we proceed along them (see Figure 5.19). Such results are typical in the geometry of curved surfaces, such as the surface of a sphere, and the mathematical techniques used in general relativity were originally developed in the context of problems of curved surfaces. As a result, talk of "curvature" is common and we routinely distinguish the "flat" spacetime of special relativity from the "curved" spacetimes of general relativity.

In sum, the models of general relativity have the form

$$<M,g>$$

where M is a four-dimensional manifold and g is a generalization of the Minkowski metric η. Since every distinct distribution of masses in the universe produces a distinct gravitational field, there will be very many different models in the theory. In particular, a nonuniform matter distribution will produce a nonuniform gravitational field. As a result, the models of general relativity will, in general, have no nontrivial symmetries, so that we cannot formulate relativity principles of the type seen in the flat Newtonian spacetime theory and special relativity.

Part III: Applications

5.10 CONFUSIONS OVER COVARIANCE

Misunderstandings of the significance of the covariance group of a theory have been responsible for more than their fair share of confusions in philosophy of space and time. Let us review two important examples.

5.10.1 The Generalization of the Principle of Relativity

One of Einstein's best known claims for his general theory of relativity is that it extends the principle of special relativity to accelerated motion. We noted in the previous section that the spacetimes of general relativity admit no nontrivial symmetries in general, so that we cannot formulate a relativity principle of the type formulated in Newtonian theory or special relativity. Thus Einstein's claim has proved increasingly difficult to defend and its defense has required stratagems of increasing complexity. (Friedman 1983 makes the case against the claim especially clear.) The simplest and most common argument for the claim is not a good one. It merely notes that general relativity is a generally covariant theory. However, general covariance by itself cannot sustain the claimed generalization of the principle of relativity since every spacetime theory we have examined in this chapter has been given generally covariant formulation. They cannot all satisfy a generalized principle of relativity!

The illusion that general covariance and an extension of the principle of relativity are synonymous depends most commonly on the simple mistake of incautiously comparing two theories formulated in different manners: general relativity in its generally covariant formulation with special relativity in a standard (i.e., nongenerally covariant) formulation. In its standard formulation, the Lorentz group is both the theory's covariance group and its symmetry group, the group of its symmetry transformations. As we have seen, the principle of relativity is associated with the symmetry group so that a theory that extends the principle would need to expand that symmetry group. In the transition to general relativity, we do expand the covariance of the theory from Lorentz covariance to general covariance, but since the geometric structure of general relativity in general admits no nontrivial symmetries, we actually reduce the symmetries admitted by the theory. Those who have failed to keep the symmetry and covariance groups of special relativity conceptually distinct easily fail to see the significance of this reduction and fall into the trap of thinking that they have also somehow automatically extended the principle of relativity. Had the two theories been compared from the start with both in their generally covariant formulations, this problem might never have arisen.

5.10.2 Conventionality of Simultaneity

Winnie (1970) showed that we can generalize a standard coordinate system of special relativity to a new coordinate system with time coordinate t_ϵ in such a way that events with equal t_ϵ are judged simultaneous by some ϵ-criterion. It is sometimes thought that this fact *by itself* is sufficient to vindicate the conventionalist claim. This is obviously false since all that has been shown is that we can extend the covariance of the theory so that it can use t_ϵ coordinate systems. We have seen that it is possible to extend the covariance of the theory even further to general covariance, which allows arbitrary coordinate systems. Indeed we have seen that we can give generally covariant formulations of every spacetime theory considered so far. If we can automatically read the t coordinate of any of these formulations as giving a criterion of simultaneity, then we could vindicate the strangest of simultaneity relations, including nonstandard simultaneity relations even in Newtonian spacetimes. What is needed is some independent means of arguing that the t coordinate of a given formulation does represent a possible simultaneity relation, such as the causal theory of time seeks to provide for t_ϵ.

5.11 MALAMENT'S RESULT

One of the most dramatic turns in the debate over the conventionality of simultaneity was provided by Malament (1977a). Contrary to most expectations, he was able to prove that the central claim about simultaneity of the causal theorists of time was false. He showed that the standard simultaneity relation was the only nontrivial simultaneity relation definable in terms of the causal structure of a Minkowski spacetime of special relativity.

Let us give a more precise version of Malament's result and outline the ingenious method he used to establish it. To begin, recall that we saw in Section 5.8 that the causal structure of a Minkowski spacetime is equivalent to its light cone structure. Recall also that the standard simultaneity relation is inertial frame dependent so that unless we specify an inertial frame in some way we should expect no interesting results at all. Malament picks out an inertial frame by specifying one of its worldlines *O* as the worldline of the *O*bserver for whom the simultaneity relation is to be defined. Thus the basic question becomes:

> What simultaneity relations are definable in terms of the light cone structure of a Minkowski spacetime and the worldline *O* of an inertially moving observer?

Malament first shows that

> The relation of standard simultaneity is definable in terms of *O* and the light cone structure.

The proof involves the construction shown in Figure 5.20. We pick any event *e* on *O* and seek the hypersurface of events *s* simultaneous to *e* in the inertial frame of *O* according to the standard criterion. We have found that hypersurface if the following condition is satisfied. Let *a* be any event on *O* prior to *e*. The set of all possible light signals emitted from *a* must intersect *s* and, when they are reflected back to *O* upon intersection, they must all arrive at *O* at the same event *b*. The hypersurface *s* is, of course, orthogonal to *O*.

Malament's central result is that

> The relation of standard simultaneity is the only binary relation definable in terms of the light cone structure and the worldline *O* provided
>
> (i) the relation cannot be trivial insofar as it relates every event to every other event, or fails to relate events on *O* to events not on *O*;
> (ii) the relation is an equivalence relation.

Condition (ii) is required if the relation is to partition the events of the manifold into disjoint sets of mutually simultaneous events such as, for example, the hypersurfaces of simultaneity of the standard case.

The proof of the result depends on the fact that the worldline *O* and the light cone structure admit certain symmetries. For example, in the rest frame of *O* these structures single out no preferred spatial direction and thus remain invariant under spatial rotation about *O*. Thus any relation defined exclusively in terms of *O* and the light cone structure will be unable to pick out a preferred direction and, therefore, must admit the same rotational symmetry. So if *p* and *q* are related by the simultaneity relation and *f* is any rotation about *O*, then the rotated events *f*(*p*) and *f*(*q*) must also be simultaneous (see Figure 5.21). We can now repeat this argument for all the

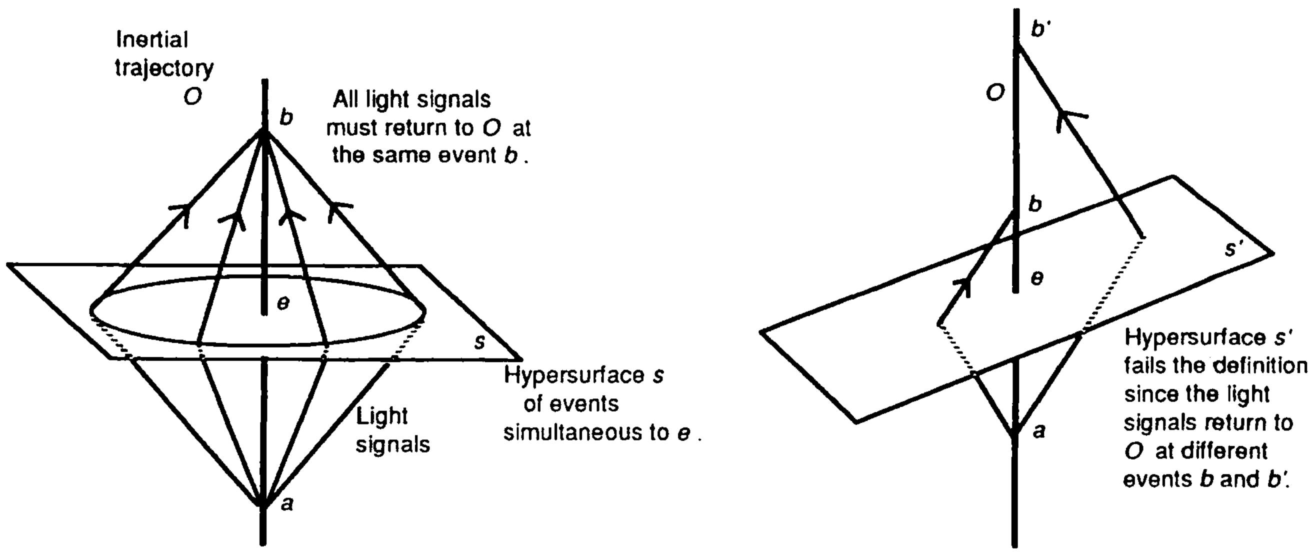

Figure 5.20 Causal definition of standard simultaneity in a Minkowski spacetime.

remaining symmetries of O and the light cone structure. The basic idea is that every symmetry of O and the light cone structure must also be symmetries of any simultaneity relation it defines. These symmetries are the translations, scale expansions (uniform magnifications and reductions) and reflections about a hypersurface orthogonal to O, all of which must map O back into itself. They are shown in Figure 5.22. Malament then showed that the standard simultaneity relation is the only binary relation satisfying (i) and (ii) and remaining invariant under these symmetries. Without going through the proof, we can easily satisfy ourselves of the plausibility of the result. Assume that the simultaneity relation is such that it will slice up the spacetime into hypersurfaces of mutually simultaneous events, as in Figure 5.22. Then it is intuitively evident that only a slicing by orthogonal hypersurfaces will remain invariant under the symmetries listed.

The major weakness of Malament's analysis lies in the sensitivity of his basic result even to small changes in the conditions assumed. The analysis depends on the assumption that the simultaneity relation be definable by the following list of structures:

light cone structure, the inertial worldline O.

It is crucial that this list be preserved since the slightest change in it seems to be sufficient to defeat Malament's basic result. For example, we could ask what simultaneity relation is definable if we add to the list another inertial worldline O' with a velocity differing from O. Through the construction of Figure 5.20, we can define at least the standard simultaneity relation of the O frame and the standard simultaneity relation of the O' frame; but the latter is a nonstandard relation with respect to the O frame. More generally, Peter Spirtes (1981, Chapter 6) has shown

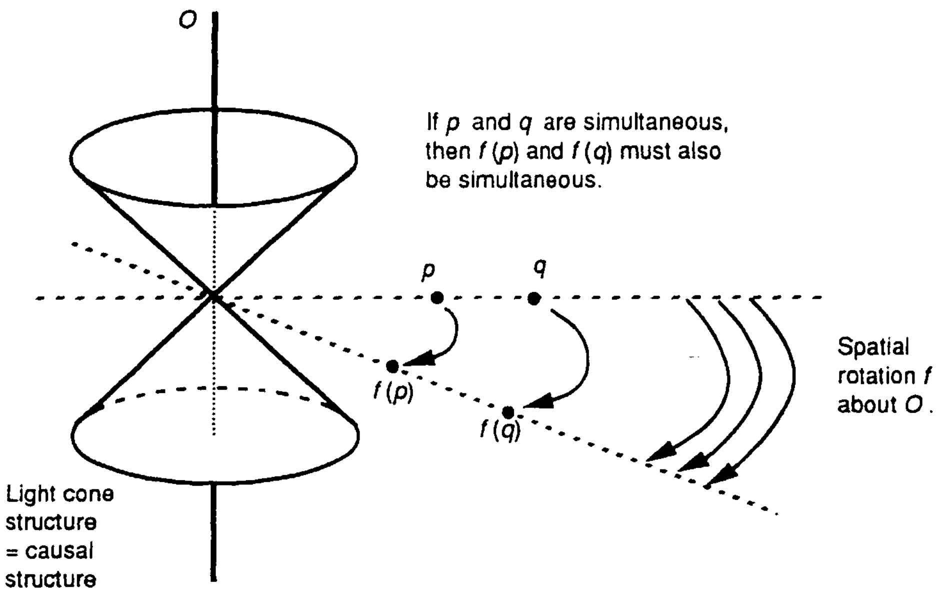

Figure 5.21 Simultaneity preserved under rotation.

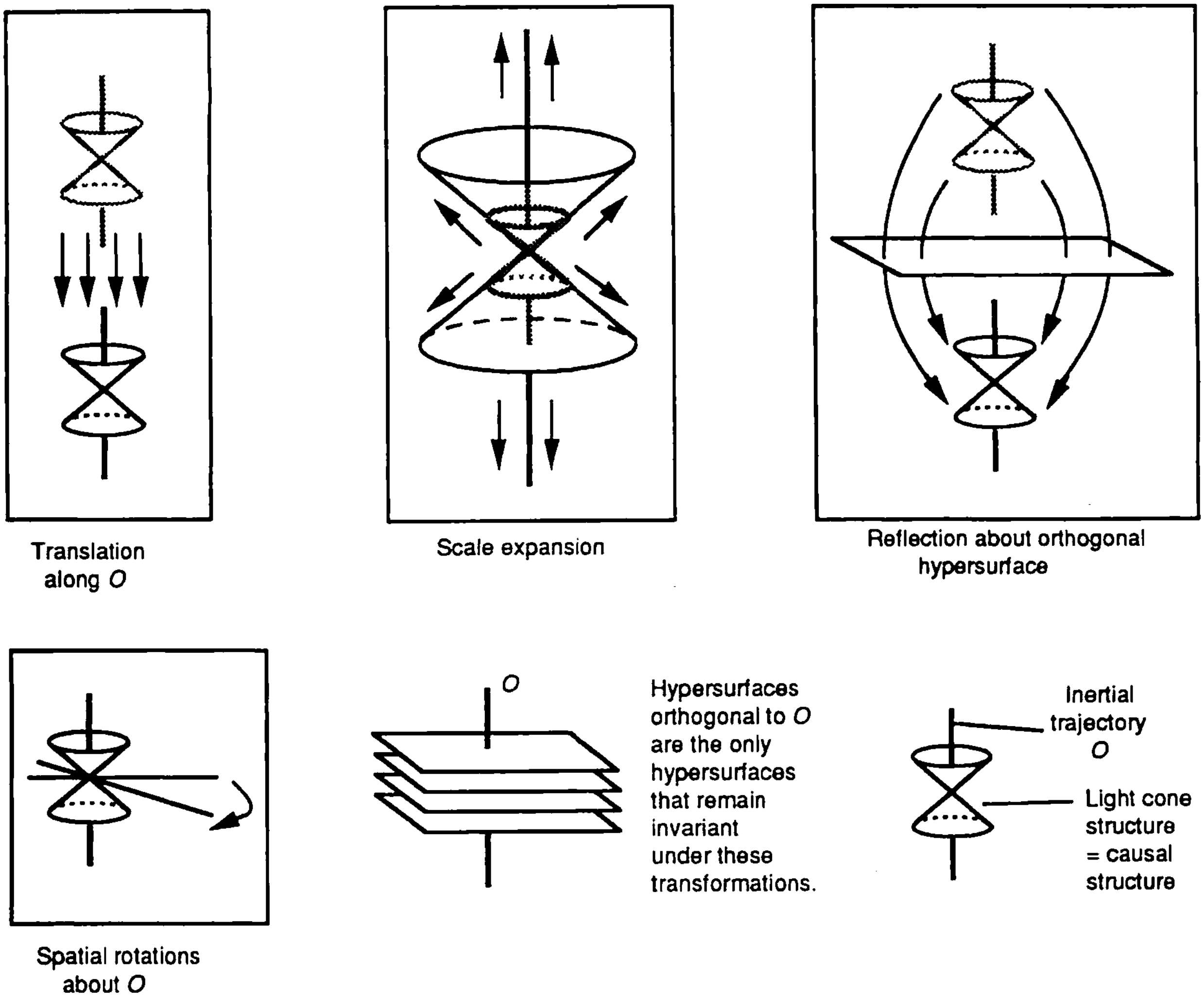

Figure 5.22 Symmetries of the light cone structure of a Minkowski spacetime and an inertial trajectory O.

that merely adding a temporal orientation to the list—that is, the ability to distinguish past from future—is sufficient to enable definition of infinitely many nonstandard simultaneity relations. However, before modifying the construal of causal definability by adding or subtracting from the list, we would need to find very good reasons for doing so.

Adolf Grünbaum (private communication) has pointed out that the need to postulate (ii), that the relation is an equivalence relation, rather than derive it, is another weakness of Malament's challenge to the causal theorists. It eliminates by decree any simultaneity relation that does not partition the spacetime into disjoint sets of mutually simultaneous events. An example of such a relation is the relation "is not causally connectible" which has been called the relation of "topological simultaneity" in the literature (Grünbaum 1973, 203). The latter relation fails to be transitive: Events A and B may each not be causally connectible to a third event, while being causally connectible to each other on a time-like worldline. Therefore this relation cannot partition the spacetime into disjoint sets of mutually simultaneous events.

5.12 REALISM ABOUT SPACETIME STRUCTURES

5.12.1 Spacetime Substantivalism and the "Hole" Argument

Isaac Newton is usually singled out as the canonical realist in the context of theories of space and time and most especially so for his treatment of the absoluteness of his absolute space and absolute time. Their absoluteness arises in a number of senses which have been dissected admirably in Earman (1989). The sense we are concerned with here is that of independence. Absolute space, as we saw in Section 5.1, and absolute time are asserted to have existences entirely independent of the things they contain. This doctrine is the "substance" view or the "substantivalist" view. It owes its somewhat unfortunate name to the view that substance is that which can exist independently. A better name, with fewer distracting connotations, might have been simply the "independence" view. Clearly, the substantivalist position can be formulated analogously for spacetime theorists.

The view is an extreme form of realism concerning spacetime. It arises fairly naturally for realists who seek to construe theories of spacetime as literally as possible. Such a construal automatically sees the divisions between the different structures of a theory as reflecting natural divisions between the actual structures of the physical world. The substantivalist position gives expression to the reality of one of the most important divisions in physical theories, that between spacetime and the matter it contains. The position has become increasingly attractive with the revival of realism in philosophy of science and the problems facing the nonrealist programs of conventionalism and relationalism in spacetime theories.

The "hole" argument (Earman and Norton 1987) is based on ideas advanced by Einstein in 1914, 1915 and 1916 and seeks to establish that acceptance of spacetime substantivalism in a very broad class of spacetime theories forces acceptance of an odious form of indeterminism. (See Chapter 6 for a discussion of determinism.) In informal terms the argument establishes that the substantivalist is forced to insist that there are differences between certain physically possible worlds, even though not just observation but the laws of the theories themselves cannot pick between them.

5.12.2 Presuppositions of the Argument

To make the argument more precise, we must settle several questions left vague. The term "spacetime" is ambiguous insofar as it is unclear as to what specific entity it refers. Let us assume that "spacetime" means the manifold M of our models so that the substantivalist attributes the substantival properties to M or to what M represents in the physically possible worlds. (Other choices are possible here, and in many such cases the hole argument can still be made to apply, as shown in Norton 1989.) The "very broad class of spacetime theories" mentioned is what we call "local spacetime theories." These are generally covariant, spacetime theories of the type considered in this chapter, including versions of Newtonian spacetime theory, special and general relativity. The most important instance of a theory to which the argument applies is general relativity, our current best spacetime theory, which is available only in local formulation. We will develop the hole argument as it applies to general relativity.

Finally we need some more precise theoretical statement of the substantivalist doctrine. The phrase "independent existence" conjures pictures to our intuitions, but without restatement it cannot be analyzed by the machinery of this chapter. Unfortunately there seems to be no precise and satisfactory construal of the doctrine. The claim that spacetime, represented by a bare manifold M, can exist independently of its contents translates naturally to the claim that there is a possible world modeled by the bare manifold M. However, this claim is routinely denied by every spacetime theory we have seen so far. They invariably require that the manifold M be supplemented by further structures in order to produce models of physically possible worlds. Fortunately we do not need a precise construal of spacetime substantivalism to complete the argument. We need only a necessary commitment of spacetime substantivalists.

That commitment arose in Leibniz's famous debate with Newton's representative Samuel Clarke. In their correspondence, Leibniz asked if the world would be changed if God had placed its bodies into space in such a way that East and West were exchanged but all other relations between the bodies were preserved. Leibniz noted that there would be no discernible difference and he urged that no change had actually been effected. However, he realized that the Newtonian substantivalist must nonetheless insist that the world would be different for its bodies would now be located in different spatial locations.

In spacetime theories, the analogue of Leibniz's spatial rearrangement of bodies retaining all other relations between them is a transformation on the manifold M and associated transformation of geometric structures defined on M. If $\langle M,g\rangle$ is a model of general relativity and h a transformation on M then h transforms g into the new metric $g' = h(g)$. From the general covariance of general relativity, we know that $\langle M,g'\rangle$ will also be a model of the theory. The two metrics g and g' will in general assign different metrical properties to the same event in M. Spacetime substantivalists must take seriously this rearrangement of properties on the spacetime manifold M and they must hold that the two models $\langle M,g\rangle$ and $\langle M,g'\rangle$ represent different possible worlds. That is, spacetime substantivalists must *deny*:

> *Leibniz equivalence*: Two intertransformable models of a spacetime theory, such as $\langle M,g\rangle$ and $\langle M,g'\rangle$, represent the same possible world.

This denial is immediately awkward for the substantivalists. Since the metrics g and g' are intertransformable, they are clones of one another. For every property of g we can find a corresponding property of g' by consulting the transformation h. These corresponding properties include all observable properties, so that both g and g' agree on all observables. (The metrics g and g' disagree only on how their properties are to be spread over the manifold M and these differences of spreading cannot be translated into observable differences.) Thus substantivalists must insist that $\langle M,g\rangle$ and $\langle M,g'\rangle$ represent two distinct worlds, even though they are worlds whose differences could not be discerned by any observation. In the heyday of logical positivism and the verifiability criterion, this conclusion alone would have been sufficient

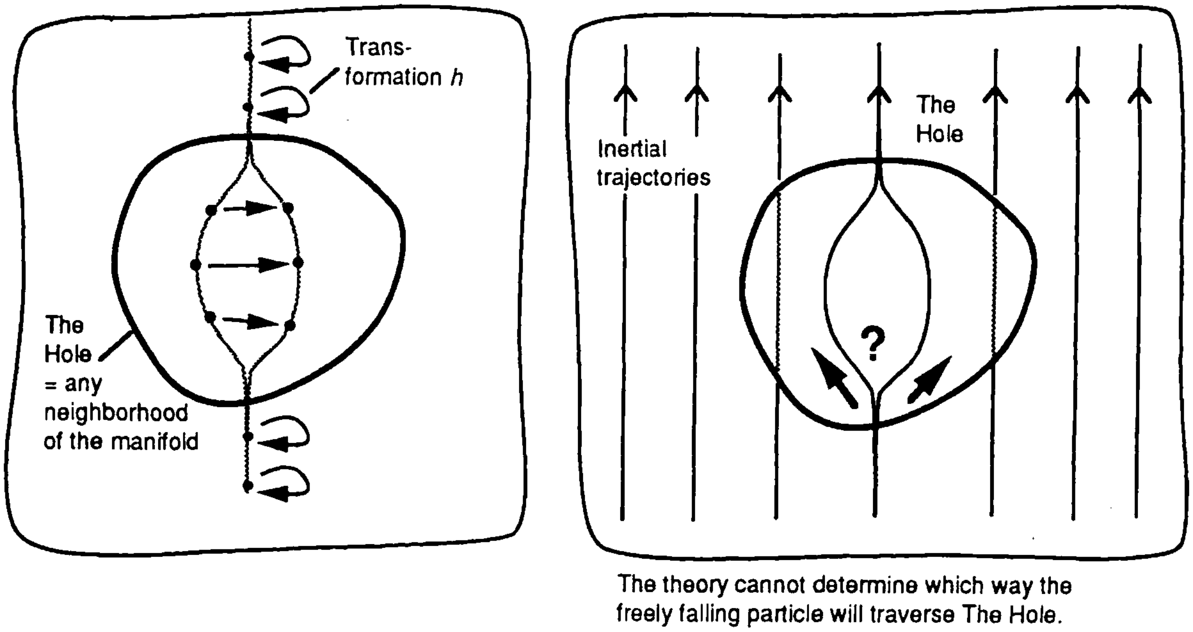

Figure 5.23 The hole argument.

grounds for rejection of the substantivalist position. The hole argument, however, leads the substantivalist to an even worse result.

5.12.3 The Argument

To arrive at the odious form of indeterminism promised, we select any neighborhood of the manifold M. We call it the "hole" for historical reasons associated with Einstein's first use of an early form of the argument. We select any transformation h on M which is the identity outside the hole but comes smoothly to differ from it inside the hole (see Figure 5.23). Then g and $g' = h(g)$ will be the same everywhere outside the hole but will come smoothly to differ within the hole. It now follows that even with a full specification of the spacetime everywhere outside the hole, the theory will be unable to tell us how the spacetime will develop into the hole. For if the model of the spacetime assigns the metric g to the manifold outside the hole, then the theory will allow the metric to develop as either g or g' into the hole and cannot determine which is the correct development.

If we recall that the metric determines the inertial trajectories of the spacetime, then we can see just how disastrous is this result. Given the fullest specification of the spacetime outside the hole, the theory will be unable to determine the trajectory along which a particle in free fall will traverse the hole, even though its trajectory before and after the hole is known exactly. As is explained in Chapter 6, this is an extremely awkward form of indeterminism, for the hole might be both of very small spatial size and temporal duration. Even given a full specification of the fields in its future, past and everywhere else in space, the theory is still unable to specify what happens inside the hole.

The substantivalist is driven to this indeterminism by the need to deny Leibniz equivalence. If the substantivalism were to be given up, Leibniz equivalence could be accepted. Then both the original model and its diffeomorphic copy could be said to represent the same physically possible world, and the indeterminate nature of the development of the fields into the hole would be a mathematical curiosity of no physical significance. Otherwise the substantivalist must adhere to the physical distinctness of two states of affairs whose distinctness is opaque to both observation and the laws of the theory in question.

DISCUSSION QUESTIONS

1. Compare the application of the verifiability criterion as described in Section 5.1 in the context of the principle of relativity with some of its other applications.
2. How are we to approach two theories of space and/or time which have identical observational consequences? Consider whether we are free to choose conventionally between them. (You may find it helpful to consider the examples of Newton's theory of space and time with and without absolute rest and Euclidean geometry with vanishing and nonvanishing universal forces.)
3. Outline some of the virtues and vices of the reduction of temporal or spatiotemporal structure to causal structure offered by the causal theory of time.
4. Adjudicate in the debate between a conventionalist and a realist over the geometry of space or the simultaneity relation in special relativity.
5. Compare the axiomatic way of formulating theories of space and time (such as used by Euclid and many others) with the model theoretic or "semantic" method used in this chapter.
6. Einstein often acknowledged that his discovery of the theories of relativity owed a debt to the reading of various philosophers, notably Hume and Mach. Read the introductory sections of Einstein ([1905] 1952b) and Einstein ([1916] 1952a) (they are not at all hard to follow!) and try to identify those parts of his development dependent on overt philosophical considerations and, if you can, pin down their source.

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