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EINSTEIN, THE HOLE ARGUMENT AND THE REALITY OF SPACE

1. INTRODUCTION

In November 1915, Einstein put the finishing touches to his general theory of relativity. Then he proclaimed that the theory, through its general covariance, "robbed time and space of the last trace of objective reality" (Einstein, 1915, p. 831). This triumphant proclamation was repeated early in 1916 in a review of the theory. The requirement of general covariance "takes away from space and time the last remnant of physical objectivity." (Einstein, 1916, p. 117).

This case, as Fine has reminded us, is one of a number of embarrassments for scientific realists who like to think that progress in science has depended at least in some measure on the realist orientation of scientific investigators (Fine, 1984, pp. 91—92). But Einstein's work on special and general relativity owed a great debt to Machian positivism and in particular Mach's non-realist[1] attitude towards Newton's absolute space and time. It is striking, for example, that Einstein's earliest and still best known exposition of the complete general theory of relativity does not begin by revealing some empirical deficiency of earlier theories. Rather he launches the theory by pointing out an "epistemological defect" in special relativity and classical mechanics, which, he tells us, was first noticed by Mach. (Einstein, 1916, p. 112.)

In the simplest of glosses, Einstein's work on relativity theory is portrayed as the relentless pursuit of the implications of Mach's non-realist view of space and time. Assertions about motion with respect to space are to be rendered meaningless unless they can be reinterpreted solely in terms of the relative motion of bodies. Special relativity was the first step. In it motion with respect to some absolute state of rest was eliminated and with it went the aether of electromagnetism. General relativity completed the process by removing the unacceptable intrinsic distinction between inertial and accelerated motion which had still lingered in special relativity. The result was a complete victory for Leibniz's relational view of space and time. In so far as one talked about space and time within general relativity (and this is done

John Forge (Ed.), Measurement, Realism and Objectivity, 153—188.

frequently!), the talk must be understood entirely instrumentally. According to general relativity, the terms space, time and spacetime do not refer to any entities in the world.

This account portrays the development of relativity theory as driven by a naive positivism and non-realism and as such it does not capture the subtlety and depth of Einstein's work towards the discovery of relativity theory.[2] To illustrate my claim I am going to tell the story of the origin and significance of an argument of Einstein which seems at first glance to be quite naively positivistic in outlook and somewhat trivial in import. I follow John Stachel in calling the argument the "point-coincidence argument" and quote the well known 1916 presentations of it. The argument is given in the wake of Einstein's statement of the requirement of general covariance, the requirement that the laws of nature must "hold good for all systems of coordinates." It reads:

> That this requirement of general co-variance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and the observed point-events happening at the same place at the same time.
>
> The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences. We allot to the universe four space-time variables x_1, x_2, x_3, x_4 in such a way that for every point-event there is a corresponding system of values of the variables $x_1 \ldots x_4$. To two coincident point-events there corresponds one system of values of the variables $x_1 \ldots x_4$, i.e. coincidence is characterized by the identity of the coordinates. If, in place of the variables $x_1 \ldots x_4$, we introduce functions of them x'_1, x'_2, x'_3, x'_4, as a new system of coordinates, so that the systems of values are made to correspond to one another without ambiguity, the equality of all four coordinates in the new system will also serve as an expression for the space-time coincidence of the two point-events. As all our physical experience can be ultimately reduced to such coincidences, there is no immediate reason for preferring certain systems of coordinates to others, that is to say, we arrive at the requirement of general covariance. (Einstein, 1916, pp. 117—118)

Presented in this form, the argument has little force. Its conclusion is not the most interesting claim of the passage, which is that space and time have lost the last remnant of physical objectivity. Fine (1984, p. 91) quite rightly calls it a "suspicious-looking verificationist argument". What I think makes the argument look suspicious is not its verificationism. Rather it is the caution with which Einstein pursues a quite trivial conclusion: that there should be no preferred spacetime coordinate systems, which acquires the lofty title of postulate or requirement of general covariance. If spacetime coordinate system has its usual meaning — a smooth but otherwise arbitrary numerical labelling of spacetime events — then it is hard to see how we could require otherwise. Certainly this requirement is a commonplace of differential geometry, in which any well formulated spacetime theory is automatically expressible in coordinate free (= generally covariant) terms. This makes the requirement essentially useless as a criterion for selecting between competing theories. For example, generally covariant formulations of Newtonian spacetime theories and of special relativity are well known.

Nevertheless Einstein's point-coincidence argument, with its verificationist turn of phrase, fascinated contemporary philosophers such as Reichenbach and Schlick, both of whom studied closely the new-born general theory of relativity. (I follow here the discussion of Friedman, 1983, Ch. 1 and Howard, 1984, Sect. 3.) Briefly, they saw in it a perfect example of the relation between theory and fact proposed by the soon to emerge logical positivist movement and in a manner essentially related to the non-realistic view of theoretical terms. For Reichenbach, for example, our freedom in choosing coordinate systems was another instance of the conventionality inherent in theory, which would surface elsewhere as the conventionality of geometry and distant simultaneity. Schlick applauded the point-coincidence argument as illustrating how we can eliminate elements which are superfluous to our theory in the sense that they are arbitrary and thus cannot correspond to anything real. For Schlick space and time were the arbitrary elements while the coincidences of the point-coincidence argument were non-arbitary. Thus Friedman identifies the arbitrariness of choice of coordinate system as "the genesis of Reichenbach's notion of '*coordinative* definition'" (p. 19) and in Einstein's 1916 statement above of the point-coincidence argument he sees "the birth of the modern observational/theoretical distinction." (p. 24)

We owe a great debt to John Stachel (1980), who discovered the key to a proper understanding of the point-coincidence argument. With the help of Einstein's correspondence from that period, he was able to identify the argument as Einstein's resolution of a grave difficulty which had helped delay the completion of the general theory of relativity by as much as three years. In the so-called "hole argument" of late 1913, Einstein had convinced himself that generally covariant gravitational field equations were incompatible with physical determinism. Late in 1915, in order to be able to readmit generally covariant gravitational field equations into general relativity, Einstein had to find an answer to the hole argument. That answer was the point-coincidence argument, which Einstein then interpreted as establishing that space and time must forfeit the "last remnant of physical objectivity."

In this paper, I shall review the hole and point-coincidence arguments and the circumstances surrounding their origins. We shall see that the really important conclusion Einstein drew from this episode was a result which I label "Leibniz equivalence". It asserts that in a generally covariant theory such as general relativity, a single gravitational field cannot be represented by a single mathematical field, but must be represented by an equivalence class of diffeomorphic fields.

We shall see that without excursions into Einstein's earlier publications and his correspondence it is impossible for readers of Einstein (1916) to understand that this result was the issue or, for that matter, precisely how the verificationism of the point-coincidence argument was to be applied. Moreover we shall see the establishment of Leibniz equivalence did not depend on the verificationist argument, but was already forced on us by a stronger consideration not mentioned in Einstein (1916). It is clear from retrospective appraisal of the hole argument, that unless we accept Leibniz equivalence, we will commit ourselves to an altogether unacceptable variety of indeterminism when we come to formulate generally covariant field theories such as general relativity.

Finally we shall see that non-realism about space and time played only a secondary role in the episode, entering only at the denouement of a much longer story. I can see no way for the point-coincidence argument to support the non-realist view. Rather it establishes forcefully an antisubstantivalist view of spacetime, which asserts not that spacetime has no reality, but no reality independent of the fields it contains. We shall see that, within four years of 1916, Einstein retracted his non-realist statements in favour of explicit antisubstantivalism.

2. THE *ENTWURF* THEORY

In August 1912 Einstein returned to Zurich. Over the preceding five years he had worked intermittently on the problem of relativizing gravitation theory and extending the principle of relativity to accelerated motion. Within less than a year, with the mathematical assistance of his friend Marcel Grossmann, he was able to sketch out virtually all the essential components of his general theory of relativity. We now call that theory the *Entwurf* ("Outline") theory after the first word of the title of Einstein and Grossmann (1913a and 1913b), in which the theory first appeared.

I now describe some elements of this theory in modern terms, terms somewhat different to those used by Einstein and Grossmann. Without this more precise terminology it would be very difficult to explicate adequately the hole and point-coincidence arguments. The theory proposed that spacetime was a four dimensional differentiable manifold on which certain fields were defined, the most important of these being the metric tensor field.

A *four dimensional differentiable manifold* is a set whose members are identified with the points or events of spacetime in the standard developments of spacetime theories. If the manifold were just a set of events, then we would have no idea of which events neighbor on which others. This information is provided by the topological structure of the manifold, which specifies which subsets of events are the open sets (neighbourhoods). Thus we have a notion of locality through which we can identify the neighbourhoods containing each event. The manifold looks locally like a four dimensional Cartesian space — that is, for any event we can always find a neighbourhood containing the event which can be mapped one—one onto some open subset of R^4.

A *coordinate system* or coordinate chart is just such a map. It labels each point p of the relevant neighbourhood with some unique four-tuple of reals, $x^i(p)$ ($i = 0, 1, 2, 3$). If one such coordinate system K is possible, then from K, it is easy to define a second coordinate system K' which assigns a different four-tuple $x^{i'}(p)$ to p. All coordinate systems, which are related by continuous, infinitely differentiable transformation equations where they overlap, belong to the manifold's altas of coordinate systems.

In terms of manifold structure alone, it is possible to define curves (smooth maps from an interval of the reals into the manifold) and their tangent vectors. But a bare manifold is unlike a Euclidean space in the sense that we cannot define length along the curves and thus have no notion of a straight line. Moreover we cannot single out preferred coordinate systems. In a Euclidean space we could distinguish preferred

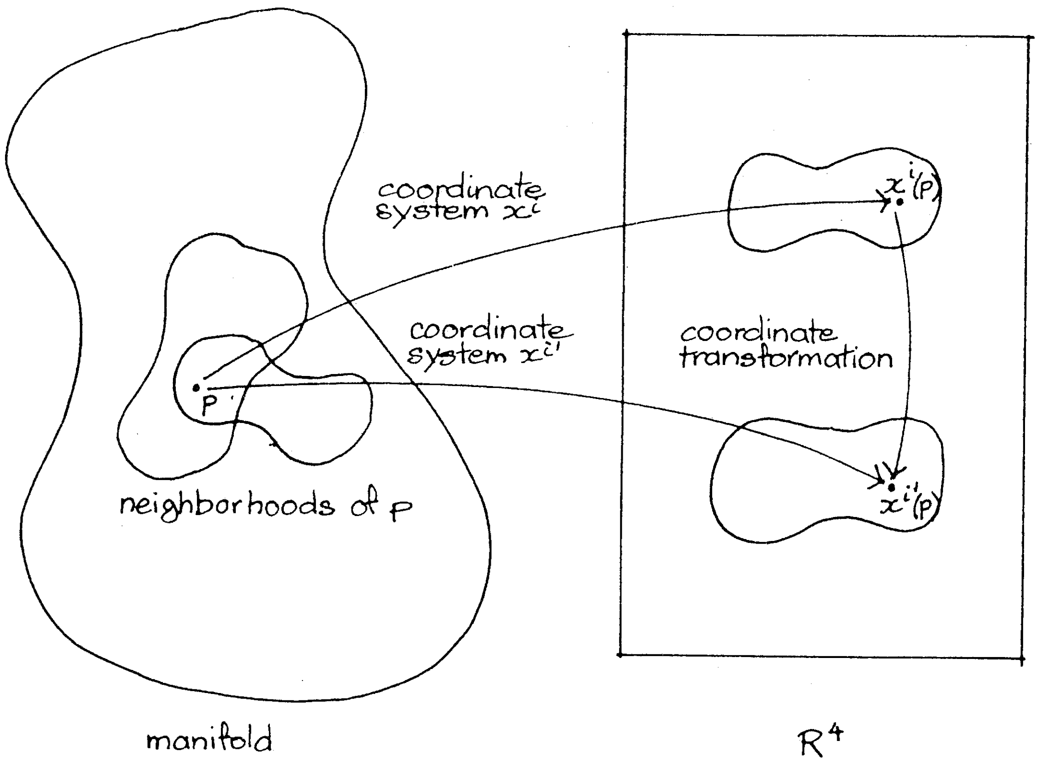

Fig. 1. A manifold with coordinate systems.

Cartesian coordinates, whose coordinate differences correspond to length, only because Euclidean spaces have extra structure enabling the defining of the length of curves.

A covariant, second rank, symmetric, Lorentz signature *metric field*, g_{ab} provides this notion of length (usually called "interval") in the *Entwurf* theory. Its Lorentz signature means that it does not assign lengths isotropically, unlike a Euclidean metric. It assigns positive lengths to curves in one direction, now identifiable as the "time-like" direction, and negative lengths to the curves of the other three "space-like" directions. Thus the Lorentz signature metric gives spacetime its light cone structure. Time-like curves are the possible trajectories or world lines of real non-zero rest mass particles. The null length curves forming the light cones are possible trajectories of light. The metrical length of a curve is given by the integral

$$\int \sqrt{g_{ac} V^a V^c}\, \mathrm{d}l$$

along the curve, where V_a and V_c are the curve's tangent vector and l the associated path parameter.

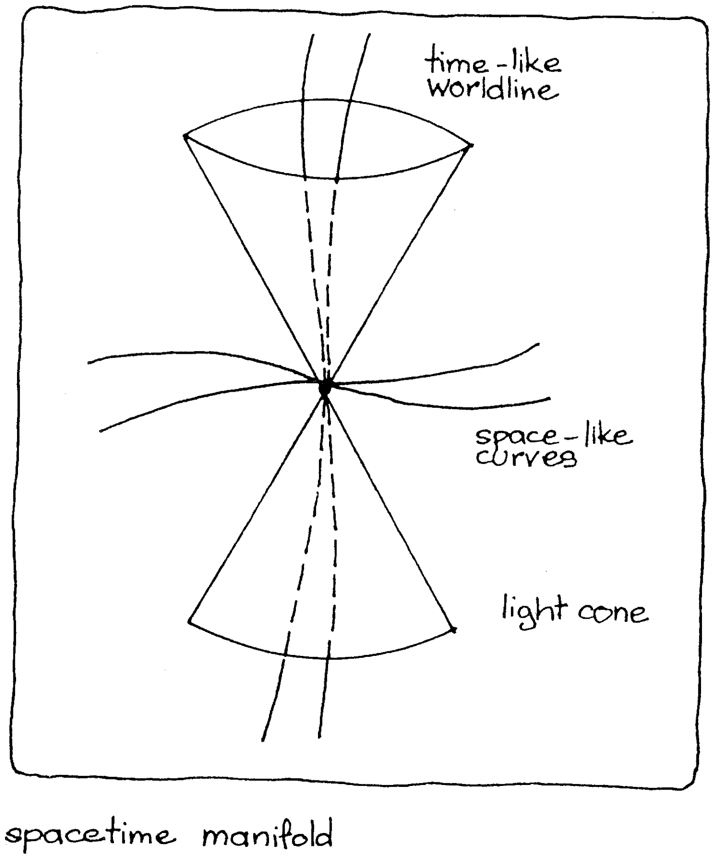

Fig. 2. Light cone structure of spacetime.

Gravitation and metrical curvature. The world lines of particles in free fall are time-like geodesics, curves of extremal interval, the analogue of Euclidean straights. If the metric is flat, we have a Minkowski spacetime, the case of special relativity. Particles in a Minkowski spacetime with initially parallel world lines never approach or diverge, analogous to the behaviour of parallel straights in a Euclidean space. This will no longer be the case if the metric tensor has non-vanishing curvature. Free particles with initially parallel world lines might now approach one another. This would be taken to be due to a gravitational action and the non-vanishing curvature of the metric associated with the presence of a gravitational field.[3]

Transformation law for tensor components. A second rank, covariant tensor, such as the metric tensor, can be represented uniquely in a given coordinate system by a 4×4 matrix of its components, g_{im}, where i, $m = 0, 1, 2, 3$.[4] The components of all such tensors obey the following transformation law under change of coordinate system. If any such tensor G_{ab} has components G_{im} in coordinate system x^i and com-

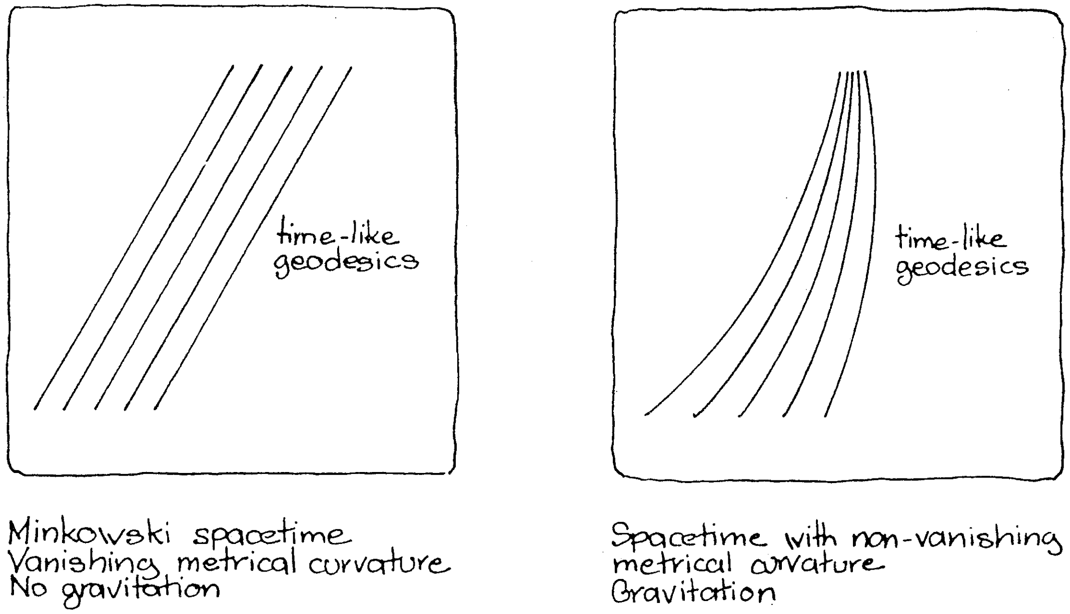

Fig. 3. Gravitation and curvature.

ponents $G_{i'm'}$ in the new coordinate system $x'^{i'}$ (i', m' = 0, 1, 2, 3) *at the same point in the manifold*, then

$$G_{i'm'} = \frac{\partial x^{i}}{\partial x^{i'}} \frac{\partial x^{m}}{\partial x^{m'}} G_{im} \tag{1}$$

where summation over repeated indices i' and m' on the right hand side is implied in accord with the Einstein convention introduced in Einstein (1916, p. 122).

This transformation law will figure prominently in the story to follow. Notice in particular that if a tensor G_{ab} has all zero components in one coordinate system x^i at p:

$$G_{im} = 0$$

then it follows immediately from the above law that it will have zero valued components in any other coordinate system $x^{i'}$ at p:

$$G_{i'm'} = 0$$

Such a tensor is a zero tensor.

The *stress energy tensor* T_{ab} is another field in spacetime which we need consider for what follows. It is a second rank, covariant tensor like g_{ab} and represents the energy and momentum of all non-gravitational forms of matter, such as electromagnetic fields or dust clouds.

As far as the above details are concerned, the *Entwurf* theory did not differ from the completed general theory of relativity. In fact the two theories agree in all but one essential aspect. The exception is crucial. The metric tensor takes the place of the scalar gravitational potential φ of Newtonian gravitation theory. As a result Einstein required in both the *Entwurf* and his final general theory that the metric tensor enter into a field equation analogous to Poisson's equation

$$\Delta\varphi = 4\pi G\rho$$

in Newtonian theory. That equation was required to have the form

$$G_{ab} = kT_{ab} \tag{2}$$

where k is a constant, analogous to the Newtonian gravitation constant G; T_{ab} is the stress energy tensor, analogous to the source mass density ρ of Poisson's equation; and G_{ab} is the gravitation tensor, which is constructed out of any combination of the metric tensor and its first and second derivatives, and which is linear in the second derivatives. This tensor is the analogue of the Laplacian of the Newtonian gravitational potential, $\Delta\varphi$.

The derivatives in question here are derivative of the components of the metric tensor with respect to the coordinates. Notice that the above constraint appears to allow very many possible gravitation tensors. This freedom is illusory, however, for if g_{im} is a tensor, then it does not follow for example that its derivatives with respect to coordinate x^n, that is $g_{im,\,n}$, will also be a tensor. $g_{im,\,n}$ cannot represent a tensor since it can readily be confirmed that it does not satisfy a transformation law analogous to (1). It turns out to be very hard to combine the coordinate derivatives of the metric tensor to yield a new tensor. The only relevant possibilities are the metric tensor itself, the Riemann curvature tensor $R^a{}_{bcd}$, its contractions R_{ab} (the Ricci tensor) and R, and tensors formed from them. The last include the Einstein tensor, $R_{ab} - \frac{1}{2}g_{ab}R$. It is now well known that the addition of the requirement of energy momentum conservation is sufficient to force the choice of G_{ab} as the Einstein tensor, the gravitation tensor of Einstein's final theory of November 1915. (He then ignored the possibility of an additive cosmological term proportional to g_{ab}.)

Einstein and Grossmann clearly knew in 1912 and 1913 that the obvious place to look for a gravitation tensor was in the contractions of the Riemann curvature tensor. They even considered the Ricci tensor as a gravitation tensor, which would have given the final field equations at least in the source free case. But, they decided, this choice did not yield the correct Newtonian limit in the case of weak, static fields and, worse than that, they could find no acceptable gravitation tensor. An acceptable account of how they arrived at this conclusion has only recently become available. See Norton (1984) and in shortened form Norton (1985a) and (1985b). There I also describe the three year odyssey of compounded error upon which Einstein embarked and which culminated in a breathless and dramatic discovery of the final field equations in November 1915. The hole argument arose as one of the episodes of this odyssey in the following way.

To deal with their failure to find a gravitation tensor, Einstein and Grossmann took a desperate measure. They had found what they believed to be an acceptable quantity to stand for G_{ab} in equation (2). But that quantity was not a tensor, since the matrix of its components did not transform according to (1) for all coordinate transformations. As a result they distinguished two types of tensor:

(a) those whose components transformed as tensors under arbitrary coordinate transformations;
(b) those whose components transformed as tensors under some limited set of coordinate transformations.

To be generally covariant, the theory would have had to have a gravitation tensor of the first type, but Einstein and Grossmann offered a tensor of the second type. It followed that the field equations of the theory held only in a restricted set of coordinate systems. They soon began work on the problem of determining precisely how large this set was. After April 1914, with his move to Berlin, Einstein had to work on this problem alone. I have conjectured (Norton, 1984, p. 295) that it was in the process of the variational calculations involved that Einstein hit upon a marvellous way of converting failure into success. That was the hole argument.

3. THE HOLE ARGUMENT

The hole argument purported to demonstrate that any generally covariant gravitational field equations in the context of the *Entwurf* theory would violate physical determinism in a severe and striking manner.[5] It aimed to show that if one had a matter distribution with a matter free spacetime neighbourhood (which Einstein called the "hole") and with the gravitational field specified everywhere outside the hole, then generally covariant field equations would be unable to determine uniquely the gravitational field within the hole, no matter how small the hole. Naturally this provided much comfort to Einstein, who could now regard his failure to find generally covariant field equations as unimportant. He need not doubt that such field equations were possible, but there was no point in pursuing them since they would be physically uninteresting.[6]

The hole argument was published four times by Einstein. In order of publication dates, they were Einstein and Grossman (1913b), pp. 260—261;[7] Einstein (1914a), p. 178; Einstein and Grossmann (1914), pp. 217—218; and Einstein (1914b), pp. 1066—1067. The first three of these were essentially the same. I quote the second:[8]

> *If the reference system is chosen quite arbitrarily, then in general the g_{mn} cannot be completely determined by the* $\mathbf{T}_{sn}$. For, think of the $\mathbf{T}_{sn}$ and g_{mn} as given everywhere and let all $\mathbf{T}_{sn}$ vanish in a region Φ of four dimensional space. I can now introduce a new reference system, which coincides completely with the original outside Φ, but is different to it inside Φ (without violation of continuity). One now relates everything to this new reference system, in which matter is represented by $\mathbf{T}'_{sn}$ and the gravitational field by g'_{mn}. Then it is certainly true that
>
> $$\mathbf{T}_{sn} = \mathbf{T}'_{sn}$$
>
> everywhere, but unlike them the equations
>
> $$g'_{mn} = g_{mn} \tag{3}$$
>
> will definitely not all be satisfied inside Φ. The assertion follows from this.
>
> If one wants a complete determination of the g_{mn} (gravitational field) by the $\mathbf{T}_{sn}$ (matter) to be possible, then this can only be achieved by a limitation on the choice of reference systems. [Einstein's italics]

Reduced to its essentials, Einstein's argument appears to run as follows:

1. Consider a metric within the matter-free hole with components g_{mn} in some coordinate system. The metric satisfies the generally covariant source free field equations $G_{mn} = 0$, for some boundary condition specification of the metric and source matter distribution everywhere outside the hole.
2. Introduce a new coordinate system within the hole which agrees smoothly with the original coordinate system outside the hold. In the new coordinate system, the components of the gravitation tensor still vanish, i.e. $G'_{mn} = 0$, since G_{mn} is a zero tensor (in accord with

the discussion in Section 2). Thus the new components of the metric tensor g'_{mn} still satisfy the source free field equations.

3. Therefore we have a case of a unique boundary condition outside the hole but two distinct fields within, both satisfying the field equations.

Einstein's argument seems to rest on a simple beginner's blunder. It is certainly the case that the *components* of the metric tensor will differ in the new and old coordinate system within the hole, so that the equality of (3) will fail within the hole. But the failure of this equality does not mean that one has arrived at a different metric tensor as is claimed in step 3. Rather we only repeat the well known result that different matrices of components can represent the same metric in different coordinate systems.

Whilst it is difficult to imagine that Einstein could commit such a beginner's blunder repeatedly on a question which had his devoted attention for nearly three years, many commentators have been unable to resist convicting him of it. The most recent is Pais, 1982, pp. 221–222. What makes this 'blunder account' untenable is the footnote Einstein appended to the sentence containing equation (3). (An equivalent footnote did not appear in the first or third versions of the argument cited.) It read

> The equations are to be understood in such a way that each of the independent variables x'_n on the left-hand side are to be given the same numerical values as the variables x_n on the right-hand side.

In the blunder account there is simply no good reason for Einstein to insist on this perverse way of reading the equation. The only reading of the hole argument compatible with it is one in which the transformation introduced in step 2 is understood in the active sense, in which case Einstein's argument becomes far from trivial.

I now review the active and passive view of transformations. The coordinate transformations discussed in the last section are generated from a smooth map from R^4 to R^4 which assigns the 4-tuple x'^n to the 4-tuple x^n. This map can be used in two ways:

Passive view: Coordinate transformation. The map is used to generate a new spacetime coordinate system x'^n from x^n. That is, it is used to relabel the points of the manifold with different coordinates.

Active view: Point transformation. The map is used to generate a another map *in the manifold* which will smoothly assign points in the manifold to other points in the manifold. Represent this induced map by h. Then the point p with coordinates x^n will be mapped onto the point hp with coordinates x'^n *in the same coordinate system*. If h is invertible and both it and its inverse are continuous and infinitely differentiable — which is the case usually dealt with — then h is called a diffeomorphism.

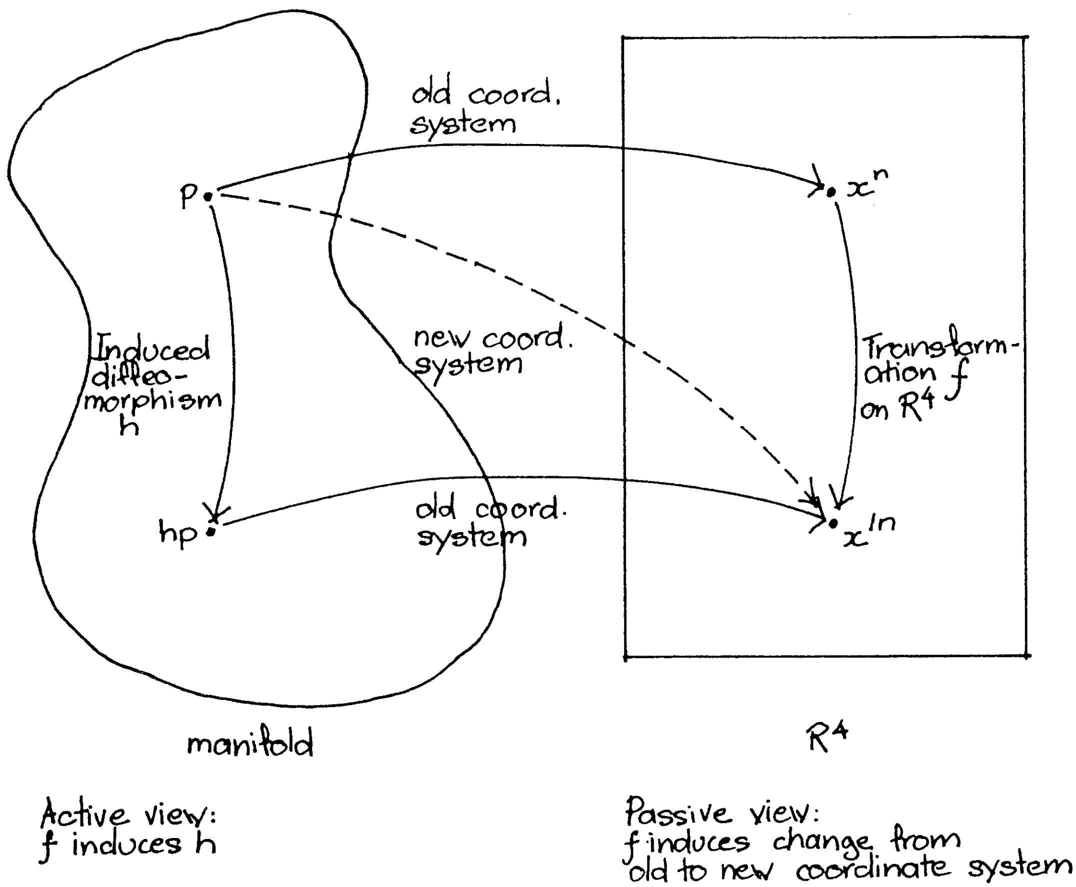

Fig. 4. The active and passive view of transformation.

Each h induces another map, h^*, the *carry along*, which maps structures defined on the manifold at a point p to structures defined on the manifold at hp. Thus h^* defines a carried along coordinate system. The carried along coordinates h^*x^n at hp are defined naturally by the requirement that they be numerically equal to the coordinates of x^n of p. Similarly the carried along metric h^*g_{ab} is defined by the requirement that:

> The components of the carried along metric h^*g_{ab} at hp in the carried along coordinate system at hp are numerically equal to the components of the original metric g_{ab} at p in the original coordinate system.

Thus if primed indices represent the carried along coordinate system and unprimed indices the original coordinate system, this amounts to requiring

$$(h^*g)_{m'n'}(hp) = g_{mn}(p) \tag{4}$$

The inverse of h^* is the "pull back".

Diffeomorphism represents the gauge freedom of tensor field equations. Recall the earlier result that if a tensor has all zero valued components in one coordinate system, then it has all zero valued components in all coordinate systems and is the zero tensor. Thus it follows from the above rule that the carry along of a zero tensor will still be a zero tensor. Therefore if a metric tensor g_{ab} satisfies a tensorial gravitational field equation $G_{ab} = 0$, it then follows that the carry along of g_{ab} will also satisfy the field equation. For the carry along of G_{ab} will still vanish.

We now ask how to test whether h^*g_{ab}, the carry along of a metric tensor g_{ab}, will be the same tensor as g_{ab}, the original metric tensor. We begin with equation (4), which does not allow immediate comparison because the matrices of components on either side of the equation belong to different coordinate systems. The easiest way to compare the carry along h^*g_{ab} and the original g_{ab} is to transform the components of the carry along in (4) from the carried along coordinate system back to the original coordinate system. To do this, we must carry out the following operation:

> *Algorithm for comparing components of g_{ab} and h^*g_{ab} in the same coordinate system.* We take the matrix $g_{mn}(p)$, which comprises the components of g_{ab} in the original coordinate system, transform it to the new coordinate system x'^n and compare the resulting matrix of components with the components of the original metric at hp. To ensure that we compare metrics at the same point in the manifold (which here is hp), we recall that the matrix of components will be a function of the coordinates and must insist that the comparison be carried out for matrices with equal coordinate values (here the coordinate values of hp).

But this operation is precisely the 'perverse' reading of equation (3) upon which Einstein insisted in the footnote to the hole argument quoted above! Thus the only reasonable conclusion is that Einstein viewed the transformation of the hole argument actively and that this recipe did genuinely yield a new metric, the carry along of the original, and that the failure of the equality in equation (4) shows that the new metric does differ from the original within the hole. Finally it follows from the above discussion that the carried along metric will satisfy the field equations if the original metric already does, so completing Einstein's argument.

Einstein seemed to realize that his first three versions of the hole argument were not transparent. In his fourth and final version, he went to great pains to remedy this defect and in particular to show that he did intend the transformation to be viewed actively.

> We consider a finite region of the continuum Σ, in which no material process takes place. Physical occurrences in Σ are then fully determined, if the quantities g_{mn} are given as functions of the x_n in relation to the coordinate system K used for description. The totality of these functions will be symbolically denoted by $G(x)$.
>
> Let a new coordinate system K' be introduced, which coincides with K outside Σ, but deviates from it inside Σ in such a way that the g'_{mn} related to the K' are continuous everywhere like the g_{mn} (together with their derivatives). We denote the totality of the g'_{mn} symbolically with $G'(x')$. $G'(x')$ and $G(x)$ describe the same gravitational field. In the functions g'_{mn} we replace the coordinates x'_n with the coordinates x_n i.e. we form $G'(x)$. Then, likewise, $G'(x)$ describes a gravitational field with respect to K, which however does not correspond with the real (or originally given) gravitational field.
>
> We now assume that the differential equations of the gravitational field are generally covariant. Then they are satisfied by $G'(x')$ (relative to K'), if they are satisfied by $G(x)$ relative to K. Then they are also satisfied by $G'(x)$ relative to K. Then relative to K there exist the solutions $G(x)$ and $G'(x)$, which are different from one another, in spite of the fact that both solutions coincide in the boundary region, i.e. *occurrences in the gravitational field cannot be uniquely determined by generally covariant differential equations for the gravitational field.*
>
> Einstein (1914b, pp. 1066–1067) [Einstein's italics.]

Einstein's use of the "$G(x)$" notation is not standard, but his purpose is clear enough. He carefully acknowledges that a mere coordinate transformation cannot produce a new field — "$G'(x')$ and $G(x)$ describe the same gravitational field." Rather he uses the transformation to generate a new field with the required property. That new field is $G'(x)$, which we can identify as the carry along of the original metric, or, more precisely, the components of the carry along in the original coordinate system. The comparison of $G'(x)$ with $G(x)$ implements exactly the above algorithm for comparing h^*g_{ab} and g_{ab}.

Thus in summary Einstein's hole argument, when read actively, has the force Einstein claimed. It amounts to the following:

1. Consider a metric g_{ab} within the matter free hole. The metric satisfies the generally covariant source free field equations $G_{ab} = 0$, for some boundary condition specification of the metric and source matter distribution everywhere outside the hole.
2. Let h be a diffeomorphism which maps points within the hole to different points within the hole and which smoothly becomes the identity map everywhere outside the hole. Because of the tensor nature (general covariance) of the field equations, the carry along h^*g_{ab} will still satisfy the field equations. But in the general case, the carry along will differ within the hole from the original metric.
3. Therefore we have a case of a unique boundary condition outside the hole but two distinct fields within, both satisfying the field equations.

Einstein concluded that such a violation of physical determinism was unacceptable and that the only way out was to deny use of generally covariant field equations.

4. THE POINT-COINCIDENCE ARGUMENT

Einstein's fourth version of the hole argument was communicated to the Prussian academy in October 1914. A year later Einstein had completely lost confidence in the *Entwurf* field equations and returned in desperation to the search for generally covariant field equations. He communicated a new set of field equations to the Prussian academy on November 4. He submitted a modified version on November 11. The following week on November 18 he submitted his celebrated explanation of the then anomalous motion of Mercury. But he still did not have the modern field equations with the Einstein tensor as gravitation tensor. These field equations — the third set to be offered by him in the course of a month — were communicated to the Prussian academy on November 25.[9]

The following month Einstein wrote to Ehrenfest and reflected upon the momentous events of the past month.[10]

It is comfortable for Einstein. Each year he retracts what he wrote the previous year; now my duty is the extremely sad business of justifying my most recent retraction.

Nowhere in Einstein's frantic communications of November 1915 to the Prussian academy had he explained how the hole argument could be reconciled with his new generally covariant field equations. This task was the "sad business" to which Einstein now turned. He addressed the fourth statement of the argument. (It had appeared as section 12 of Einstein (1914b) and its first three paragraphs were quote above.) Einstein continued:

In §12 of my work of last year, everything is correct (in the first three paragraphs) up to the italics at the end of the third paragraph. One can deduce no contradiction at all with the uniqueness of occurrences from the fact that both systems $G(x)$ and $G'(x)$, related to the same reference system, satisfy the conditions of the grav. field. The apparent force of this consideration is lost immediately if one considers that

(1) the reference system signifies nothing real
(2) that the (simultaneous) realization of two different g-systems (better said, two different grav. fields) in the same region of the continuum is impossible according to the nature of the theory.

In the place of §12 steps the following consideration. The reality of the world-occurrence (in opposition to that dependent on the choice of reference system) subsists *in spacetime coincidence*.* For example the intersection

[Footnote] *and in nothing else!

points of two different world lines are real, as is the assertion that they do *not* intersect one another. Those assertions, which refer to physical reality, are not lost then through any (unambiguous) coordinate transformation. If two systems of $g_{\mu\nu}$ (or [more] gen.[erally], variables used for describing the world) are so constituted, that one can obtain the second from the first merely through a space-time-transformation, then they refer to exactly the same thing [voellig gleichbedeutend]. For they have all timespace coincidences in common, i.e. all that is observable. This consideration shows immediately how natural is the requirement of general covariance. [Italics in the original.]

The argument developed above is the point-coincidence argument, which we can now identify as Einstein's answer to his hole argument. Seen in this context it is clear why it is nearly impossible for modern readers to understand the point-coincidence argument if they only read the well known version given in Einstein (1916), which was quoted above in the introduction.

First, modern readers will not be able to see why there is a need for any such argument at all for general covariance, for it is a minimum requirement of any well formulated modern spacetime theory. In particular there is no hint in Einstein (1916) of the long period of doubt about general covariance which preceded the paper, let alone any mention of the hole argument.[11] Second, it is by no means clear that the argument seeks to establish anything more than the following: a spacetime coordinate system is a smooth but otherwise arbitrary labelling of events with four numbers; therefore no theory can suppose that any coordinate system is distinguished or preferred independently of the other structures defined on the manifold. Why, we must ask, would Einstein seek to derive this entirely straightforward result from contentious assertions about reality being constituted of spacetime coincidences? Finally, it is not clear even after reading Einstein's letter to Ehrenfest, that the point-coincidence does reconcile the hole argument with general covariance.

In short I will show that we can retain these objections only as long as we read the transformation it invoked passively. If we read it actively — and I shall urge that there is good reason to do so — then Einstein's argument becomes cogent and makes a strong case for the results claimed. The argument will be broken up into two steps. The first argues for what I call "Leibniz equivalence"; from it, the second seeks to establish the naturalness of general covariance, now understood in an active and non-trivial sense.

I begin by making the argument more precise. Represent a model of a generally covariant gravitation theory as the ordered triple $\langle M, g_{ab}, T_{ab}\rangle$, where M is a four dimensional manifold, g_{ab} a Lorentz signature metric and T_{ab} a stress energy tensor. In accord with the usual convention, g_{ab} and T_{ab} represent two tensor fields in coordinate free fashion. Of course each tensor can be represented by a matrix of components g_{ik} and T_{ik}. The corresponding model of the theory based on components in coordinate system K is represented by $\langle M, K, g_{ik}, T_{ik}\rangle$.

The basic assertion of the point-coincidence argument is made most clearly towards the end of the passage quoted above from Einstein's letter to Ehrenfest: two systems of spacetime quantities represent the same physical system if they are related by a coordinate transformation, for then each yields identical observables, that is, spacetime coincidences.

Point-Coincidence Argument (*Passive Reading*)

Thesis: Two models $T_1 = \langle M, K_1, g_{ik}, T_{ik}\rangle$ and $T_2 = \langle M, K_2, g'_{ik}, T'_{ik}\rangle$

represent the same physical system in case T_1 becomes T_2 under coordinate transformation from K_1 to K_2.

Justification: The transformation from K_1 to K_2 preserves spacetime coincidences, which are the only observables.

Under this passive reading, the thesis becomes trivially true. It merely reminds us that coordinate transformations do not alter quantities, but only the matrices of components which represent them. The justification offered does not establish this trivial thesis. It is just irrelevant to it.

If we read the transformation actively — that is, as a diffeomorphism induced by the coordinate transformation — we have:

Point-Coincidence Argument (Active Reading)

Thesis (Leibniz equivalence): Two models $T_1 = \langle M, g_{ab}, T_{ab}\rangle$ and $T_2 = \langle M, g'_{ab}, T'_{ab}\rangle$ represent the same physical system in case there exists a diffeomorphism h such that the carry along of T_1 is T_2. Then we have

$$\langle M, g_{ab}, T_{ab}\rangle = \langle hM, h^*g_{ab}, h^*T_{ab}\rangle$$

Justification: The diffeomorphism h preserves spacetime coincidences, which are the only observables of the system.

T_1 and T_2 are said to be diffeomorphic. Thus a convenient expression for the thesis of the active point-coincidence argument is

Leibniz equivalence: Diffeomorphic models represent the same physical system.

On this active reading, the argument is far from trivial. A model and its carry along are quite distinct mathematical structures. That they represent the same physical system is a claim which requires justification. A verificationist justification is provided. Observables are claimed to be preserved under the carry along, so that a model and its carry along agree on all observables.[12] To insist that a model and its carry along represent different physical systems, is to insist that there are physical systems which differ in some property, even though there can be no possible observational verification of the difference.[13]

Einstein supports the claim that observables are preserved under carry along by asserting that all observables can be reduced to coincidences, that is, to the relative points of intersection of physical systems. What is not preserved is the locations of these coincidences in the manifold. But these locations are in principle unobservable. Consider, for example, a model in which the system of fields representing a ray of light strikes a system of fields representing a photographic plate at its midpoint. Then that coincidence will be preserved in an arbitrary carry along of the model, even though the coincidence will be located at a different point in the manifold. Moreover that it is located at a different point has no observational consequences at all.

The active reading of the argument (but *not* the passive reading) does release Einstein from the problem of the hole argument. Recall that the indeterminism established by the hole argument was a consequence of our ability to take a solution of a tensor field equation and produce arbitrarily many diffeomorphic replicas, which still satisfied the field equation for the same boundary conditions but were nevertheless distinct from the original. Leibniz equivalence eradicates such indeterminism by asserting that all these diffeomorphic replicas represent the same physical system. The fields within the hole are *mathematically* underdetermined, but not *physically*, underdetermined since the allowed fields all represent the same physical situation. Thus the generation of diffeomorphic copies of the original solution within the hole amounts to the exercising of a gauge freedom akin to that of electrodynamics. Given any solution of Maxwell's equations in terms of a scalar and a vector potential, we can generate arbitrarily many more mathematically distinct solutions by a change of gauge, but each solution still represents the same electric and magnetic field.

What grounds are there for reading the point-coincidence argument activity? There are several. To begin, coherence points directly to the active reading. Only the active reading is not trivial and the justification offered actually relevant to the thesis. Only the active reading does what Einstein claimed, namely, resolve the hole argument. Some of Einstein's contemporaries — Schlick for example[14] — clearly read the point-coincidence argument actively. The two versions of the point-coincidence argument quoted so far are most naturally read passively by modern readers. We cannot allow this to rule out the active reading as Einstein's intended reading. We have already seen in the case of the hole argument that Einstein simply failed to make clear in two of four versions that he intended the transformation to be viewed actively. In another, as we have seen, he flagged this fact just by an opaque footnote. In the fourth and final version he made his intention clear only by use of a clumsy and non-standard notation.[15]

The strongest evidence for the active reading arises from the scepticism of another of Einstein's contemporaries. Ehrenfest was not convinced by Einstein's letter of 26 December 1915 of the admissibility of general covariance. He presented Einstein with a counterexample in a letter which I believe is no longer extant. Einstein responded in a letter to Ehrenfest of 5 January 1916 (EA 9 372). Reconstructing the counterexample from Einstein's response, it dealt with the system of a star, an aperture and a photographic plate, illuminated through the aperture by the star. Einstein explained that "Your difficulty has its root in the fact that you instinctively treat the reference system as something 'real'." He then continued:[16]

Your example somewhat simplified: you consider two solutions with the same boundary conditions at infinity, in which the coordinates of the star, the material points of the aperture and of the plate are the same. You ask whether "the direction of the wave normal" at the aperture always comes out the same. As soon as you speak of "the direction of the wave normal *at the aperture*," you treat this space with respect to the functions $g_{\mu\nu}$ as an infinitely small space. *From this and the determinateness of the coordinates of the aperture it follows that the direction of the wave normal AT THE APERTURE for all solutions* are the same.[17]

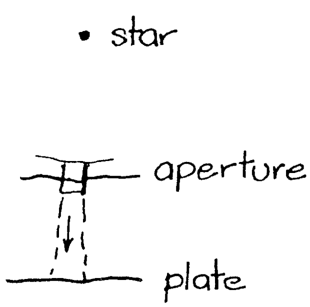

Fig. 5.

This is my thesis. For more detailed explanation [I offer] the following. In the following way you recover all solutions allowed by general covariance in the above special case. Trace the above little figure onto completely deformable tracing paper. Then deform the tracing paper arbitrarily in the plane of the paper. Then make a carbon copy back onto the writing paper. Then you recover e.g. the figure

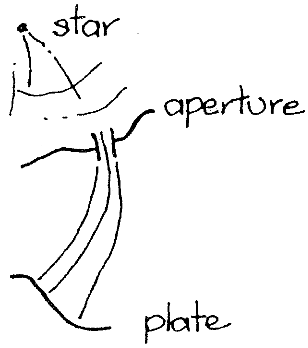

Fig. 6.

When you relate the figure once again to orthogonal writing paper coordinates, the solution is mathematically different from the original, and naturally also with respect to the $g_{\mu\nu}$. But physically it is exactly the same, since the writing paper coordinate system is only something imaginary. The same point of the plate always receives the light. If you carry out the distortion of the tracing paper only in the finite and in such a way that the picture of the star, the aperture and the plate do not lose continuity, then you recover the special case to which your question relates.

The essential thing is: as long as the drawing paper, i.e. "space", has no reality, then there is no difference whatever between the two figures. It [all] depends on coincidences . . .

[Italics in the original.]

Einstein makes clear here that he intends the transformation used in the point-coincidence argument to be read actively as a diffeomorphism. This diffeomorphism is represented appropriately by a distortion of the tracing paper. Its carrying along of structures is represented very graphically by the carrying along of lines of a drawing by the distortion. The comparison of the two structures in question is clearly intended to be carried out in the same coordinate system, the orthogonal system of the writing paper, as required in an active (but not passive) reading of the argument. Einstein then continues in the following paragraph to give an *active* account of what it is for a theory to lack general covariance:

If the equations of physics were not generally covariant, then you certainly could not carry out the above argument; but relative to the writing paper system the same laws would not hold in the *second* figure as in the first. Then to this extent both would still not be equally justified. This difference falls away with general covariance.

[Italics in the original.]

In more modern terms we would say that a theory is generally covariant in the active sense if it satisfies the following condition:

General covariance of a theory (active reading): If T is a model of the theory, then a carry along of T under an arbitrary diffeomorphism is also a model of the theory.

Since the relation of being diffeomorphic is an equivalence relation, it follows that the set of models of a generally covariant theory can be divided into equivalence classes of diffeomorphic models. It is important to see that the requirement of general covariance is not the same as Leibniz equivalence. The former provided for the existence of equivalence classes of diffeomorphic models within the set of models of the theory. The latter requires that each member of a given equivalence class represents the same physical system.

For comparison, we can formulate the general covariance of a theory in the passive sense as follows. The formulation is specific to theories with models of the form $\langle M, g_{ab}, T_{ab}\rangle$. but its generalization is obvious.

General covariance of a theory (passive reading). If $T_1 = \langle M, K_1, g_{ik}, T_{ik}\rangle$ is a model of a theory, then so is any model $T_2 = \langle M, K_2, g_{i'k'}, T_{i'k'}\rangle$ *where* $g_{i'k'}$ and $T_{i'k'}$ are the matrices of components produced by transforming g_{ik} and T_{ik} from K_1 to K_2.

The two requirements of general covariance are not equivalent. It is easy to find examples of theories which satisfy one requirement but not the other. Consider, for example, a version of special relativity whose sole model is a particular Minkowski spacetime $\langle M, n_{ab}\rangle$, where M is a four dimensional manifold and n_{ab} a Minkowski metric. In component terms it has a set of models, each of the form $\langle M, K, n_{ik}\rangle$, which contains just all coordinate systems K defined on M and all the component representations of n_{ab}. This theory is generally covariant in the passive sense. But the theory is not generally covariant in the active sense since by stipulation none of the diffeomorphic replicas of $\langle M, n_{ab}\rangle$ are models of the theory.

Fortunately the two requirements agree in a number of important cases. For example, consider a general relativity-like gravitation theory whose models are the set of all triples $\langle M, g_{ab}, T_{ab}\rangle$ which satisfy the field equation

$$H_{ab} = G_{ab} - kT_{ab} = 0 \tag{5}$$

were G_{ab} is some generally covariant gravitation tensor. The theory is obviously generally covariant in the passive sense. It is also generally covariant in the active sense. For the H_{ab} tensor of any model is the zero tensor. Therefore the H_{ab} tensor of the carry along under arbitrary diffeomorphism of the model will also be a zero tensor, given our earlier result that the carry along of a zero tensor is always a zero tensor. Note that the active general covariance of this class of theories, restricted to the case in which g_{ab} is source free, was the central result discussed in Section 3 in the context of the hole argument. Writing the field equation in the form of (5), enables us to drop the restriction to the source free case.

If we read general covariance actively, we can now take the final step and see how Einstein's argument proceeds from Leibniz equivalence to general covariance. From Leibniz equivalence we have: given a model of a theory which represents some physical system, we can generate arbitrarily many diffeomorphic replicas which represent the observables of the same system equally well. The requirement of general covariance "is a natural one," to quote Einstein (1916, p. 117), since it just allows that all of these diffeomorphic replicas are also models of the theory. Without Leibniz equivalence, the requirement of general covariance is not at all natural. It is simply disastrous, as the Einstein of 1915 and 1916 well knew. For the diffeomorphic copies admitted by general covariance need no longer represent the same physical system and one arrives immediately at the radical indeterminism of the hole argument.

5. WHAT DO THE HOLE AND POINT-COINCIDENCE ARGUMENTS ESTABLISH?

Leaving aside the historical issues, let us ask what are we warranted to conclude from these two arguments. I think there is only one clear and unambiguous conclusion which we can draw and which has direct impact on the application of general relativity: Leibniz equivalence. This equivalence is now incorporated as a matter of course into some of the better modern texts on general relativity, although there is no acknowledgement of Einstein's original adoption of it. (See Hawking and Ellis (1973, p. 56), Sachs and Wu (1977, p. 27) and Wald (1984, p. 438).)

But Einstein clearly felt in 1915 and 1916 that some kind of conclusion about the reality of spacetime could also be recovered. I consider three possibilities. The first is literally Einstein's proposal of 1915 and 1916, the non-realist proposal, which I will argue is *not* established by Einstein's arguments. The second is a proposal by John Stachel and the third is due to John Earman and myself.

Non-Realism About Spacetime

Einstein urges (1916, p. 117) that we conclude that general covariance "takes away from space and time the last remnant of physical objectivity." Non-realism about spacetime is the direct reading of Einstein's assertion and presumably the one he indended. It claims that the term "spacetime" (or correspondingly "space" and "time") have no referent in the physical world. The claim should be tightened just a little, since what is really at issue is not whether the English word "spacetime" has a referent, but a theoretical structure known as "spacetime" in general relativity. Here I take the manifold to be that structure and the non-realist claim about spacetime to be that the manifold refers to nothing in the physical world.

But what supports the claim is not clear. General relativity read literally posits that the actual world and other possible worlds are represented by spacetime manifolds with fields, such as g_{ab} and T_{ab}, defined on them. In the model representing the actual world, these fields refer to real physical fields. Correspondingly the points of the manifold refer to real physical events.

The hole and point-coincidence arguments complicate the issue by making it impossible to determine which point of the manifold refers to which physical event without considering the fields defined on the manifold. The trouble is that according to Leibniz equivalence the same manifold can figure in two different (diffeomorphic) models which represent the same physical system. Imagine for example that the point p of the manifold M refers to some event — say the collision of two cars — in the model $\langle M, g_{ab}, T_{ab}\rangle$. Then the same point p will in general not refer to the same event in the diffeomorphic model $\langle M, h^*g_{ab}, h^*T_{ab}\rangle$. Rather the different point hp will refer to that event.

This difficulty however does not establish the non-realist claim. The relativization of reference is not the same as the elimination of reference entirely.

Spacetime Events Lose Individuation

The loss of individuation of spacetime events is the basic conclusion which Stachel urges we draw from the hole and point-coincidence arguments.[17a] As I understand it, Stachel's conclusion amounts to the difficulty mentioned above: the physical events to which points of the manifold refer cannot be determined from manifold structure alone. Their reference is determined by the fields defined on the manifold.

Recall that a spacetime coordinate system gives us a numerical labelling which enables us to distinguish points of the manifold from one another. So presumably Einstein had in mind such a loss of individuation when he sought to clarify his view to his correspondents in 1915 and 1916. He stressed to Ehrenfest (26 December 1915) that "the reference system signifies nothing real" and similarly to Besso (3 January 1916) that the coordinate "system K has no physical reality."[18] (Speziali, 1972, pp. 63—64) He returned to precisely this point in his letter of 6 January 1916 to Ehrenfest, describing a failure to grasp it as the root of Ehrenfest's objection to general covariance.

Thus Stachel (1985, Section 4) writes of the hole argument:

> The main difficulty here was to see that the points of the space-time manifold (the "events" in the physical interpretation) are not individuated *a priori* but inherit their individuation, so to speak, from the metric field.

Stachel's response (1985, Section 6) to this loss of individuation is to cease representing physical events by points of the manifold in the case of spacetime theories without absolute objects, such as general relativity. In this case he represents physical events by structures in the fibre bundle formed from the manifold and geometric objects definable on it. (Specifically they are a set of maps from the manifold into cross-sections of the bundle.) In this way, the ambiguity of reference can be avoided since the geometric objects which determine that reference are automatically incorporated in the structure. I refer the reader to Stachel's paper for details of this construction and of his general proposal concerning spacetime theories with and without absolute objects.

Refutation of Spacetime Substantivalism

John Earman and I (Earman and Norton, forthcoming) have argued that the hole and point-coincidence arguments amount to a decisive refutation of the doctrine of spacetime substantivalism for a large class of spacetime theories. When the arguments are suitably generalized, that class includes general relativity as well as spacetime formulations of Newtonian theory and special relativity. I limit the discussion here to the case of general relativity.

According to this doctrine, spacetime is held to have an existence independent of anything it contains. The doctrine is best known through Newton's views towards absolute space and absolute time, whose properties are asserted to be entirely independent of the matter they contain. In particular one can have Newtonian absolute spaces and times devoid of matter. An exactly analogous formulation of the doctrine is not possible in the spacetime case within general relativity. For general relativity posits that every spacetime has both a manifold and a metric. By hypothesis, we cannot have a spacetime, understood to be the manifold, without the metric field it must contain. This renders spacetime substantivalism false by hypothesis in general relativity. Clearly this analysis resolves the question too cheaply, for spacetime substantivalism is not usually regarded as analytically false in general relativity.[19]

At this point the natural move is to seek an acceptable reformulation of the doctrine of spacetime substantivalism. Is it captured, we might ask, by the assertion that spacetime is not reducible to other structures; or that we must quantify unavoidably over spacetime events; or in Stachel's notion of no independent individuation of the points of the manifold? Fortunately we do not need to embark on this laborious quest. For our purposes it suffices that spacetime substantivalists must all agree on a simple acid test. The test is best known through Leibniz's challenge to Newtonian space substantivalists: would God have created a different universe if he had placed all the masses in it reversed East to West, but otherwise preserving all relations between them? Newtonian space substantivalists must concede that the new universe would be different to the old one, since the bodies in the new one are at quite different spatial locations, even though there would be no observable difference between the two universes.

The spacetime analogue of reflecting systems of masses East-West in space is a carry along by diffeomorphism over the manifold. Correspondingly, spacetime substantivalists, irrespective of the precise formulation of their views, must agree that diffeomorphic models of a spacetime theory represent different physical systems, for the fields are now located at different points in the manifold. We can express this by:

Leibniz test for spacetime substantivalism: Spacetime substantivalists must deny Leibniz equivalence.

But if spacetime substantivalists must deny Leibniz equivalence, then they face dire consequences. The hole argument forces them to agree that general relativity, with generally covariant gravitational field equations, is subject to what Earman and I call "radical local indeterminism". That is, the metric field within any neighbourhood of the spacetime manifold, no matter how small, is not uniquely determined by even the most complete specification of the fields outside that neighbourhood.[20] And in the case of the point-coincidence argument, they must insist that it is possible for there to be distinct systems which no possible observation could distinguish.

Thus we can summarize the import of the hole and point coincidence arguments for spacetime substantivalists in the form of two dilemmas:

Indeterminism dilemma (*Hole argument*): Spacetime substantivalists must either
(a) accept radical local indeterminism in general relativity, or
(b) deny their substantivalism.

Verificationist dilemma (*Point-coincidence argument*): Spacetime substantivalists must either
(a) accept that there are distinct systems which are observationally indistinguishable, or
(b) deny their substantivalism.

It is hard to imagine that even the most hardened of spacetime substantivalists could cling onto their doctrine in the face of these dilemmas. Perhaps they may do so in the case of the verificationist dilemma, given that verificationism is no longer fashionable. But surely the spectre of radical local indeterminism in the other dilemma is far too high a price to pay for a doctrine that adds nothing predictively to general relativity.

6. FROM NON-REALISM TO REALISM

If the Einstein of 1915 and 1916 held to non-realism about spacetime, he did not retain this belief for very long. By 1920 he had clearly shifted from non-realism (spacetime has no existence) to antisubstantivalism (spacetime has no existence *independent* of the fields it contains). This was a fortunate development since, as we have seen, the former view was not supported by his arguments, whereas the latter is most strongly supported. He wrote:

> There can be no space [spacetime] nor any part of space without gravitational potentials; for these confer upon space its metrical qualities, without which it cannot be imagined at all. The existence of the gravitational field is inseparably bound up with the existence of space. (Einstein, 1920, p. 21)

Since non-realist claims about spacetime disappeared from Einstein's writings from this time onwards, I conjecture that this antisubstantivalism was the conclusion drawn ultimately by him from the hole and point-coincidence arguments and general covariance — and perhaps even what was intended all along in his 1915 and 1916 non-realist remarks.

Antisubstantivalism appears frequently in Einstein's writings of the 1950's, even though sometimes it appears in the form of the slogan, no space without metric field. (See Einstein (1953) and, in his correspondence, letters to D. W. Sciama, 28 December 1950 (EA 20 469), to G. Sandri, 24 June 1950 (EA 20 449) and to M. Fischler, 9 September 1954 (EA 11 023).) The best known version of the claim is in the 1952 appendix, "Relativity and the Problem of Space," to Einstein (1917), his popular exposition of relativity theory (p. 155):

> In accordance with classical mechanics and according to the special theory of relativity, space (space-time) has an existence independent of matter or field . . . . On the basis of the general theory of relativity, on the other hand, space as opposed to "what fills space," which is dependent on the coordinates, has no separate existence . . . . If we imagine the gravitational field, i.e., the functions g_{ik}, to be removed, there does not remain a space of the the type (1) [Minkowski spacetime], but absolutely *nothing*, and also no "topological space." For the functions g_{ik} describe not only the field, but at the same time also the topological and metrical structural properties of the manifold . . . . There is no such thing as an empty space, i.e. a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field.
>
> [Einstein's italics.]

Einstein's stress here on viewing spacetime as a property of the metric field rather than an independent entity makes it possible for us to characterize the view as something slightly stronger than antisubstantivalism. The view is a relational view of spacetime. That is, spacetime arises as an abstraction from the spatiotemporal properties of other things.

Readers of Einstein (1916) may well be able to see how he could modify his non-realism about spacetime to antisubstantivalism. But surely they will be surprised by the hopes expressed in 1930 by an Einstein deeply embroiled in the search for a unified field theory (Einstein, 1930, p. 184):

> We may summarize in symbolical language. Space, brought to light by the corporeal object, made a physical reality by Newton, has in the last few decades swallowed aether and time and seems about to swallow also the field and the corpuscles, so that it remains as the sole medium of reality.

What complicates the whole discussion and lends an aura of contradiction to it is the fact that the term "spacetime" (or else "space" or "time") refer to different theoretical structures in different contexts.

In the analysis of spacetime substantivalism given by Earman and myself, spacetime is identified with the manifold. But it is harder to determine precisely what theoretical structure stands for "spacetime" in various of Einstein's writings. Presumably the Einstein of 1916 took spacetime to be the manifold. But the Einstein of 1930, who expresses the hope that space would become "the sole medium of reality," surely took space to be manifold plus metric or manifold plus the geometric structure of his unified field theory.

In the context of his slogan, no space without metric field, Einstein seems to take "space" to be certain spatiotemporal properties of a manifold with metric. Consider for example Einstein's (1953) remark that in a generally covariant field theory, "that which constitutes the spatial character of reality is then simply the four-dimensionality of the field"; and his remark in a letter to D. W. Sciama of 28 December 1953 (EA 469): "'Space' exists only as the continuum property of physical reality (field), not as a kind of container with independent existence, into which physical things are placed:" The difficulty with this reading of "space" is to make precise exactly which properties are in question. Stachel (Stachel, 1985, Section 6) makes the only attempt of which I am aware to deal with this problem.

The story of Einstein's change of viewpoint from non-realist (or perhaps just antisubstantivalist) to realist about spacetime is a fascinating one. Since it involves issues well beyond the scope of this paper, I can only mention a few of its highlights here.

The Einstein of 1915 and 1916, who rejoiced in the loss of objectivity of space and time, had by his own later admission (Einstein, 1946, p. 27) not appreciated fully the picture of reality demanded by a true field theory. Then he had sought to explain the origin of inertial forces solely in the interactions of bodies, alloting to fields a purely intermediate role. (Einstein, 1916, Section 2) The crucial insight, which he attributed to Mach, was that an epistemologically satisfactory mechanics could not admit inertial spaces as causes.

H. A. Lorentz, whom Einstein revered as a father figure, must have played some role in changing Einstein's mind. They corresponded extensively in the 1910's over relativity. Einstein conceded to him (15 November 1919, EA 16 494) that he had been hasty in concluding the non-existence of the aether from special relativity. He should only have concluded the non-reality of an aether *velocity*. The aether belonged in general relativity in so far as that theory posited spacetime as a bearer of physical qualities. Those qualities are the metric field. Thus Einstein began to portray general relativity as an aether theory and the term aether figured prominently in some of the titles of Einstein's papers. (See Einstein (1918, p. 702; 1920; and 1924).)

In particular, in a 1920 lecture at Leiden read before Lorentz, Einstein conceded that the Machian analysis of the origin of inertia no longer leads us to seek an account of inertia solely in the interactions of distant bodies, since we should no longer be prepared to posit action at a distance. Rather we are led to an aether, which "not only *conditions* the behavior of inert masses, but *is also conditioned* by them. Mach's idea finds its fullest expression in the aether of the general theory of relativity." (Einstein, 1920, p. 18; Einstein's italics)

Gradually Einstein replaced the term "aether" by "space" and with it the shift from non-realist to realist view of spacetime completed. Einstein now allowed that his Machian critique did not require a non-realist view of spacetime but the elimination of its preferred causal status. He summarized his changed viewpoint (Einstein, 1927, p. 260):

> The general theory of relativity formed the last step in the development of the programme of the field-theory ... Space and time were thereby divested not of their reality but of their causal absoluteness — i.e. affecting but not affected — which Newton had been compelled to ascribe to them in order to formulate the laws then known.

7. CONCLUSION

My story began in 1916 with a rather unconvincing verificationist argument from Einstein for general covariance and an associated non-realist claim about space and time. Even though the argument and claim are much cited and quoted, we found that they are rarely understood, largely because Einstein failed to include in his presentation virtually all the ingredients necessary for this understanding.

The notion of general covariance at issue was not merely the passive form of invariance of laws under arbitrary coordinate transformation. It was general covariance in the active sense under which any model of a theory belongs to an equivalence class of all possible models diffeomorphic to it. The crucial result was what I called Leibniz equivalence, that each member of one such equivalence class represents the same physical system. Einstein's verificationist argument makes good sense as an argument for Leibniz equivalence. Einstein did not mention the vital link which connected Leibniz equivalence to general covariance. Leibniz equivalence released him from the conclusion of his earlier hole argument, which was that general covariance would lead to a radical and unacceptable form of indeterminism in his gravitation theory. He also did not mention that the threat of this indeterminism in the hole argument could now be turned into an argument for Leibniz equivalence, which to modern eyes is stronger than the verificationist argument he offered. Perhaps Einstein felt that in 1916 an appeal to verificationism would be more readily accepted. Certainly any such expectation was vindicated by the enthusiastic response of such contemporary philosophers as Reichenbach and Schlick.

The non-realist claim about space and time entered only at the last moment of this episode and was dropped by Einstein within five years. We could find no argument within the episode to support this non-realism. Rather we extracted two dilemmas for those who hold to a related view, spacetime substantivalism, the view that spacetime has an existence independent of the fields it contains. These dilemmas force the rejection of that view. We found antisubstantivalist claims concerning spacetime common in Einstein's later work.

Einstein strove to express his ideas as simply and clearly as possible. Unfortunately sometimes his efforts backfired on him and he simplified his ideas to the point that they become unintelligible to even a diligent reader, as we have seen in the case here. I have not addressed the question of whether Reichenbach's or Schlick's reading of Einstein's work suffered from this difficulty. My story ends with a broader challenge to historians and philosophers of science: which other of Einstein's claims and arguments have been misunderstood for this reason?

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NOTES

* I wish to thank the Hebrew University of Jerusalem, Israel for its kind permission to quote the material in this paper from Einstein's unpublished writings, and Don Howard for discussion and comments on an earlier draft of this paper.

[1] A non-realist holds that "space" and "time" have no referents in the physical world. I would have preferred to use the term "anti-realist", but it has already been used by van Fraassen (van Fraassen, 1980, pp. 9—11) for a view which is agnostic about the existence of these referents.

[2] Einstein objected to this gloss also. For example he corrected Ehrenfest in correspondence of 1919 by noting that the novelty of special relativity in 1905 was not epistemological (non-existence of a resting aether) but empirical (equivalence of all inertial systems with respect to light). He allowed that epistemological demands came into play in 1907 when he commenced work on general relativity. But here too the empirically determined equality of inertial and gravitational mass played a significant role. A. Einstein to P. Ehrenfest, 4 December 1919, EA 9 451. (EA 9 451 refers to the document with control number 9 451 in the duplicate Einstein Archive, Mudd Manuscript Library, Princeton.)

[3] I have argued at length (Norton, 1985c) that this modern view was not Einstein's. He did not associate the presence of a gravitational field just with non-vanishing metrical curvature, but with the presence of a metric of any curvature. Thus a Minkowski spacetime for Einstein was already a special case of a gravitational field bearing spacetime. This was one of the crucial insights gleaned by Einstein from his principle of equivalence.

[4] I follow the usual modern conventions concerning indices. A sub- or superscripted $a, b, c, d, \ldots$ is used to represent the rank and type of a geometric object according to the abstract index convention. Thus g_{ab} represents a second rank covariant tensor. Sub- or superscripted $i, k, m, n, \ldots$ take values 0, 1, 2, 3 and are used to represent matrices of components of geometric objects. Thus a second rank, covariant tensor g_{ab} has components $g_{00}, g_{01}, \ldots, g_{23}, g_{33}$ in some coordinate system. These components are represented by g_{im}, where i, m are understood to take all values 0, 1, 2, 3.

[5] This was not Einstein's first argument against the physical acceptibility of general covariance. He had already argued against it as early as August 1913 on the basis of the limited covariance of the stress energy tensor of the gravitational field. See Norton, 1984, pp. 284—86.

[6] He did insist however that if his *Entwurf* field equations had any physical content, then they must have a generally covariant generalization. Einstein, 1914a, pp. 177—178.

[7] But the argument did not appear in the original separatum of this article, Einstein and Grossmann (1913a).

[8] For notational continuity, I have replaced Einstein's Greek indices by Latin indices both here and in the later version of the argument. Similarly $\mathbf{T}_{mn}$ is the stress energy tensor *density*, $\sqrt{-g}\ T_{mn}$, which Einstein denoted with a Gothic $\mathfrak{T}$.

[9] This episode is outlined in Norton (1984), (1985a) and (1985b) in which the first explanation of Einstein's need for three separate version of the field equations in this month is offered.

[10] A. Einstein to P. Ehrenfest, 26 December 1915, EA 9—363. Einstein presents

essentially the same arguments to Besso in slightly briefer form in A. Einstein to M. Besso, 3 January 1916, in Speziali (1972, pp. 63—64).

[11] Obviously I do not mean that Einstein was trying to suppress this episode. It would have been well known to any contemporary who had been following his theory in the literature. There was just no need for him to remind his readers of the embarrassing confusions of the previous three years.

[12] Thus the active reading makes sense of the remark of Einstein to Ehrenfest quoted above, where the passive reading does not: "Those assertions, which refer to physical reality, are not lost then through any (unambiguous) coordinate transformation."

[13] The denial that such observationally indistinguishable systems are different was called "Leibniz equivalence" above, since this was precisely the point Leibniz made to Clark in their celebrated correspondence when he asked how the world would differ if God had placed the bodies of our world in space some other way, only changing for example East into West. (Alexander, 1956, p. 26)

[14] Schlick's version, as quoted in Friedman (1983, p. 23), is very clear and simple — and unambiguously active.

[15] We can draw a useful moral here. If Einstein talks of coordinate transformation but his discussion is incoherent, it is worth considering the possibility that he may really mean the corresponding point transformation.

[16] The two figures shown have been redrawn after the sketches included in the original letter.

[17] I render the double underlining of "at the aperture" by uppercase italics.

[17a] For more details see Stachel 1985. Torretti (1983, Section 5.6) reviews the hole argument, the point-coincidence argument, Stachel's proposal and then offers what I believe amounts to Leibniz equivalence as a prefered alternative.

[18] The other point stressed in both letters was that it is impossible to realize simultaneously two different gravitational fields in the same neighbourhood of the manifold. If Einstein intends that the two fields are diffeomorphic (which is not clear), then I read this remark as a somewhat awkward statement of Leibniz equivalence.

[19] For general discussion of space, time and spacetime substantivalism, see Sklar (1977).

[20] Note that the argument has been generalized by dropping the requirement that the hole be matter free. The construction now requires that both g_{ab} and T_{ab} be carried along by the diffeomorphism and the field equations of form (5) used.

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