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The Inductive Significance of Observationally Indistinguishable Spacetimes

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Results on the observational indistinguishability of spacetimes demonstrate the impossibility of determining by deductive inference which is our spacetime, no matter how extensive a portion of the spacetime is observed. These results do not illustrate an underdetermination of theory by evidence, since they make no decision between competing theories and they make little contact with the inductive considerations that must ground such a decision. Rather, these results express a variety of indeterminism in which a specification of the observable past always fails to fix the remainder of a spacetime. This form of indeterminism is more troubling than the familiar indeterminism of quantum theory. The inductive inferences that can discriminate among the different spacetime extensions of the observed past are here called "opaque," which means that we cannot readily see the warrant that lies behind them.

1. Introduction[1]

The thesis of the underdetermination of theory by evidence, in its strong and interesting form, asserts that all evidence necessarily fails inductively to fix theory. In a recent paper (2008),

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[1] I am grateful to Claus Beisbart and John Manchak for helpful discussion on an earlier draft of this note..

I have joined with a current of thought in philosophy of science that urges that the thesis is groundless. A claim of assured underdetermination has, I argue, only been established in one, defective account of inductive inference, a crude form of hypothetico-deductivism. Other, more tenable accounts of induction do not sustain it and commonly rule against the claim. A second line of support for the thesis employs "observationally equivalent" or "empirically equivalent" theories. They are pairs of theories that agree on all possible observations. All evidence, it is then urged, must leave us powerless to decide between the two theories. In response, I have argued that the sorts of examples that can be developed in the confines of the journal literature must fail. For, if observational equivalence can be demonstrated so easily, the two theories must turn out to be so close in structure that we cannot preclude the possibility that they are merely variant formulations of a single theory. Of course, evidence ought not to pick apart variant formulations of what are factually the same theory.

The existence of observationally indistinguishable spacetimes in general relativity was brought to the attention of philosophy of science in papers by Clark Glymour (1977) and David Malament (1977). The basic claim is simple. An observer at any event in a spacetime is depicted as having full knowledge of all that transpires in the temporal past of that event. The temporal past is that set of events from which ordinary processes, propagating at less than the speed of light, may reach the observer's event. An observer in this spacetime may find that exactly the same observable past arises somewhere in a second spacetime. The first spacetime of the observer is observationally indistinguishable from the second if this finding is assured no matter where the observer may be in the first spacetime.

Our understanding of observationally indistinguishable spacetimes was decisively furthered by recent work by John Manchak (2009). He proved what had formerly been conjectured. That is, he showed that any well-behaved[2] spacetime always has many geometrically distinct, nemesis spacetimes from which it is observationally indistinguishable. Moreover the nemesis spacetimes will be "locally" the same as the observer's spacetime. In the first spacetime, one might have a condition that holds at each event, such as the Einstein

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[2] The theorem obviously excludes spacetimes in which the entire spacetime is observable from one event. They are, we are assured, "bizarre," because they include closed timelike curves, that is, curves that realize time travel.

gravitational field equations; or, more simply, a different condition that just asserts everywhere vanishing geometric curvature. The locality clause of the theorem assures us that the nemesis spacetimes will satisfy these same conditions.

The theorem and its proof involve some sophisticated spacetime geometry. But the basic idea behind them is very simple. A loose analogy, shown in Figure 1, illustrates it. Imagine that you are an ant on an infinite, flat (Euclidean) sheet of paper and that all you can survey is the surrounding 10,000 square foot patch. By surveying the patch, no matter where you might happen to be, you cannot distinguish your sheet from a nemesis sheet, which consists of a copy of the original sheet of paper rolled into a cylinder with a circumference of one mile.[3]

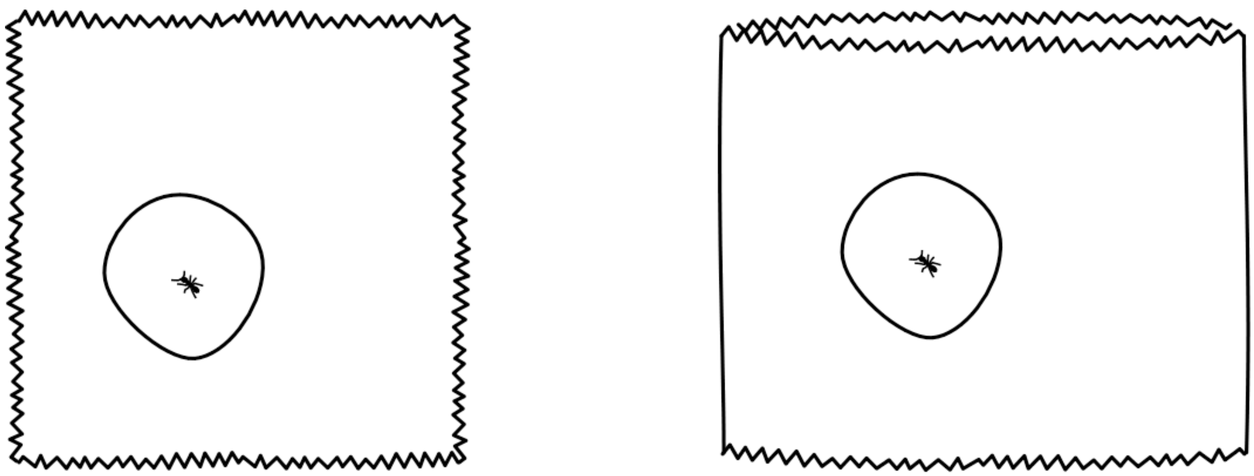

Figure 1. Ants on a sheet of paper

The similarity in the terms "observationally equivalent theories" and "observationally indistinguishable spacetimes" requires some thought.[4] Are the observationally indistinguishable spacetimes illustrations of the thesis of the underdetermination of theory by evidence? In this note, in Section 2, I will argue that they are not. Here I agree with the assessment of the opening remarks of Manchak (2009). He notes the distinctness of his result from the skeptical thesis in which acceptance of some particular scientific claim is resisted in the face of all evidence by revision to background theories. Rather, in Section 3, I will urge that these results on

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[3] If you think that rolling the sheet into a cylinder is mere topological trickery, then you will not be happy with the spacetime literature either. For the proof of the theorem and many of the examples depend essentially on vastly more extravagant topological trickery.

[4] It requires more thought than I gave it in writing footnote 13 of Norton (2008).

observationally indistinguishable spacetimes exemplify a different sort of failure manifested by physical theories, a form of generic indeterminism in general relativity that will prove to be more troubling than the familiar indeterminism of quantum theory. While we may have no means to distinguish deductively among different cosmic futures in the cases considered in this literature, I will urge in Section 4 that we can pick among them with quite familiar sorts of inductive arguments. Nonetheless, in Section 5, I will urge that inductions required for this discrimination are troubling in that they are what I shall call "opaque." That is, we cannot see through the inductive inferences to an unproblematic warrant. In this regard, they are unlike the inductive inferences that pick among the possible futures delivered by the indeterminism of quantum theory.

2. What the significance is not

The assurance of observationally indistinguishable spacetimes in general relativity fails to bear on the thesis on the underdetermination of theory by evidence in two ways.

First, the indistinguishability does not pertain to theory. We are not presented, for example, with general relativity and some competitor theory, indistinguishable from it. Rather, what we cannot distinguish is whether *this* spacetime is the one of our observations or whether it is *that* one. There is a slight ambiguity in the use of the term "theory." One might conceive an individual spacetime as a theory in its own right. The most prominent example would be Minkowski spacetime, whose spacetime geometry is, in effect, synonymous with the special theory of relativity. However this use is unnatural in general relativity, in which the particular spacetimes are models of the general theory.

Second, the indistinguishability is driven largely by deductive inference, whereas the thesis of the underdetermination of theory by evidence is concerned essentially with a failure of the determining power of inductive inference. Here is how it works. Many other spacetimes are logically compatible with what we can observe in spacetime. So we cannot deduce from what we observe which is our spacetime. As some sort of principled limit to the power of science, this sort of deductive indistinguishability is rather uninteresting. If we take it seriously, we will have to discard essentially all our science. No observation of energy conservation in our part of the world will enable us to distinguish our world from worlds in which energy conservation obtains and then suddenly fails at some time or other in the future. Of course inductive inference would indicate against the second sort of world. If we have always seen energy conserved, we would infer inductively that it will continue to be conserved.

Manchak's theorem, however, is stronger. It does preclude certain sorts of inductive inferences from distinguishing the spacetimes. Our observable spacetime is four-dimensional and has a Lorentz signature metrical structure. We are allowed the inductive inference that this will persist in the unobserved part. More generally, we are allowed to infer inductively to the persistence of any local condition, such as the obtaining of the Einstein gravitational field equations, in both the observer's and the nemesis spacetimes. These inductive inferences, the theorem shows, will still not enable us to separate the spacetimes, for both will agree on them.

What is not shown, however, is whether other inductive inferences would enable us to separate the two spacetimes. It is essential to the theorem that the observer's spacetime and its nemesis are factually distinct. What needs to be shown is that these factual differences cannot be exploited by an inductive inference that can separate the two spacetimes. I will suggest below in Section 4 that what appear to be quite routine inductive inferences are capable of exploiting these factual differences to discriminate a spacetime from its nemesis. In Section 5, however, I will urge that this routine appearance is deceptive in that the warrants of these inductive inferences are unclear.

3. What the Significance is

We can discern a more secure import of the results on observationally indistinguishable spacetimes by noting that the results amount to this: we fix some part of the spacetime and, within the context of some physical theory like general relativity, the rest of the spacetime remains undetermined. Characterized this way, we can see that the result is a form of indeterminism.

We have grown used to indeterminism in physical theories. It arises whenever the full specification of the present fails to fix the future. Indeterminism is routine in standard, collapse versions of quantum theory. The full specification of the present state of a radioactive atom does not determine when it will decay in the future.

Determinism, in one sense, arises commonly (but not always) in the spacetimes of general relativity. For example, it arises in the Robertson-Walker spacetimes of relativistic cosmology. If we fix the spacetime geometry and matter fields of the universe at one moment of cosmic time, they are fixed by the theory for all times. This time on which we fix the present state forms what is known as a Cauchy surface and it figures essentially in the mathematical theorems that delimit when Einstein's gravitational field equations have a physically unique solution. Versions of determinism defined in terms of these surfaces are mathematically quite useful. For the entirety of the history of the spacetime can be captured as the deterministic evolution in time of these surfaces.

Mathematical convenience, however, is not always what makes determinism or its failure interesting. In the case of quantum theory, its indeterminism is interesting and was shocking in the 1920s since it implied a profound limit on what we could know about the future. It told us that, in principle, no matter how much we knew about the present state of some suitably chosen quantum system, we could not know assuredly what it would do in the future.

This kind of principled epistemic limit on what we can and cannot know is not captured well by seeking to implement determinism in terms of the Cauchy surface "nows" of cosmic time in relativistic spacetimes. For no observer can observe the entirety of one of these surfaces. Rather, what an observer can access at one moment is delimited better by the observer's temporal past. Even though it represents an excessively optimistic assessment of our observational abilities, take what we know assuredly to be just everything within our temporal past. Then, even with this optimism about our abilities, the results on observationally indistinguishable spacetimes place powerful constraints on just what can be inferred directly from our spacetime theories about the remaining unobserved parts of our spacetime. They tell us that, even with the assistance of local spacetime theories, we can never assuredly fix a unique extension to the portion we have observed. In this regard these results are the appropriate analog of the indeterminism of quantum theory.[5]

However, there is a strong disanalogy to the indeterminism of quantum theory. Both forms of indeterminism express an impossibility of the past *deductively* determining the future. They differ markedly when we consider the possibility of *inductive* determination of the future.

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[5] Claus Beisbart has pointed out to me that, aside from Manchak's result, there is a familiar expectation of this sort of indeterminism. Fixing one's temporal past leaves open the possibility of influences propagating to one's future from spatial infinity or even just from spatially elsewhere in the region of spacetime outside one's past light cone.

While inductive discrimination is possible in both cases, as we shall see below, they employ rather different sorts of inductive inferences.

4. Some Cosmic Inductions

Inductive inferences can discriminate a spacetime from an observationally indistinguishable nemesis arising in the results on observationally indistinguishable spacetimes. A simple example illustrates the types of induction required. Consider a Minkowski spacetime. It is observationally indistinguishable from a "half Minkowski spacetime"; that is a Minkowski spacetime in which half has simply been excised. This excised half is the "t=0" hypersurface, in a standard coordinate system, and all events to its future. The observational indistinguishability depends on the fact that every observer's past in either spacetime is identical to every other observer's past in either spacetime; they are all geometric clones of one another. Figure 2 illustrates a typical observer's temporal past in each spacetime.

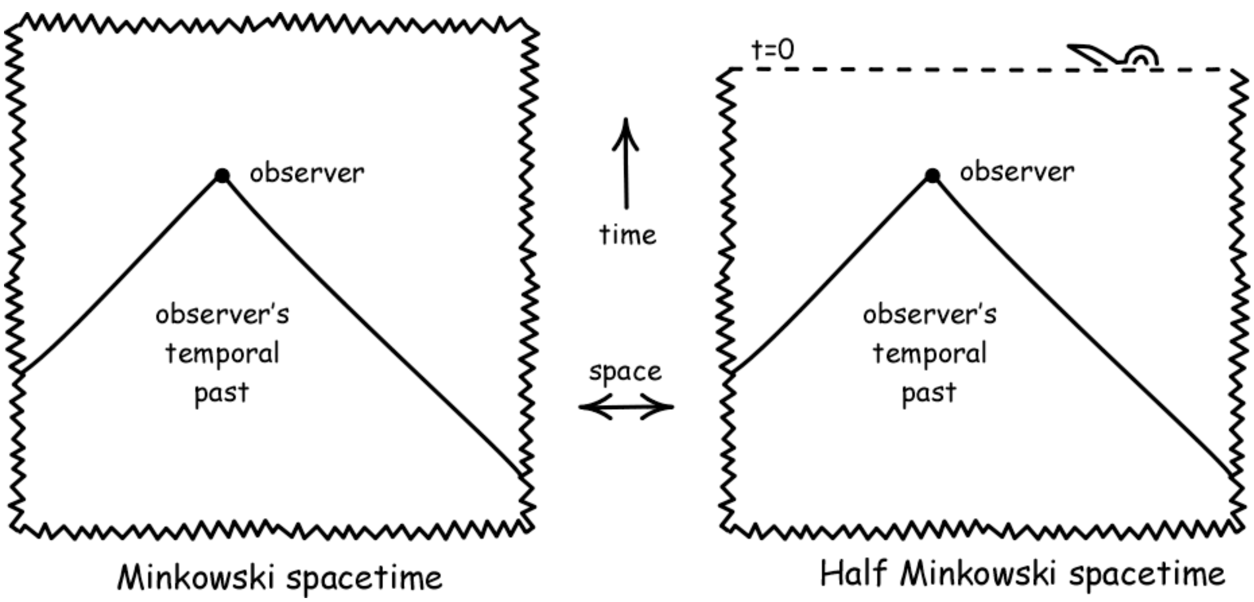

Figure 2. Minkowski and half Minkowski spacetimes

We now notice the following about timelike curves of any inertial observer in either spacetime. No such observer would ever detect a failure of the observer's world line to extend by a millisecond of proper time.[6] Indeed every observer could call upon a history in which they have

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[6] The parsing of the sentence is important. If the observer were in the half Minkowski spacetime, there would be cases of the worldline not extending by a millisecond. These would arise when

done the experiment of waiting a millisecond many times and have always found that their worldline was extended by a millisecond. The natural inductive inference would be that all future terminated inertial worldlines can always be extended by one millisecond of proper time. But that condition can only be met in the full Minkowski spacetime. Hence, even though the two spacetimes are observationally indistinguishable as far as deductive discriminations are concerned, this induction indicates in favor of the full Minkowski spacetime. The millisecond extension of the worldlines of two inertial observers is illustrated in Figure 3.

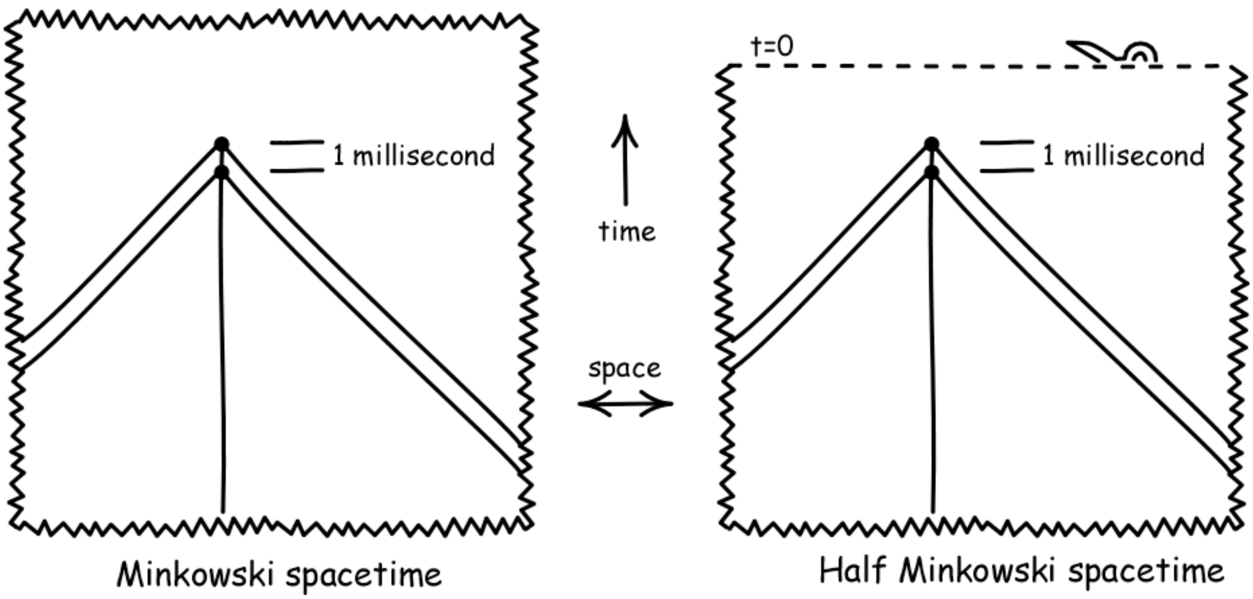

Figure 3. Millisecond extensions of inertial observers' worldlines

Of course the use of the extendable half Minkowski spacetime calls upon a peculiar spacetime. It is standard to rule out such spacetimes since they can be extended merely by adding more pieces. But then I find these extendable spacetimes only a little more peculiar than the constructions used to generate indistinguishable spacetimes. Where the nemesis of a Minkowski spacetime was created by subtracting spacetime structure, more common examples in the literature create the nemeses by adding. The ingenious chain construction of Manchak's proof requires us to build an infinity of duplicate spacetimes and then stitch them together in an infinite chain by what amounts to wormholes. In the case of a full Minkowski spacetime, observers

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the observer's worldline ran into the non-existent t=0 excision. However exactly because the observer has run into it, that observer ceases to exist and no record of the failure is registered by that or any other observer.

would never detect any such wormholes in the portions of spacetime they observe. They remain unsure of whether, as time passes, such a wormhole link to the duplicated Minkowski spacetimes will eventually enter the growing portion that they can observe. Deduction cannot rule out the possibility. Induction can; these off structures have never been seen, so one expects never to see them.

These examples in which spacetime structure is added can be multiplied. A familiar case is a two-dimensional de Sitter spacetime and versions of it spatially unrolled into larger spacetimes of twice, thrice, etc. the spatial size of the original spacetime.[7] This de Sitter spacetime can be pictured as a two-dimensional hyperboloid in a three dimensional Minkowski spacetime. Its spatial slices are circles and the unrolling just consists of replacing them by larger circles of twice, thrice, etc., the circumference. The original and unrolled versions are depicted in Figure 4.[8]

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[7] Since the spacetime has only one spatial dimension, perhaps it is too simple. The sorts of spatial duplications described are harder to implement with three dimensional spaces. The simplest case arises with a topologically toroidal Euclidean space. It is created by taking a cube of Euclideian space and identifying opposite faces. The space can be unrolled by connecting its faces to duplicate cubes of Euclidean space.

[8] The figures are misleading in so far as it appears that the doubling is achieved by a uniform expansion of the spacetime. That would not serve present purposes. It would alter the spacetime curvature at every point of the spacetime, so that the temporal pasts in the two spacetimes would no longer be isometric. Rather the doubling is effected by a cutting and pasting that leaves local spacetime structure unaffected. It proceeds as follows. Take a de Sitter spacetime "1" and copy of it, de Sitter spacetime "2". Cut each spacetime along a timelike geodesic that then exposes edges "1L" and "1R" in spacetime 1 and "2L" and "2R" in spacetime 2. Glue 1L to 2R and 1R to 2L to form the doubled de Sitter spacetime.

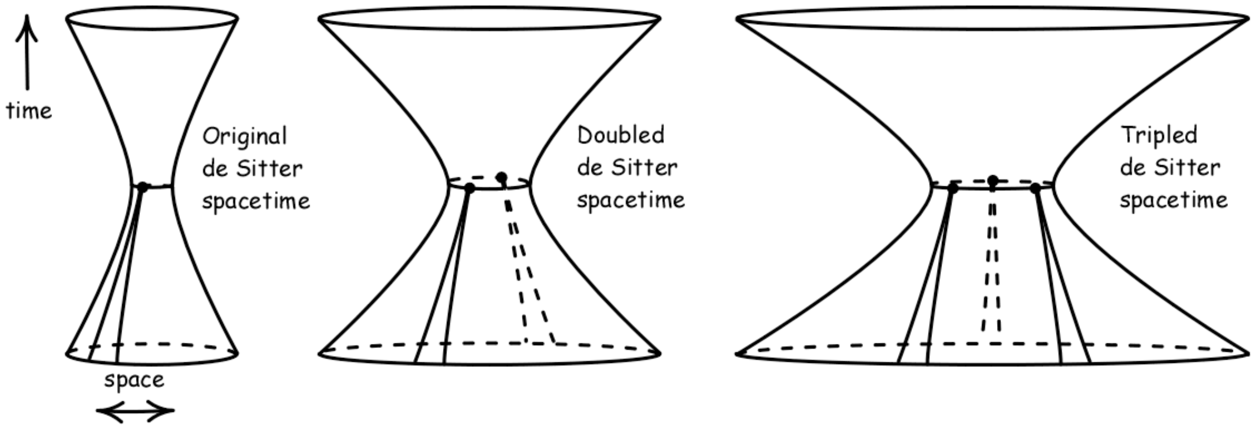

Figure 4. Two dimensional de Sitter spacetimes

The unrolled versions have the curious property of harboring spatial regions that are duplicated twice, thrice, etc. according to the extent of the unrolling. This property is illustrated in the figure by the presence of a single observer's temporal past in the original de Sitter spacetime; and then a duplicate of it in the doubled de Sitter spacetime; and two duplicates in the tripled de Sitter spacetime. The effect would be rather like the multiple duplications of the room one sees in parallel mirrors in a funhouse, although the duplication would not be observable in the spacetime case. A spacetime with no duplications and a spacetime with 27 duplications will be observationally indistinguishable by deductive means. However Occam's razor motivates an inductive inference to the first spacetime.

5. The Opacity of Cosmic Inductions

While we can discriminate inductively among possible futures in both cases, the indeterminism arising through observationally indistinguishable spacetimes is more troubling than the indeterminism of quantum theory. The difference arises through the differing character of the inductive inferences in the two cases. In the case of quantum theory, the warrant for the inductive inferences is quite clear and unproblematic. In the spacetime case, it is hard to see through the inductions to the warrant that lies behind them. In so far as warrants can be found, they are suspect. As a label for this property, I will call these latter inductions "opaque."

In the case of quantum theory, while the future is not generally assured, we can pick inductively among the possible futures, for the theory supplies physical chances for the alternatives. Take the radioactive decay of an atom. We are equally sure that the atom will or will not decay over a single half-life; both outcomes have the same physical chance of 1/2. We can be very sure, inductively, that decay will have happened if we wait ten half-lives of the atom; the physical chance of decay in ten half lives is of $1-(1/2)^{10} = 0.999$. These inferences from the known past to the unknown future in quantum theory are grounded in the theory itself. From it, we can infer to definite probabilities of future occurrences and quite often to probabilities so high that we are certain of the occurrences, for all practical purposes. We can see through these inductions to the physical theory that grounds them; in this sense, they are "transparent."

The inductions arising in observationally indistinguishable spacetimes are of a different kind.[9] Relativity theory provides no physical chances to weight the different spacetime extensions that it allows for our temporal past. The theory itself merely declares that the various alternatives are possible and nothing more. It leaves to us the task of determining if one or other of them is somehow preferred. We must look outside the physical principles of cosmology to decide this question.

This is a natural project for inductive inference. However the examples of Section 4 above reveal no single, principled inductive approach that can be applied across the many cases of indeterminism. Rather we must consider each on a case by case basis and hope that we can find principled grounds in each. Take the extrapolation of the extendability of observed spacetime curves to all spacetime curves. Can it be grounded in an inductive principle that asserts that what has always been *so*, will always be *so*? Such a bare, universal principle is quite untenable. It can only apply to some things that have been *so*, otherwise we rule out the possibility of any novel changes in the future. In a cosmology with a future "big crunch" singularity, we will have the same records of assured millisecond extensions, yet our inertial trajectories will not be indefinitely extendable. A full examination of the evidence will reveal to us that the singularity is in our future and we will know ahead of time not to apply the inductive principle. So the inductive principle must be modified with further clauses to preclude its inappropriate use. But what could these clauses be, if we are to secure a universal inductive

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[9] The problems rehearsed here for cosmology are not new. They have been long discussed in the context of justifying the cosmological principle. Its justification requires an inductive inference from the large scale, spatial homogeneity and isotropy of the observed part of spacetime to all spacetime. For a recent discussion, see Beisbart and Jung (2006).

principle applicable beyond the one case? The danger is that we reduce the principle to a blatant circularity, that is, we solemnly declare that it applies except when it does not.

We face similar problems when we try to rule out the funhouse mirror duplications of the unrolled de Sitter spacetimes or the extravagantly duplicated spacetimes, connected by wormholes. We would like to ground the inductive inference in a principle like Occam's razor. However, the idea behind it, that simplicity is often accompanied by truth, is more a convenient and fallible rule of thumb than a guarantee. These problems are deepened by an absence of any clear rules as to just what counts as simple.[10]

I have long harbored dissatisfaction with the evident failure of any universal inductive principle such as the ones just listed. My solution has been to propose that we abandon the search for universal, formal approaches to inductive inference. Rather, in a material theory of induction, I urge (2003, 2005) that inductive inferences are not warranted by general principles but by facts. A fitting application of this material approach is the inductive inferences just seen in quantum theory. The laws of quantum theory are the facts that warrant the inductive inferences. The law of radioactive decay warrants a near certain inference to radioactive decay in ten half-lives.

What is troublesome from the material perspective is the absence of warranting facts for the inductions in the spacetime case. Take the case of extendability. It seems natural to infer inductively to the fully extended Minkowski spacetime rather than the extendable half Minkowski spacetime; or, more generally, to avoid admitting holed spacetimes that are created from other spacetimes by excising even quite small parts. However it is very hard to specify just what facts ground the inference. That we have never seen holes in spacetime does not settle the matter. By their construction, there cannot be an observable trace of holes, if that is what our spacetime has. That remains true even if our world tubes pass directly through the hole. We would cease to be for the portion of our world tubes coinciding with the excision. However the portion of our world tubes in the future of the hole would be reconstituted precisely with all the

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[10] I set aside Bayesian analyses, for, in the end, all they will do is take the basic notions of one or more of these principles and use them to determine prior probabilities and likelihoods. The resulting analysis will be no more secure than the principles used to set the prior probabilities and likelihoods, although this will be harder to see since the principles used will be hidden behind the complications of the computational machinery.

memories and other traces of the excised spacetime. If observed facts do not ground the inductive inference, what of physical laws? We could cite the common postulate in relativity texts that spacetimes are inextendable. However that postulate is merely the supposition of precisely what is at issue and is distinctive as being dispensable from a physical perspective. It is present as much for mathematical convenience.

In his (manuscript), Manchak reports the justifications usually given for assuming inextendability. They amount to invoking Leibniz's principle of plenitude. "Why, after all, would Nature stop building our universe … when she could just as well have carried on?" Manchak quotes from the writings of the mathematical physicists Robert Geroch as a typical justification.

One cannot help but be struck by how tenuous the grounding has become. We are now to secure our inductions in abstract metaphysics. The principle of plenitude itself is sufficiently implausible that we need to prop it up with anthropomorphic metaphors. We are to image a personified Nature in the act of creating spacetime, much as I might be painting my fence on the weekend. Just as I might not want to stop when there is one board remaining unpainted, so Nature is supposedly loath to halt with a cubic mile-year of spacetime still uncreated. If the complete arbitrariness of the principle of plenitude is not already clear, we might pause to apply it elsewhere. We are supposed to prefer spacetimes without duplications by virtue of a metaphysics of simplicity. Yet surely the metaphysics of plenitude would direct the opposite result. Why would Nature, guided by the slogan "make all you can make," eschew yet another duplication of the spacetime if it is possible?

All these inductive inferences are opaque in that we cannot see through them to their warrants. If we seek to justify them by means of general inductive principles, we resort to principles that are clearly false, or, if not, so hedged as to be inapplicable. If we seek to justify them materially in facts, we arrive almost immediately in the dubious, abstract metaphysics of plenitude and simplicity. This circumstance is to be contrasted with the transparent inductive inferences in the quantum context. Their grounding is found directly in the laws of quantum theory; and we can in turn satisfy ourselves of those laws by tracing back further warrants in the experimental and theoretical foundations of quantum theory.

In sum, we have what appears to me to be an intractable problem. On the one hand, it seems completely unjustified to presume that wormholes we have never seen in our past spacetime will appear in the future. It seems completely unjustified to presume that processes we observe here are duplicated many times over in an unrolled spacetime, when those duplications are by construction, necessarily invisible to us. It seems completely unjustified to assume that there are holes in spacetime, when the spacetime would, by construction, look identical to us if there were no holes. Indeed, even if our world tubes had no past, we would have memories of a past that never was. The inductive inference from those memories to the reality of the past seems irresistible, as do the inductive inferences that reject spatial duplications and future wormholes to new universes. To deny these inductive inferences would, in other contexts, be denounced as delusional. We routinely dismiss as desperate zealots those who tell us our universe was created last Wednesday complete with all records of an ancient past.

Yet, on the other hand, when we try to display the proper warrant of those inductive inferences we favor, whether the warrant is in general principles or material facts, the ground crumbles around our feet.

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