Quantum Emergence:

Entangling the Universe

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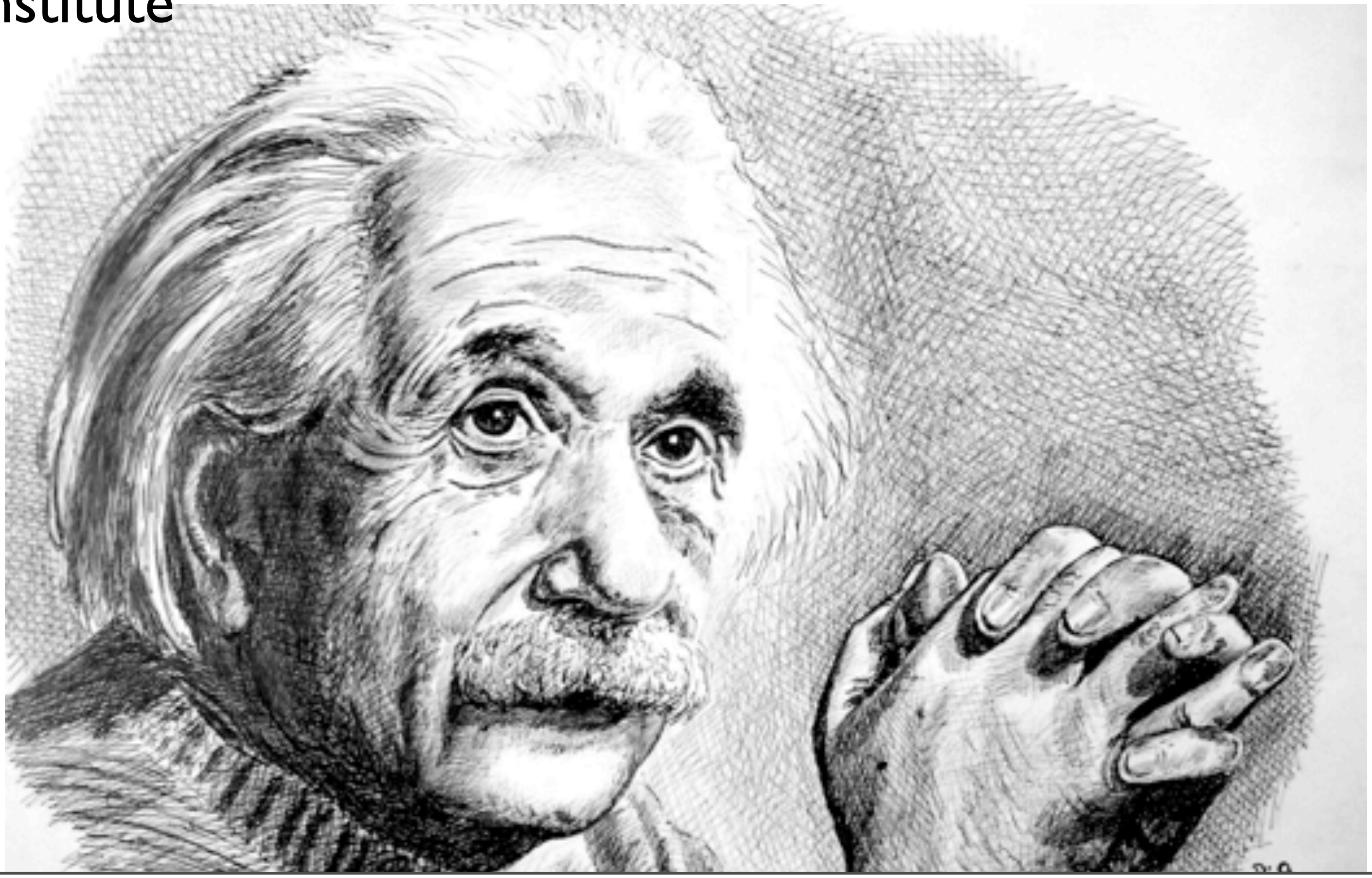


abstract

This talk is about emergence as seen by a theoretical physicist. Simply stated, I see an emergent result as any scientific conclusion that is a subtle or unexpected result of the basic postulates of a scientific field. The talk starts by describing some of the ways this can happen. It continues with a detailed discussion of entanglement in quantum mechanics.

Quantum entanglement is an idea that was added to the basic quantum theory ten years after that theory was put together in 1924-5. Its impact began to be felt only twenty-five years later with the work of John Bell. Since then the entanglement concept has formed the basis for entire fields of science. For example, it has had a dominating influence on "hard" condensed matter physics.

Possible Views of Emergence:

I. An Emergent result is anything that surprises the investigator.

Christiaan Huygens by Bernard Vaillant, Museum Hofwijck, Voorburg

In 1665, the scientist and clockmaker Christiaan Huygens noticed that two pendulum clocks hanging on a wall tended to synchronize the motion of their pendulums. A similar scenario occurs with two metronomes placed on a piano: they interact through vibrations in the wood and will eventually coordinate their motion.

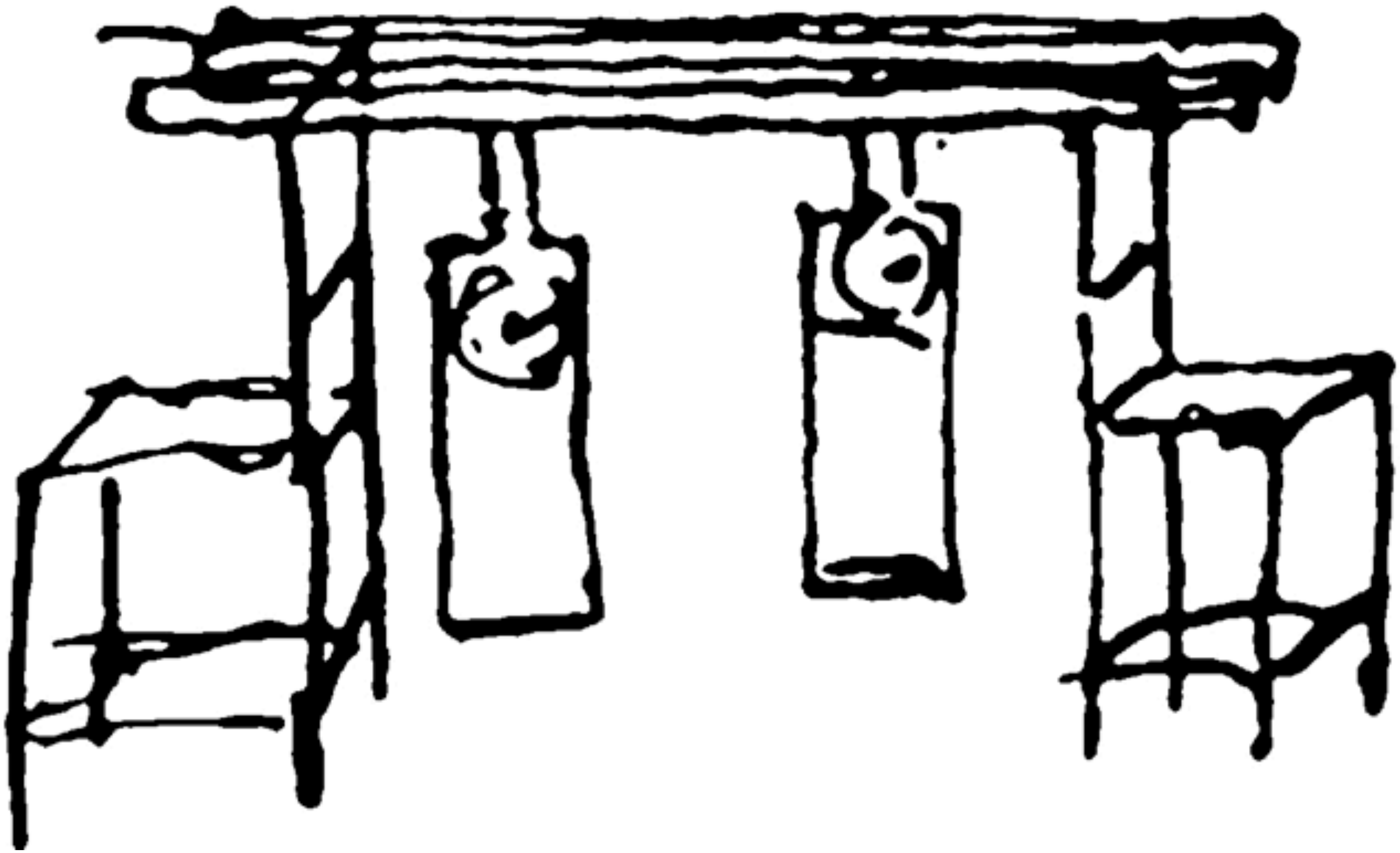
This result is somewhat surprising since the coupling between two clocks or two metronomes is likely to be very weak.

Huygens then looked further into his accidental discovery by setting up an experiment to demonstrate the synchronization phenomenon.

The material on Huygens is taken from an unpublished work by Mogens Jensen and LPK.

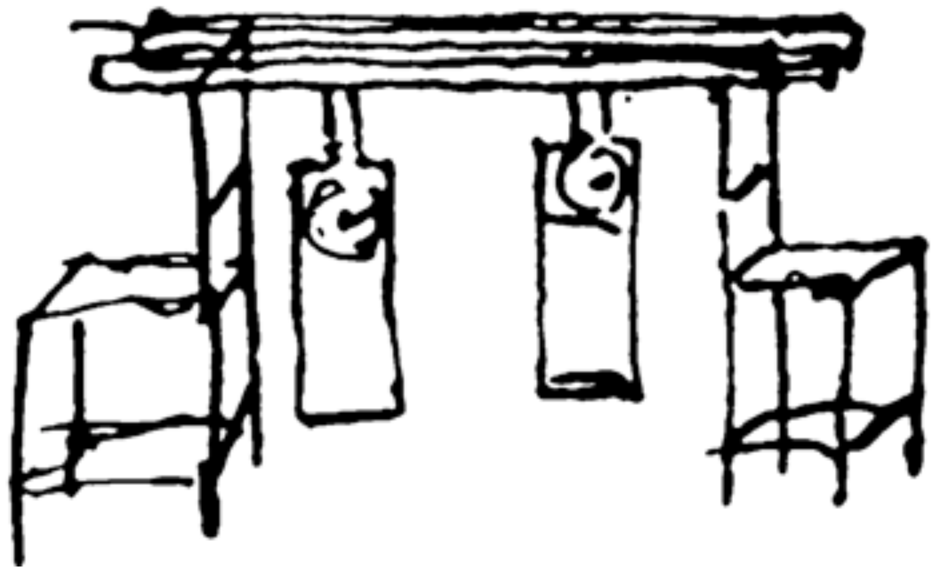
Huygens' Experiment

The observed synchronization was duly reported to the Royal Society in London. The letter included Huygen's picture:



I wonder whether Huygen's crude picture was intended to remind us that this phenomenon can be observed using only the simplest of experimental setups.

In modern language, Huygens has set up a dynamical system, and observed that the synchronized state of this system is an attractor, a region of phase space that attracts neighboring orbits.



This attraction arises because of friction, not included in the simplest description of two pendulums. In this situation friction is weak, but, after many clock-periods, it has an important cumulative effect. A general way to calculate this kind of motion came with the analysis of "secular perturbations" by Poincaré, Lindstedt, and others more than 200 years after Huygens' observation.

Seen in this way, we can classify Huygen's result as quite surprising.

Poincaré, H. (1957) [1893], *Les Méthodes Nouvelles de la Mécanique Célèste* **II**, New York: Dover Publ.,
A. Lindstedt, Abh. K. Akad. Wiss. St. Petersburg 31, No. 4 (1882)

Another Definition:

II. A result is emergent if it does not fit into the theoretical framework of the investigator

In that sense, as time went on, Huygen's result became more emergent. Classical mechanics was translated into Hamiltonian mechanics. The Hamiltonian mechanics of finite system is a theory that cannot include friction. This theory has no attractors.

There are different ways that a result can fail to fit. I give examples

New Math

Zeno's paradox:
In a race, Achilles give a tortoise a head start. He then goes through a succession of efforts. His first effort halves the distance between him and the tortoise. He makes a second effort, which shortens by half the new distance to the tortoise. His third effort similarly shortens the just-previous distance. Achilles is not too bright, but he does catch on that it will take him an infinite number of such efforts to reach the tortoise. In despair he drops out of the race. Did he do the right thing?

Achilles conclusion is, of course, incorrect. He needs to learn the math that comes with the limiting process. With that will come the emergent result that Achilles can indeed pass the tortoise.

This emergence demands a new definition of mathematics, less finitetistic than before.

"More is Different:" Limiting Processes

The title of Anderson's article is somewhat misleading. He argues that an infinite limit may yield results obtainable in no finite system. He thought about the possibility of qualitative changes in behavior that occur in phase transitions. Such a qualitative change can occur in a material with an infinite number of particles, but happen in no finite system.

So in qualitative properties, infinite is not equal to finite, but many is the same as few.

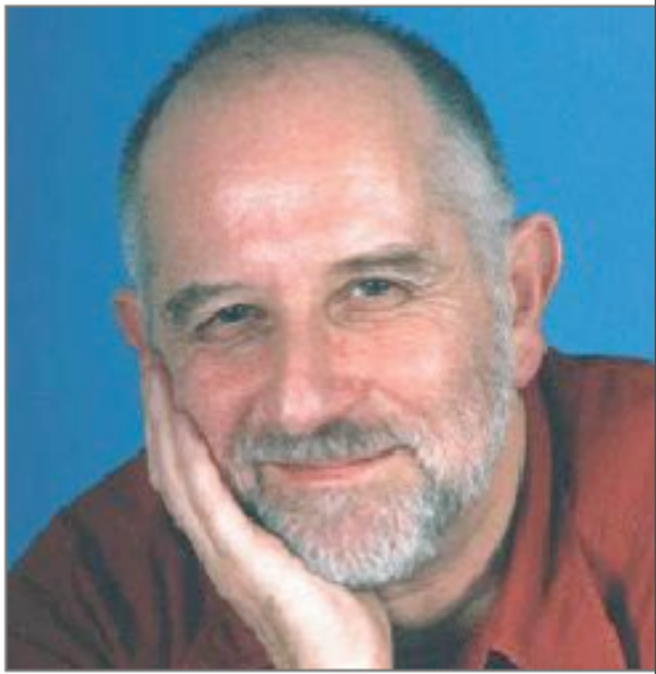
Anderson's examples, like Zero's can be understood by studying limiting processes.

The work of Anderson and others has led to a major change in how physicists view physics. Now, a sharp distinction is drawn between qualitative change in a system, and changes that are only quantitative. The former require cooperation by an infinite number of degrees of freedom.

P. W. Anderson, More is different, *Science* **177**, 393-396 (1972)

A further addition: Michael Berry

Michael Berry and Robert Batterman have stressed the philosophical implications of limiting processes.
Berry has looked at examples from Twentieth and Twenty-First Century Physics that might be approached through *singular perturbations*. These are the small terms that can nonetheless produce essential, qualitative changes. These usually arise when the highest order term in a differential equation is, in effect, multiplied by a small parameter.



Examples include
• the transition from wave optics to ray optics
• the transition from classical mechanics to quantum mechanics
• phase transitions studied as a limiting process in a large system

Berry,M.: Singular limits. Phys. Today **55** 10–11 (2002)

Berry,M.: Asymptotics,singularities and the reduction of theories. In:Prawitz, D.,Skyrms, B.,Westerstaahl, D. (eds.) Proceedings of the 9th International Congress of Logic, Methodology and Philosophy of Science, pp. 597–607 (1994)

Battermant,R.W.:The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction and Emergence. Oxford University Press, Oxford (2002)

The most exciting emergence arises when the proposed theory does not fit within the intellectual boundaries of the existing theory.

A clear example is when the Alverez's, father and son, wished to explain dinosaur extinctions as the result of a comet colliding with the earth. Many paleontologists could not abide this extraterrestrial explanation.

Similarly, Lord Kelvin could not accept geological scientists' estimates of the age of the earth because earth temperature could not be explained in terms of the energy sources of nineteenth Century Physics.

In evolutionary theory, many biologists have fought hard against the idea that bacteria have a social life. Cooperative behavior does not look sufficiently "Darwinian" to them.

Quantum Theory

The rest of this talk will deal with my candidate for a "poster boy" of emergence: The notion of entanglement. This idea was first put forward by Schrödinger in response to Einstein's stringent and intelligent objections to quantum mechanics.

The old quantum theory, put together by Einstein, Planck, Bohr and others was a patchwork designed to explain experimental facts, but built upon a weak intellectual and mathematical basis. Nobody could view it as the final word.

Then in 1924–1925 Schrödinger and Heisenberg put forward two different versions, of a new theory that had considerable mathematical solidity. These two theories turned out to be different versions of the same "new quantum theory." The new theory then formed the basis for much of the physics of the subsequent ninety years.

Werner Heisenberg (1925). "Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen". *Zeitschrift für Physik* **33** (1): 879–893. An English translation may be found in B.L. van der Waerden, trans., ed. (1968). *Sources of Quantum Mechanics*. New York: Dover. pp. 261–276

Schrodinger, Erwin (1926). "Quantisierung als Eigenwertproblem". *Annalen der Phys* **384** (4): 273–376. Bibcode:1926AnP...384..361S.

A reminder

Heisenberg's quantum mechanics dealt with operators including operators for coordinates, like $\mathbf{x,y,z}$, and momenta, like $\mathbf{p_x}$. The major new feature is that these operators did not all commute

$\mathbf{x\, p_x - p_x\, x} = \mathbf{i}\, \hbar$

but some do

$\mathbf{x\, y - y\, x} = 0$

The x-p commutation relation gives an uncertainty principle

$\Delta x\, \Delta p \geq \hbar/2$

saying you cannot know the values of $\mathbf{x}$ and $\mathbf{p}$ at the same time.

Schrödinger's quantum mechanics dealt with wave functions like $\psi(x,y,z)$ and the Schrödinger equation

$[-\hbar^2\nabla^2/(2m) + V(x,y,z)]\, \psi(x,y,z) = E\, \psi(x,y,z)$

Probabilities of physical quantities are obtained from $|\psi|^2$

Quantum Theory: 90 years old; unchanged but still evolving

The quantum theory put together by Dirac and Heisenberg has remained unchanged since the 1920s. Even the concept of entanglement, which has formed the basis of much new thinking, goes back to the work of Schrödinger in 1935 and 1936.

However, entanglement has forced us to new and emergent views of the roles of localization, separability and information in physical theory.

These new ideas have important applications in the possibilities of quantum electronics and quantum computing. Equally important is the possibility that the new ideas will permit some reconciliation of quantum theory and gravity together. It may also give us with an extended understanding of astrophysics.

Alisa Bokulich, Reexamining the Quantum-Classical Relation, Cambridge University Press (2008).

Schrödinger, E., 1935. "Discussion of Probability Relations Between Separated Systems," *Proceedings of the Cambridge Philosophical Society*, 31: 555–563; 32 (1936): 446–451.

Separability

Almost every theoretical physics paper starts with a statement something like: we write down the Hamiltonian/Lagrangian for our system as... Then the authors write a descriptor for a very small part of the world's degrees of freedom, independently of all the rest. It is far from obvious that one can properly do this. Aristotle and Mach have argued against this practice, as have all purveyors of a holistic view of the world.

Philosophers can well ponder how (or whether) the world can be separated into its parts while still maintaining its most essential properties .

David Kaiser, How the Hippies Saved Physics, Norton New York, 2011.
Fritjof Capra, The Tao of Physics, Bantam, New York, 1975

Localization

Most of the Hamiltonians or Lagrangians used by physical scientists are sums over terms each of which refers to one particular region of space or space-time. These structures obey the ancient prejudice that "action at a distance" is impossible.

In classical physics, those theories that include long-ranged interactions often can be "cured" of that difficulty by the introduction of measurable fields-- electromagnetic, gravitational, ... -- that then eliminate all non-locality by permitting the theory to be expressed in terms of partial differentials equations. In that way one can ensure that one gets flow of information only between neighboring points.

For "action at a distance" theorists these fields can be viewed as hidden variables enabling one to restore locality to classical electromagnetism nd gravitational theory. Since their values may be measured as fully as everything else in a non-quantum theory, in a classical perspective we may believe that they have a reality comparable to that of magnets and electric charges.

Localization, continued

However, quantum theory is different: its variables--- wave functions, vector potentials,… --- cannot be measured in their entirety. Only parts of their information are accessible at any one time. In that sense they are different from anything classical. At first, it was thought that this difference was benign. Then, little by little it was realized that the emergent properties of quantum theory strongly impinge upon our view of "reality."

A discussion of the history of quantum theory and of these ideas parallel to the one given below can be found in Bub, Jeffrey, "Quantum Entanglement and Information", The Stanford Encyclopedia of Philosophy (Summer 2015 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2015/entries/qt-entangle/>.

Einstein, Podolsky, Rosen (EPR)

attempted to define for us what must be "real." They said

- A physical quantity will be real if and only if it can be measured without changing its value.
- If one can show that measurements of A are real and then show the same for measurements of B then both objects are real.
- If I do something here, the reality of a far-away object cannot change immediately. (localization!)
- standard quantum theory must be correct.

They then reach a contradiction by considering the quantum theory of two particles on a line respectively described by coordinates and momenta, $\mathbf{x_1}, \mathbf{p_1}$ and $\mathbf{x_2}, \mathbf{p_2}$. They show how, by measuring $\mathbf{x}_1$, they can find the value of $\mathbf{x_2}$. Equally, a measurement of $\mathbf{p_1}$ can tell them the value of $\mathbf{p_2}$. However quantum theory (the Heisenberg uncertainty principle) says that one cannot simultaneously measure $\mathbf{x}$ and p. Contradiction! Quantum theory is dead!??

Albert Einstein, Boris Podolsky, Nathan Rose "Can the quantum mechanical description of physical reality be considered complete," Physical Review **47** 777-780 (1935).

EPR...The maths

Because $\mathbf{x_1}$-$\mathbf{x}_2$ commutes with $\mathbf{p_1}$+$\mathbf{p_2}$ it is legal to write down any normalizable wave function of the form $\psi(x_1-x_2, p_1+p_2)$.
Let use choose a wave function that is highly peaked around separations x_1-x_2 close to L and p_1+p_2 close to 0.

$\psi(x_1-x_2, p+ p_2). =\Delta(x_2-x_1 -L)\ \Delta(p_1+p_2\ )$

This can represent a particle which has split into two parts in a decay process.

- Measure x_1. The value of x_1 gives us at once the value of $\mathbf{x_2}$. as x_1+L.

We conclude that $\mathbf{x_2}$ is real

- Instead we could decide to measure $\mathbf{p_1}$. The value of $\mathbf{p_1}$ gives us at once the value of $\mathbf{p_2} = -p_1$. We conclude that $\mathbf{p_2}$ is equally real.

However, in quantum theory, $\mathbf{x_2}$ and $\mathbf{p}_2$ do not commute. We cannot know them simultaneously. We have reached a contradiction. Something is wrong with the EPR assumptions.

Asher Perez, *Quantum Theory Concepts and Methods*, Klewer, Dordrecht (1995) pp 148-149.

where does the trouble arise?

It arises when we look at a long sequence of repetitions of the basic experiment, with a comparison of results after each repetition

- There is no problem when Alice and Bob both measure positions. The positions are perfectly determined and perfectly correlated $x_1 = -L + x_2$

- There is no problem when Alice and Bob both measure momenta. The momenta are perfectly determined and perfectly correlated, $p_1 = -p_2$

The problem arises for the times that Bob measures a momentum and Alice a position or vice versa. Then using the equations in red we could give numerical values to all four variables. This contradicts the uncertainty principle

EPR concluded

All of this remains true even if L is so large that relativity indicates that no communication between is possible during the duration of the observations at x_1 and x_2.

Localization seems to have failed. Perhaps there is a "hidden variable" that influences events at both x_1 and x_2. It seems unlikely, but maybe such a variable can rescue localization.

David Bohm's theory

David Bohm put together a theory that realized one of the EPR desires. It is a "hidden variable" theory correlating the behavior particles far away from one another using a wave function that depends upon the coordinates of all of the particles. All the consequences that can be drawn from the wave function are the same as those of the usual quantum theory. However, actions upon one set of particles would, in this theory, still produce almost instantaneous effects upon particles far away. Thus, hidden variable theories can work but at least this one cannot fit Einstein's view of reality.

Bohm, D., 1952, "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables, I and II," *Physical Review* **85**: 166-193.
Bohm, D., 1953, "Proof that Probability Density Approaches $|\psi|^2$ in Causal Interpretation of Quantum Theory," *Physical Review* **89**: 458-466.
Stamford Encyclopedia of Mechanics, Bohmian Mechanics (2006).
http://plato.stanford.edu/entries/qt-entangle/ (2015).

Bell's Theorem

All this work on entanglement did not have much impact in the years from 1935 through the 1960s. Then, John Bell published what has become a very influential paper. It is about *classical mechanics*, but its implication is crucial for quantum theory. It used variables that take on the values plus/minus one and calculated correlations among quite separated measurements, and gave the maximum possible strength of these correlations.

No classical hidden variables could make these correlations stronger than Bell's estimate. But, a brief calculation showed than quantum spins indeed had a stronger correlation.

So, there was a sharp distinction between the implications of quantum and of classical theory. Experiment could decide.

John Bell, Physics . "On the Einstein-Podolsky-Rosen Paradox" 1 195 (1964).
John Bell Rev. Mod. Phys. 38 447 (1965).

The proof is in the pudding

So quantum theory gives results that would be impossible in any situation defined by classical mechanics. Maybe classical mechanics is right?

That possibility seems to have been ruled out by experiments involving atoms and photons, especially those of Alan Aspect and Anton Zeilinger. They observed the predicted correlations and found correlation-values that exceeded the maximum possible in classical mechanics, but matched the calculations for the usual quantum theory.

However, in the view of many physicists, classical mechanics has not been superseded. It is not obsolete; classical mechanics is still needed to give meaning to quantum measurements*.

Classical mechanics also describes pretty much everything we see around us.

*Asher Peres, "Quantum Theory Concepts andMethods", Kluwer Academic Publishers (1995). chapter 12, Measurement.

Entangled Wave Functions

In teaching quantum theory in the 1960s most of us described two simultaneously existing quantum systems by writing down their wave function as

$\psi(1,2) = \varphi(1)\eta(2)$. (unentangled form)

Here 1 and 2 respectively stand for all the position, spin, etc. variables that may appear in a wave function for each one of the systems. This is not the form that will arise when two particles, described by the above wave function scatters and thereby change the values of such variables as momentum or spin. They then became entangled. Such entanglement will arise whenever particles interact, scatter, or decay. In that case the wave functions will have the form

$\psi(1,2) = \Sigma_{j,k}\, \varphi_j(1)\eta_k(2)\, \psi_{jk}$ (entangled form.)

The differences between the two cases are quite profound. The EPR wave function was of the entangled form.

Entangled Wave Functions, continued

$\psi(1,2) = \varphi(1)\eta(2)$.   (unentangled)
$\psi(1,2) = \Sigma_{j,k}\, \varphi_j(1)\eta_k(2)\, \psi_{jk}$ (entangled form)
The problem with quantum localization arises from the fact that an experimenter at the position of "1" can project any part of the wave function that has to do with her location.

In the unentangled case no matter what is projected at "1" the wave function at "2" will remain proportional to $\eta(2)$. Since the shape of the wave function determines what is measured at "2," observations at "2" are completely independent of what happens at "1." This is the separability that we expect to be built into physics.

The entangled case is different.

Entangled Wave Functions, continued

$\psi(1,2) = \varphi(1)\eta(2)$.   (unentangled)

$\psi(1,2) = \Sigma_{j,k}\, \varphi_j(1)\eta_k(2)\, \psi_{jk}$ (entangled form)

The entangled case is different. Say that Alice, the person at "1" filters her system demanding that the filter picks out situations in which her subsystem is in mode j. We can now infer that the wave function for the combined system is

$\psi(1,2) = \varphi_j(1)\ \Sigma_k\, \psi_{j,k}\ \eta_k(2)$

Then we know that Bob, observing this same system at "2," he will find that these observations are consistent with a wave function $\Sigma_k\, \psi_{j,k}\ \eta_k(2)$. So, if there is more than one term in the sums over j and k. what happens at 1 immediately affects the situation at 2.

If these observers are at separate places, they cannot know at the time of observation what the other observer has seen. However, if they later communicate they will find that their separate observations of this system are consistent with quantum theory and with this wave function. Indeed, we have spooky action at a distance.

Onward to averages

If we are dealing with an isolated system with wave function |ψ>= ψ(1), we can calculate the average of the observable, X as <ψ| X |ψ> or more explicitly, considering that observables may be represented as matrices as

$$\langle X\rangle = \int d1\, d1' \quad \psi^*(1')\, X(1',1)\, \psi(1)$$

You should not be surprised to see that the wave function fully determines all averages….in an isolated system.

That's what we do in a quantum course. In a statistical physics course, we might also describe a quantum situation, one in which the system is in equlibrium at temperature T, but we would not make direct use of wave functions. Instead we would use a probability density matrix ρ(1,1') and calculate the average as

$$\langle \mathbf{X}\rangle = \int d1'\, d1\;\; X(1',1)\, \rho(1,1') = \text{trace}\, \mathbf{X}\, \boldsymbol{\rho}$$

The density matrix is here used for everything, along with the statistical mechanical formula

$\boldsymbol{\rho}$ =exp(-**H**/**T**) /Z(T) with **H** being the Hamiltonian and T temperature.

Now what should we do in our studies of more general quantum mechanical cases, use wave functions of use the density matrix?

von Neumann, John (1927), "Wahrscheinlichkeitstheoretischer Aufbau der Quantenmechanik", *Göttinger Nachrichten* **1**: 245–272

Onward to averages: use ψ or ρ ?

The old-fashioned answer is to use ψ. That will work for unentangled cases or even for entangled ones in which we know the entire wave function.

Now visualize an entangled system, containing him and her in separated regions. Her part, "1", is known probabilistically. There is a probability p_j of falling into the jth. configuration. Then, there is no wave function for his part of the world. Instead we must use a density matrix

$$\rho(2,2') = \Sigma_j \, p_j \, [\Sigma_k \, \psi_{j,k} \;\; \eta_k(2)] \, [\Sigma_m \, \psi_{j,m} \;\; \eta_m(2)]^*$$

with all averages being given by

$$<\mathbf{X}> = \int d2 \, d2' \;\rho(2,2') \, X(2',2)$$

For our purposes today, the formulas don't matter much. The point is that a part of an entangled system cannot be described by a wave function!. We must amend the courses we teach and start thinking in terms of density matrices.

density matrices have emerged

as the proper description of any system connected to an outside world that is only partly known. Of course, this is the usual situation.

There is an important extra emergence built into the use of density matrices:

We can use the familiar statistical mechanical formula

$$S = \text{trace}\,(\rho \ln 1/\rho)$$

to define entropy for all entangled situations. In the unentangled case, the system is in a single state and the entropy is zero. Now entropy can be used as a measure of entanglement.

A bridge has emerged connecting statistical physics, thermodynamics, and quantum physics. It is a huge extension of the concept of entropy.

Entropy can be defined for a part of a system

Let the world be divided into two parts, "1" and "2" with a density matrix, ρ , that depends on both kinds of variables. If we sum over the possibilities of part 1 we can form a probability for 2

$\rho_2 = \text{trace}_1\ \rho$

and an entropy for part 2 that is

$S_2 = - \text{trace}_2 [\rho_2 \ln \rho_2]$

In this way we can define an entropy for any part of the world, in equilibrium or not.

This is a huge extension beyond the original thermodynamic definition of entropy that only holds in equilibrium. This definition melts into the thermodynamic one in the appropriate limit.

The bridge is not a surprise

That there is an intimate connection between statistical mechanics and quantum theory has been known for ages. The new thing is that this connection can be applied to entirely new situations. An important example of this connection arose when Stephen Hawking pointed out that black holes gave out thermal radiation and Bob Wald produced a whole thermodynamics of black holes.

This kind of result encouraged people to study the propagation of entanglement in all kinds of quantum systems. Since entropy seems as real as steam engines and chemical plants, Einstein's questions about reality seem to have been answered by giving entanglement its own reality.

L. Kadanoff and G. Baym, Quantum Statistical Mechanics, W.A. Benjamin, New York (1963).

Quantum Theory has a Law of Conservation of Entropy

However, entropy is contained in the matter that is sucked into black holes. Eventually, it is believed, these hole will radiate,shrink, and disappear. What happens to the entropy that has been drawn in?

A fashionable way of asking this question is to note that changes in the information content of something can equally be described as entropy changes. So one moves from a view of quantum theory to one of information theory.

From messages to information

The development of a new understanding of quantum theory partly accelerated and partly responded to a new interest in an old concept, "information." Information theory is about where data is stored, how it may be moved, and how it may be accessed. In the Twenty-First Century, we might say that quantum theory is all about information. We might also say that the Twenty-First Century is all about information.

We can view any message as a string of bits. Each bit is a variable with two possible values. As shown in the work of Claude Shannon and others, the most efficient way to deal with a message is to deal directly with its bits. Quantum mechanics is well founded to do this. Hence for many purposes we view messages as streams of bits of information.

James Glieck, The Information: A History, a Theory, a Flood, Doubleday (2011)

However the basic storage unit for information is different in different descriptions of the world.

- In a classical view it is the bit: a variable which can take on two values. (a bit, an observable = yes or no)
- A variable, σ, that takes on two values is called a qubit. In the quantum theory of an isolated system it is described by a wave function ψ(σ) that includes two complex numbers. An observer of this isolated qubit can measure one real number.
- In the quantum theory of an entangled system, the state of a qubit, σ, is described a two by two density matrix, containing three real numbers... but only one of these is available at a time.

What you can do with these is also quite different.

With entanglement you can send secret messages and know whether anyone has tried to listen in.

It is believed that you can do a subset of calculations much faster with an entangled device than with a standard computer built on classical mechanics calculations, given that the two devices have the same number of bits available.

In theory, entanglement enables electronics much faster and more sensitive than anything now available.

Entanglement has thus begun the development of exciting new technologies:

Quantum communications to safeguard governments from public scrutiny.

Quantum computers to speed calculations. In particular they are hoped to be able to simulate quantum systems much better than anything now available. It is expected that they will be much better able to break codes and passwords than anything now available.

Quantum devices, however, are prone to contamination from unplanned interaction with an outside world. A whole field of quantum error correction has arisen to deal with this difficulty.

Quantum materials

If quantum electronics is to work a host of new materials must be developed. For example, one must construct quantum memories capable of holding and controlling entangled qubits. At first sight, shielding the qubits from unwanted interactions seems next to impossible. Nonetheless, groups of promising technologies are being studied and these are fast becoming the largest part of condensed matter physics.

For example, topological insulators are materials in which the flow of current on the surface is controlled by ordering within the bulk. This then offers the possibility that memories might be built into topological properties of the material. Topological properties are structural properties which will not change under gradual deformation. Hence the data in memories might well persist for a very long time.

Quantum glasses

Glasses are another kind of material that resist change. They are usually produced by cooling a liquid very quickly.

Recently David Huse and others have proposed that an isolated interactive quantum system might well fall into a state quite different from that of the usual thermodynamic equilibrium. With many possible different configurations, this glass might serve as a quantum memory.

A related but even more radical notion is that all glasses are glassy because of quantum entanglement.

So entanglement has started to dominate a large portion of condensed matter physics.

David A. Huse, Rahul Nandkishore, and Vadim Oganesyan, Phenomenology of fully many-body-localized systems, **arXiv:1408.4297**

A little problem

As black holes radiate they evaporate. When they are gone, where is the entropy of bodies that have fallen into them?

Relativists say who cares, why conserve entropy?

Field theorists say unitarity in quantum theory demands that we do so.

Field theorists say that, as a body passes into the body of a black hole it must burn up and radiate.

Relativists say that general relativity indicates that nothing special happens as a body passes the horizon of the hole?

This contradiction is expected to be a window, which when opened will give deep insights into the nature of the two theories.

Entangling the Structure of Space-time

Another proposal for fixing this dilemma goes by the nickname ER=EPR. EPR is our old friend Einstein, Podolsky, Rosen. It described entanglement. ER describes a line of thinking by Einstein and the same Rosen. In this idea tiny black holes, or perhaps a line formed by a skinny wormhole connect entangled quantum objects. This far-out proposal makes entanglement a source of structure in space-time.

A parallel proposal goes in the same direction. The work came from two condensed matter physicists: Shinsei Ryu and Tadashi Takayanagi who notice a correlation between gravitational curvature and entanglement in a special situation involving a black hole formed in anti-deSitter space. Is this a general thing? So perhaps space-time curvature is related to entanglement.

The idea is far out, but it comes from two of the deepest and most accomplished people in theoretical physics, Juan Maldacena and Leonard Susskind. https://www.quantamagazine.org/20150424-wormholes-entanglement-firewalls-er-epr/

Entanglement Described

Louisa Gilder in "The Age of Entanglement" says

When two particles interact, in doing so, they lose their separate existence. No matter how far they move apart, if one is tweaked, measured, observed, the other seems to instantly respond, even if the whole world lies between them.

This is, in my view a reasonable statement of the theory. She concludes

And no one knows how.

This last seems to me to be a reflection of an incorrect view of reality. Of course, no one "really" knows how anything happens.

Louisa Gilder, *The age of entanglement*, Alfred A. Knoph, New York (2008). page 3.

Emergence in its strongest form:

Previously, we based a very large fraction of our quantum teaching upon wave functions and included density matrices as an interesting footnote.

Now, the density matrix is central and the wave function is, perhaps, a topic for the introduction.

I emphasize that the basic theory has not changed by one iota.

Leonard I. Schiff Quantum Mechanics McGraw-Hill (1968)

Asher Peres, "Quantum Theory Concepts and Methods", Kluwer Academic Publishers (1995).

John Preskill, Quantum Notes, WWW.theory.caltech.edu/people/preskill/ph229/notes/chap4.pdf