret  $\Lambda$  as an exotic form of matter with  $\rho_\Lambda = \frac{\Lambda}{8\pi G}$

so:  $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho_\Lambda}{3} = \text{CONSTANT}$

solve  $a(t) = a(0) \exp(Ht)$

Rapidly accelerating ("inflationary") expansion

De Sitter spacetime

# Flatness Problem Solved

$$(\Omega(t) - 1) = \frac{k}{(\dot{a})^2} = \frac{k}{[a(t) \exp(Ht)]^2} \sim \exp(-2Ht)$$



!!!!



$(\Omega(t) - 1)$  driven towards 0.

Allow inflation to proceed long enough so that  $\Omega(t) - 1$  gets so close to zero that it is still there after non-inflationary phase to present.

# monopole problem solved

Inflationary expansion dilutes monopole density ← expect now to see none



# Horizon Problem solved

$r(t)$  = co-moving  
coordinate  
of most distant  
particle visible at  $t$

Stabilizes to constant  
value  
during inflation.  
Visible universe becomes  
small part of  
thermally equilibrated  
universe

For  $k=0$ , light follows  $ds^2=0=c^2dt^2-a^2(t)dr^2$

$$dr = \frac{c dt}{a(t)}$$

Inflation  
from time:  
0 to  $t_1$

change  
in  $r(t)$  =  $\Delta r = \int_0^{t_1} \frac{c dt}{a(t)} = \frac{c}{a(0)} \int_0^{t_1} \frac{dt}{\exp(Ht)}$

$$= \frac{c}{a(0)} \cdot \frac{1}{H} \left[ \exp(-Ht) \right]_0^{t_1}$$

$$= \frac{c}{a(0)H} \left[ 1 - \exp(-Ht_1) \right] \rightarrow \frac{c}{a(0)H}$$

But during this phase inflation  
stretches the thermally equilibrated  
portion of spacetime to a  
much larger region

# Structure Formation in Inflationary Cosmology

- matter field driving inflation  
modelled by quantum "inflaton" field.
- Quantum perturbations in  
inflaton field frozen in by  
rapidity of inflation



later seed structure  
formation.