Einstein and Nordström: Some Lesser-Known Thought Experiments in Gravitation

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Late in 1907, Einstein turned his attention to the question of gravitation in his new theory of relativity. It was obvious to his contemporaries that Newton's theory of gravitation required only minor adjustments to bring it into agreement with relativity theory. Einstein's first published words on the question (Einstein 1907b, part V), however, completely ignore the possibility of such simple adjustments. Instead he looked upon gravitation as the vehicle for extending the principle of relativity to accelerated motion. He proposed a new gravitation theory that violated his fledgling light postulate and related the gravitational potential to the now variable speed of light. Over the next eight years, Einstein developed these earliest ideas into his greatest scientific success, the general theory of relativity, and gravitation theory was changed forever. Gravitational fields were no longer pictured as just another inhabitant of space and time, like electric and magnetic fields. They were part of the very fabric of space and time itself.

In light of this dazzling success, it is easy to forget just how precarious were Einstein's early steps toward his general theory of relativity. These steps were not based on novel experimental results. Indeed, the empirical result Einstein deemed decisive—the equality of inertial and gravitational mass—was known in some preliminary form as far back as Galileo. Again, there were no compelling theoretical grounds for striking out along the path Einstein took. In 1907, it seemed that any number of minor modifications could make Newtonian gravitation theory compatible with Einstein's new special theory of relativity. One did not have to look for the relativistic salvation of gravitation theory in an extension of the principle of relativity. Einstein himself would later label the motivations for his new approach "epistemological" (Einstein 1916, section 2).

Through the years of his struggle to develop and disseminate general relativity, one of Einstein's greatest strengths was his celebrated mastery of thought experiments. If you doubted that merely uniformly accelerating your coordinates could create a gravitational field, Einstein would have you visualize drugged physicists awakening trapped in a box as it was uniformly accelerated through gravitation-free space (Einstein 1913, pp. 1254–1255). Would not all objects in the box fall just as though the box were unaccelerated but under the influence of a gravitational field? Was not a state of uniform acceleration fully equivalent to the presence of a homogeneous gravitational field?

As vivid and compelling as Einstein's thought experiments proved to be, they still could not mask the early difficulties of Einstein's precarious speculations. Even a loyal supporter, Max von Laue, author of the earliest textbooks on special and general relativity, had objected to Einstein's idea that acceleration could produce a gravitational field. How could this be possible, he complained, since this gravitational field would have no source masses.[1] Einstein's evolving theory had to compete with a range of far more conservative and more plausible approaches to gravitation, and it was to these that physicists such as von Laue looked for a relativistic treatment of gravitation.

We must ask, therefore, about Einstein's own attitude toward these alternatives. In particular, what of the possibility of a small modification to Newtonian gravitation theory in order to render it Lorentz covariant and thus compatible with special relativity? Had Einstein considered this possibility? What reasons could he give for turning away from this conservative but natural path? It turns out that Einstein had considered and rejected this conservative path in the months immediately prior to his first publication of 1907 on relativity and gravitation. He felt such a theory must violate the equality of inertial and gravitational mass. He was forced to revisit these considerations in 1912 with the explosion of interest in relativistic gravitation theories. He first continued to insist that a simple Lorentz covariant gravitation theory was not viable. In the course of the following year, however, he came to see that he was wrong and that there were ways of constructing Lorentz covariant gravitation theories compatible with the equality of inertial and gravitational mass.

After an initial enchantment and subsequent disillusionment with Abraham's theory of gravitation, Einstein found himself greatly impressed by a Lorentz covariant gravitation theory due to the Finnish physicist Gunnar

Nordström. In fact, by late 1913, Einstein had nominated Nordström's theory as the only viable competitor to his own emerging general theory of relativity (Einstein 1913). This selection came, however, only after a series of exchanges between Einstein and Nordström that led Nordström to significant modifications of his theory.

Einstein's concession to the conservative approach proved to have a silver lining; under continued pressure from Einstein, Nordström made his theory compatible with the equality of inertial and gravitational mass by assuming that rods altered their length and clocks their rate upon falling into a gravitational field so that the background Minkowski space-time had become inaccessible to direct measurement. As Einstein and Fokker showed in early 1914 (Einstein and Fokker 1914), the space-time actually revealed by direct clock and rod measurement had become curved, much like the space-times of Einstein's own theory. Moreover, Nordström's gravitational field equation was equivalent to a geometrical equation in which the Riemann–Christoffel curvature tensor played the central role. In it, the full contraction, the curvature scalar, is set proportional to the trace of the stress-energy tensor. What is remarkable about this field equation is that it comes almost two years *before* Einstein recognized the importance of the curvature tensor in constructing field equations for his own general theory of relativity! In this regard, the conservative approach actually anticipated Einstein's more daring approach.

Einstein now had an answer to the objection that general relativity introduced an unnecessarily complicated mechanism for treating gravitation, the curvature of space-time. He had shown that the conservative path led to this same basic result: Gravitational fields come hand-in-hand with the curvature of space-time.

Elsewhere, I have given a more detailed account of Einstein's response to the conservative approach to gravitation and his entanglement with Nordström's theory of gravitation (Norton, 1992). My purpose in this chapter is to concentrate on one exceptionally interesting aspect of the episode. As in Einstein's better-known work on his general theory of relativity, the episode was dominated by a sequence of compelling thought experiments.[2] These experiments concentrate the key issues into their simplest forms and present them in a way that makes the conclusions emerge convincingly and effortlessly. In this chapter I will review this sequence of thought experiments as it carries us through the highlights of the episode.

In particular, we will see how one of the more arcane areas of special relativistic physics proved decisive to the development of relativistic gravitation theory. It emerged from the work of Einstein, von Laue, and others that stressed bodies behave in strikingly nonclassical ways in relativity theory. For example, a moving body can acquire energy simply by being subjected to stress, even though it may not be deformed elastically by the stress. Nonclassical energies such as these provided Einstein with the key for incorporating the equality of inertial and gravitational mass into relativistic physics.

1. First Thought Experiment: Masses Falling from a Tower

The bare facts of Einstein's initiation into the problem of relativizing gravitation theory are known. In late September 1907, Einstein accepted a commission from Johannes Stark, editor of *Jahrbuch der Radioaktivität und Elektronik*, to write a review article on the principle of relativity.[3] That review (Einstein 1907b) was submitted a little over two months later, on December 4, 1907. Its concluding part contained the earliest statement of what came to be the principle of equivalence and of the bold conjectures about gravitation that followed from it. What we know only from later reminiscences by Einstein is that, in this brief period between September and December, he considered and rejected a conservative Lorentz covariant theory of gravitation.[4]

Einstein recalled that he knew how one could take Newton's theory of gravitation and render it Lorentz covariant with small modifications to its equations. Newton's theory is given most conveniently in the usual Cartesian coordinates (x, y, z) by the field equation

$$\nabla^2\phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi = 4\pi G\rho \tag{1}$$

for the gravitational field potential ϕ generated by a mass density ρ, where G is the gravitational constant, and by the force equation

$$\mathbf{f} = -m\nabla\phi \tag{2}$$

for the gravitational force $\mathbf{f}$ on a body of mass m. The adaptation to special relativity of the field equation to which Einstein alluded was obvious. One simply replaces the Laplacian operator ∇^2 of (1) with the manifestly Lorentz covariant d'Alembertian $\Box^2$ to recover

$$\Box^2\phi = \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 4\pi G\nu, \tag{3}$$

where ν is an invariant mass density and t the time coordinate. An analogous modification of (2) would also be required. Einstein (1933, pp. 286–287) continued to explain that the outcome of his investigations was not satisfactory.

> These investigations, however, led to a result which raised my strong suspicions. According to classical mechanics, the vertical acceleration of a body in the vertical gravitational field is independent of the horizontal component of its velocity. Hence in such a gravitational field the vertical acceleration of a mechanical system or of its center of gravity works out independently of its internal kinetic energy. But in the theory I advanced, the acceleration of a falling body was not independent of its horizontal velocity or the internal energy of the system.
>
> This did not fit with the old experimental fact that all bodies have the same acceleration in a gravitational field. This law, which may also be formulated as the law of the equality of inertial and gravitational mass, was now brought home to me in all its significance. I was in the highest degree amazed at its existence and guessed that in it must lie the key to a deeper understanding of inertia and gravitation. I had no serious doubts about its strict validity even without knowing the results of the admirable experiments of Eötvos, which—if my memory is right—I only came to know later. I now abandoned as inadequate the attempt to treat the problem of gravitation, in the manner outlined above, within the framework of the special theory of relativity. It clearly failed to do justice to the most fundamental property of gravitation.

The result that troubled Einstein in the theory he advanced came from the relativistic adaptation of the force law (2). As Einstein pointed out in his reminiscences, this adaptation could not be specified so unequivocally. We can proceed directly to the result, however, if we use four-dimensional methods of representation not available to Einstein in 1907. The natural adaptation of (2) is

$$F_\mu = m\frac{\mathrm{d}U_\mu}{\mathrm{d}\tau} = -m\frac{\partial\phi}{\partial x_\mu}, \tag{4}$$

where F_μ is the gravitational four-force acting on a body of rest mass m with four-velocity U_μ; τ is the proper time.[5] We can now apply (4) to the special case of a body whose three-velocity $\mathbf{v}$ has, at some instant of time, no vertical component in a static gravitational field. If the gravitational field at that instant at the mass acts along the z-axis of coordinates, so that the z-axis is the vertical direction in space, then it follows from (4) that the vertical acceleration of the mass is given by

$$\frac{\mathrm{d}v_z}{\mathrm{d}t} = -\left(1 - \frac{v^2}{c^2}\right)\frac{\partial\phi}{\partial z}. \tag{5}$$

We see immediately that this vertical acceleration is reduced as the horizontal speed v is increased, illustrating Einstein's claimed dependence of the rate of fall on horizontal velocity.

The "old experimental fact," which this result contradicts, surely belongs to the famous fable in which Galileo drops various objects of different weights from a tower. Einstein and Infeld (1938, pp. 37–38) certainly identify this story when they wrote:

> What experiments prove convincingly that the two masses [inertial and gravitational] are the same? The answer lies in Galileo's old experiment in which he dropped different masses from a tower. He noticed that the time required for the fall was always the same, that the motion of a falling body does not depend on the mass.

We can combine these ingredients to make explicit the thought experiment suggested by Einstein's analysis. Masses are dropped from a high tower, some with various horizontal velocities and some with none. According to (5), the masses with greater horizontal velocity fall slower, contradicting Einstein's expectation and the familiar classical result that they should all fall alike. See Figure 1.

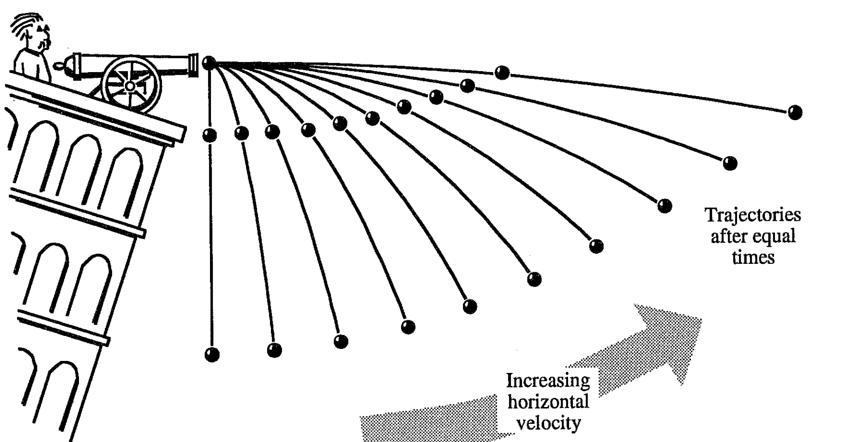

Figure 1. Vertical fall slowed by horizontal velocity in a Lorentz covariant theory of gravitation.

2. Second Thought Experiment: Spinning Tops and Heated Gases

It is not so obvious why Einstein found the outcome of this first thought experiment to be so troubling that he felt justified in abandoning the search for a Lorentz covariant theory of gravitation. The dependence is a minute effect, second order in v/c. Indeed, one might well wonder how even the most ingenious experimentalist could compare the rate of fall of a mass with that of another whizzing past at a horizontal velocity close to the speed of light. Even if this were possible, the experiment had surely not been done in 1907. How could Einstein reject this minute effect as incompatible with an "old experimental fact" whose traditional origins lay with Galileo?

The answer resides in the fact that Einstein derived the dependence of vertical acceleration on the "horizontal velocity or the *internal energy* of the system." What Einstein meant by this was made clear in 1912 when the Finnish physicist Gunnar Nordström published the first of a series of papers on a Lorentz covariant, scalar theory of gravitation (Nordström 1912). The essential assumptions and content of Nordström's theory were contained in equations (3) and (4) above. Nordström did correct, however, a problem with (4). It turns out that this force law can only hold for a mass moving so that the rate of change of the gravitational potential along its world line is zero.[6] (This condition holds instantaneously for the special case used to derive [5].) Thus the force law (4) requires modification if it is to apply to masses along whose trajectories ϕ is not constant. Nordström found two suitable modifications. He favored the one in which the rest mass m of the body is assumed to vary with the gravitational potential ϕ. In particular, he readily derived the dependence

$$m = m_0 \exp\left(\frac{\phi}{c^2}\right), \tag{6}$$

where m_0 is the value of m when $\phi = 0$.

By October 1912, when Nordström sent his paper to *Physikalische Zeitschrift*, Einstein's novel ideas on gravitation had become a matter of public controversy. In July, Einstein found himself immersed in a vitriolic dispute with Max Abraham, who saw in Einstein's admission of a variable speed of light a "death blow" to relativity theory (Abraham 1912). In his response, Einstein (1912, pp. 1062–1063) published his 1907 grounds for abandoning Lorentz covariance in the most general form he could manage. In any Lorentz covariant gravitation theory, he argued, be it a four-vector or six-vector theory, gravitation would act on a moving body with a strength that would vary with velocity. Any such theory was unacceptable, since it violated the requirement of the equality of inertial and gravitational mass.

Therefore it is not at all surprising that Nordström attracted Einstein's attention when he published just such a theory. Einstein's reaction was so swift that Nordström was able to mention it in an addendum to his original paper! The addendum began (Nordström 1912, p. 1129):

> *Addendum to proofs.* From a letter from Herr Prof. Dr. A. Einstein I learn that he had already earlier concerned himself with the possibility

> used above by me for treating gravitational phenomena in a simple way. He however came to the conviction that the consequences of such a theory cannot correspond with reality. In a simple example he shows that, according to this theory, a rotating system in a gravitational field will acquire a smaller acceleration than a non-rotating system.

Einstein's reflection on the acceleration of fall of a spinning system is actually only a slight elaboration of the situation considered in the first thought experiment above. Each element of a suitably oriented spinning body in a gravitational field has a horizontal velocity. Thus, according to (5), which obtains in Nordström's theory, each element will fall slower than the corresponding element without that velocity. What is true for each part holds for the whole. A spinning body falls slower than the same body without rotation.

This example now makes clear Einstein's remark about internal energy. When the body is set into rotation, its parts gain kinetic energy, so its overall energy and its inertia are increased. However, through (5), there is a decrease in the gravitational force acting on it, so that its acceleration of fall is decreased. That is, its rate of fall decreases as the internal energy and inertia increases. Presumably Einstein thought the spinning body just one example of a general effect of this type. In much later reminiscences, Einstein used the example of a kinetic gas.[7] As the gas is heated, each molecule moves faster and thus falls more slowly. Thus the aggregate of molecules, the heated gas, falls more slowly than a colder gas. These two examples comprise the second thought experiment. See Figure 2.

Einstein's result in this form is a far greater threat to Lorentz covariant theories of gravitation such as Nordström's, for it points to effects that might well be experimentally testable. Perhaps the effect might transcend detection by a Galileo-like timing of the fall of spinning tops or hot gases, but would it escape an apparatus similar to that of the Eötvos experiment? Nordström seemed to think so, for he continued his appendix by dismissing Einstein's argument on the basis of the effect being "too small to yield a contradiction with experience." This dismissal depended on a rather bold assumption: that there are no common systems of matter in which a great part of the internal energy, and thus inertia, is due to the kinetic energy of internal motions. Such systems, if they existed, would fall markedly slower than others according to Nordström's theory. Nordström may well have been right that no measurable effect would arise from the spinning of a body, but could he be sure that the energy of commonplace matter did not already have a significant kinetic component? The fundamental theory of matter was then in a state of turmoil and scarcely able to assure him either way. A more prudent Einstein was unwilling to take the risk. Should it turn

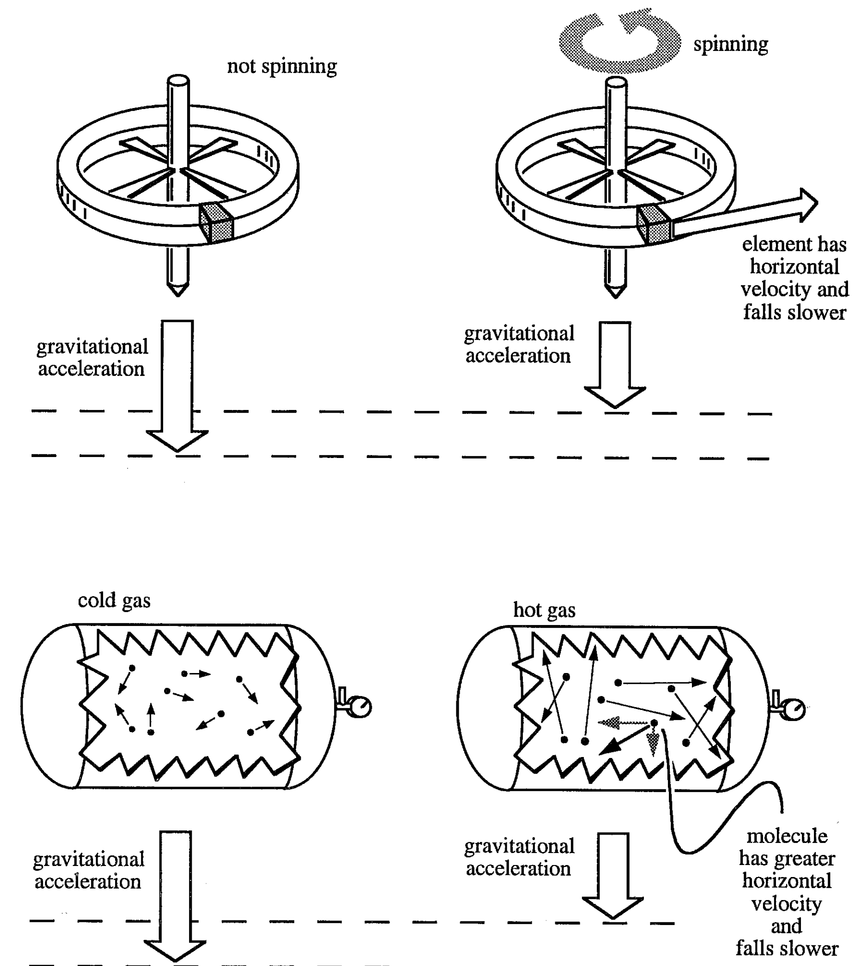

Figure 2. Spinning bodies fall slower than when not spinning. Hot gases fall slower than cold gases, in Nordström's theory.

out that a significant part of the total energy of various types of ordinary matter was due, in different proportion, to an internal kinetic energy, then Nordström's theory might well be refuted by simple observations of the fall of different substances from a tower.

By the time of submission of his next paper on the theory in January 1913, Nordström had become more wary (Nordström 1913a). While still insisting (p. 878) that no observable effect would arise in the case of spinning bodies, he was prepared to raise the question of whether the "molecular motions of a falling body" would influence the rate of fall. He did not state directly that the effect might be measurable, but the effect did worry him, since he began to speculate on a way of incorporating the effect into his theory.

3. Third Thought Experiment: The Energy of a Stressed Rod

Nordström's paper of January 1913 was devoted to a question that would ultimately completely alter the direction of development of his theory. The paper asked which quantity represented the inertial mass of a body. The question was far from trivial. Recent work in the relativistic theory of continua had shown that there were inertial effects that arose when a body was stressed for which there were no classical analogs. Nordström observed (1913a, p. 856) that it had proved possible to ignore this question and develop a complete mechanics of extended bodies without explicitly introducing the concept of inertial mass. This luxury could no longer be afforded, he continued, when one worked in a relativistic gravitation theory, because of the very close connection between inertial and gravitational masses. One had to represent the inertial mass of a body in a way that allowed for inertial effects in stressed bodies that cannot be attributed directly to an individual mass.

The body of results to which Nordström referred had reached its mature form in the work of von Laue (1911a, 1911b). There von Laue essentially presented the modern theory of relativistic continua, introducing the notion of the general stress-energy tensor of matter. The results to which Nordström alluded took the following form. If one applied a stress to a body *without deforming it or setting it into motion*, then both the energy and momentum of the body would remained unchanged in its rest frame. However, if one viewed this same process from a frame of reference in which the body was in motion, then the energy and momentum of the body might change. For example, if the body was influenced by a shear stress[8] p^0_{xy} in its rest frame and then viewed from a frame of reference moving at velocity v in the x direction, then in that frame the body would acquire a momentum in the y direction. The momentum density g_y due to the stress is given by[9]

$$g_y = \gamma \frac{v}{c}\, p^0_{xy}. \tag{7}$$

If the stress was a normal stress p^0_{xx} in the rest frame, then, when viewed in the relatively moving frame, the body would have acquired both energy and an x-directed momentum. The energy density W and momentum density g_x acquired is given by

$$W = \gamma^2 \frac{v^2}{c^2}\, p^0_{xx}, \qquad g_x = \gamma^2 \frac{v}{c^2}\, p^0_{xx}. \tag{8}$$

These are the effects for which there are no classical analogs. They proved decisive in the relativistic analysis of a number of celebrated thought experiments and real experiments, most notably the Lewis and Tolman bent lever and the Trouton–Noble capacitor.[10]

One of the clearest and earliest analyses of these nonclassical effects is due to a thought experiment of Einstein (1907a, section 1; 1907b, section 12) and was given in the context of his discussion of the inertia of energy. He imagined an extended body at rest carrying a charge distribution. He then imagined that, at some definite instant in its rest frame, the body comes under the influence of an external electromagnetic field. The net external forces are assumed to balance so that the body remains at rest. The effect of the continued action of the forces, however, is to induce a state of stress in the body. Einstein now redescribed this process from a frame in which the body moved uniformly. Because of the relativity of simultaneity, the body does not come under the influence of the external field at one instant. For a brief period, some charge elements are under the influence of the field and some are not. During this period, the external forces exerted by the field do not balance, so that there is a net external force exerted on the body. Work is done on or by the force as the body moves, and there is a net transfer of energy. This energy is the energy described in (8) and associated with the induction of a stressed state in the body.[11]

The beauty of this thought experiment is that it derives the effects of equations (8) directly from the most fundamental, nonclassical effect of special relativity, the relativity of simultaneity. Forces applied simultaneously in one frame of reference need not be seen as applied simultaneously in another. The resulting temporary imbalance leads to an energy and momentum transfer in the latter frame only and these transferred quantities emerge as those of (8). Einstein's analysis is mathematically quite complicated, however, since he considers a body of arbitrary shape and charge distribution. Recapitulating Einstein's analysis for a simpler case is sufficient to reveal the essential physics. That case is a rod of uniform cross section with equal charges at either end. This is the third thought experiment. See Figure 3.

The rod has rest length l, cross-sectional area A, and extends from $x' = 0$ to $x' = l$ in its rest frame (x', t'). At a specific instant $t' = 0$ in its rest frame, the rod comes under the influence of a field that applies equal but

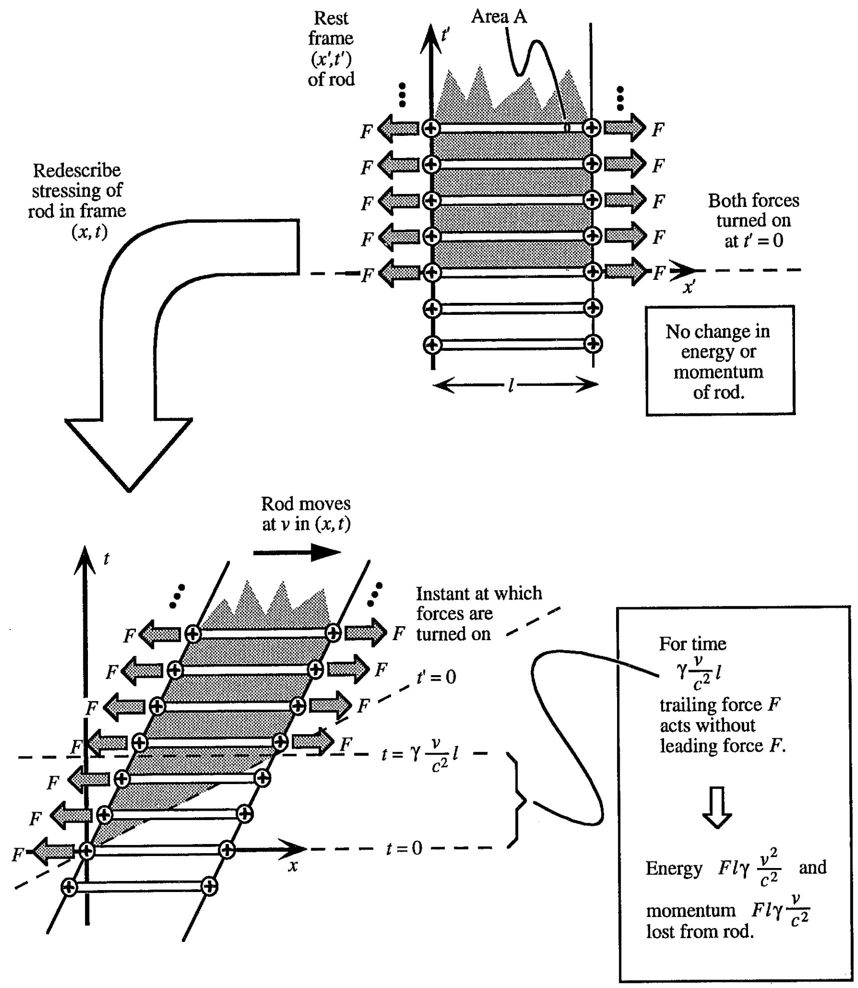

Figure 3. Stressing a moving rod changes its energy and momentum.

oppositely directed forces F to the charges. For concreteness, assume the forces are directed away from the rod along its length. The forces induce a tensile stress on the rod in its rest frame[12]

$$p^0_{xx} = -F/A.$$

If we redescribe this stressing of the rod in a frame (x, t) [13] in which the rod moves at velocity v in the $+x$ direction, we find that the two forces are not activated simultaneously because of the relativity of simultaneity. The force F on the trailing end is activated at a time $\gamma \frac{v}{c^2} l$ earlier than the force F

on the leading end. For this short time period the external force F on the trailing end is not balanced by the other external force. As a result, work is done by the motion of the rod against the force. The resulting loss of energy from the rod is $Fl\gamma \frac{v^2}{c^2}$ and the loss of momentum $Fl\gamma \frac{v}{c^2}$. Recalling the above expression for p^0_{xx} and that the volume of the rod in the frame (x, t) is $V = Al/\gamma$, we recover expressions for the energy E and x-momentum G_x gained by the rod in the process of being stressed:

$$E = \gamma^2 \frac{v^2}{c^2} p^0_{xx} V \quad \text{and} \quad G_x = \gamma^2 \frac{v}{c^2} p^0_{xx} V.$$

Division of these expressions by the volume V yields (8).

4. Fourth Thought Experiment: Radiation in a Massless, Mirrored Box

In his paper (1913a), Nordström had asked the right question. What quantity represents the total inertial mass of a body, including contributions to its inertial properties that arose from stresses? He sought his answer in the form of the source density ν for equation (3), and he looked in the right place for his answer. He expected this density to be a quantity derived from the stress-energy tensor $T_{\mu\nu}$, recently introduced by von Laue. After extensive discussion, he settled upon $1/c^2$ times the rest energy density of the source matter as his source density ν. The rest frame required for this choice was the instantaneous local rest frame of a continuous matter distribution—"dust"—which Nordström assumed contributed to the source matter. We would now express Nordström's choice in manifestly covariant form as

$$\nu = -\frac{1}{c^2} T_{\mu\nu} B_\mu B_\nu, \tag{9}$$

where B_μ is the four-velocity vector field of the continuous distribution of matter.

Nordström's answer was close to the correct answer—but not close enough, as was pointed out by Einstein, in section 7 of his physical part of Einstein and Grossmann (1913).[14] He reported that von Laue himself, also in Zurich but at the University of Zurich, had pointed out to Einstein the only viable choice, the trace of the stress-energy tensor

$$T = T_{\mu\mu}.$$

Einstein proposed to call this scalar "Laue's scalar." What was distinctive about this choice was that it enabled a gravitation theory that employed it to satisfy the requirement of the equality of inertial and gravitational mass, at least "up to a certain degree," as Einstein put it. This degree included examples such as those in the second thought experiment above, as we shall now see.

The key result that enabled satisfaction of this equality was due to von Laue. Von Laue (1911a) had found a single general solution to a range of problematic examples within relativity theory. They all involved systems whose properties appeared to violate the principle of relativity. For example, on the basis of classical electromagnetic theory, Trouton and Noble (1903) believed that a charged, parallel-plate capacitor would experience a net turning couple if it was set in motion with its plates oblique to the direction of motion—although their experiment yielded a celebrated null result. Again, Ehrenfest (1907) had raised the possibility that a nonspherical or nonellipsoidal electron could not persist in uniform translational motion unless forces are applied to it. In both cases the projected behavior would provide an indicator of the uniform motion of the system, violating the principle of relativity.

What these examples had in common was the presence of stresses within the systems and, with the proper treatment of these stresses, the threat to the principle of relativity evaporated. Von Laue noticed that these systems were all what he called "complete static systems," that is, they maintained a static equilibrium in inertial frames of reference without interacting with other systems.[15] The basic result characterizing these systems was that, in their rest frames,

$$\int p^0_{ik}\,\mathrm{d}V^0 = 0, \tag{10}$$

where the integral extends over the rest volume V^0 of the whole body. It follows from (10) that the energy and momentum of a complete static system transforms under Lorentz transformation exactly like the energy and momentum of a point-mass. Since the dynamics of a point-mass was compatible with the principle of relativity, so was the dynamics of a complete static system, and one could not expect a violation of the principle of relativity in the dynamics of these systems.

Von Laue's analysis was very general and powerful because it needed to ask very little of the inner structure of the systems. All one needed to know was whether the system was a complete static system. If it was, one could ignore the further details and simply imagine a black box drawn around the system. Its overall dynamics was now determined.

In effect, what Einstein was able to report in Einstein and Grossmann (1913, section 7) was that von Laue's machinery could be applied directly

to the problem of selecting a gravitational mass density. If one chose T as the gravitational mass density, von Laue's result (10) entailed that the total gravitational mass of a complete stationary system in its rest frame was equal to its inertial mass. For, using (10), for such a system we have[16]

$$\begin{aligned} \text{gravitational mass} &= \int T\,\mathrm{d}V^0 = \int (p^0_{11} + p^0_{22} + p^0_{33} + T^0_{44})\,\mathrm{d}V^0 \\ &= \int T^0_{44}\,\mathrm{d}V^0 = \text{total energy} = \text{total inertial mass}, \end{aligned} \tag{11}$$

where I follow Einstein in simplifying the analysis by neglecting factors of c^2, so that energy and inertial mass become numerically equal.

The power and subtlety of this rather beautiful result stood out clearly in the example that Einstein employed in his discussion. This example is our fourth thought experiment. The trace T for electromagnetic radiation vanishes. Thus it would seem that electromagnetic radiation can have no gravitational mass.[17] But what of a system of electromagnetic radiation enclosed within a massless box with mirrored walls? Would such a system have any gravitational mass? The radiation itself would not, although that radiation would exert a pressure on the walls of the box. These walls would become stressed and, simply because of this stress, the walls would acquire a gravitational mass. Since it is a complete static system, we need do no direct computation of the distribution of stresses in the walls. The result (11) tells us immediately that the total gravitational mass of the system in its rest frame is given by the system's total inertial mass. See Figure 4.

The same reasoning can essentially be applied to the spinning bodies and heated gases of the second thought experiment, if they are set in a gravitation theory that uses T as its source density. Molecules of gas with horizontal motion will fall slower than those without this motion, thus they do have a smaller effective gravitational mass. They exert a pressure on the walls of the containing vessel, however, which becomes stressed. These stresses alter the value of T and thereby contribute to the gravitational mass. Since (11) applies here, we read immediately from it that the gravitational mass of a gas enclosed in a vessel in its rest frame is given by the inertial mass of the whole system.

Similarly, the individual masses comprising a spinning body do have a smaller effective gravitational mass because of their motion, but the spinning body is stressed by centrifugal forces. We know from (11), without calculation, that the contribution of the stresses to the total gravitational mass exactly compensates for the reduction due the motion of the individual masses. As before, the total gravitational mass is given by the total inertial mass.

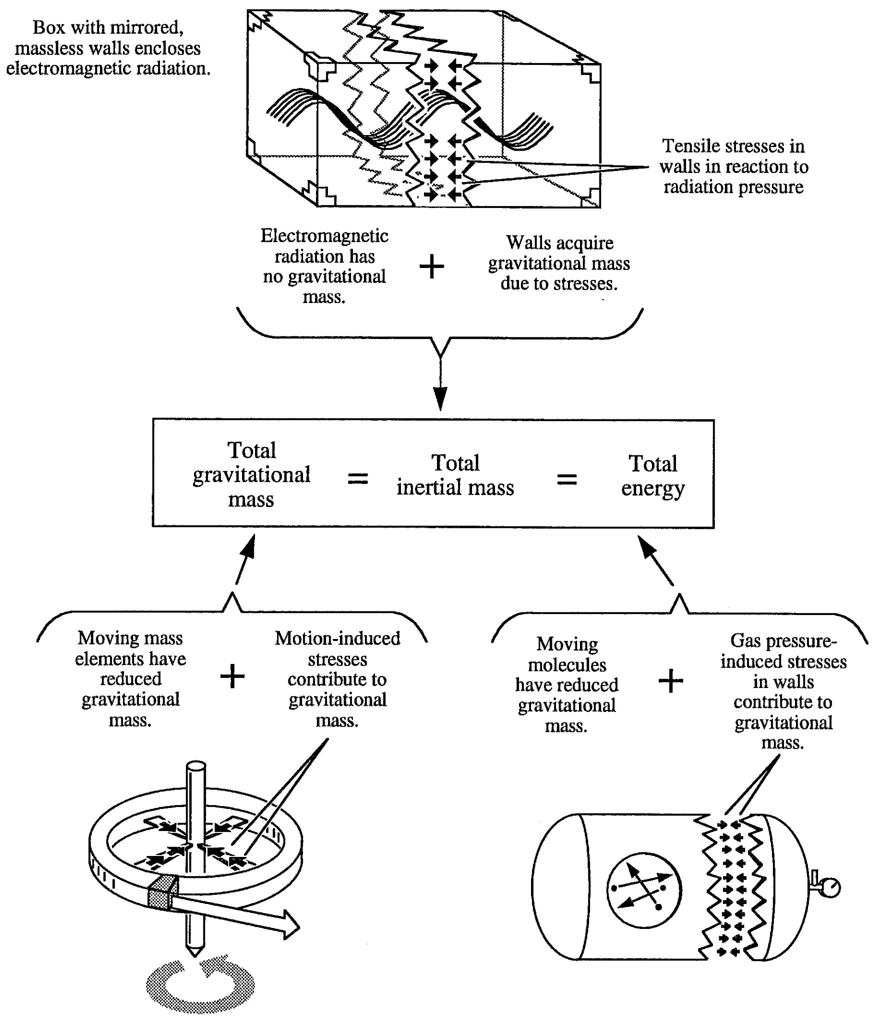

Figure 4. Equality of inertial and gravitational mass for complete stationary systems in a gravitation theory with source density T.

5. Fifth Thought Experiment: Lowering and Raising Radiation

At this point, one might anticipate that Einstein would have to capitulate and cease his opposition to Lorentz covariant gravitation theories. His objection to these theories had been that they failed to satisfy the requirement of equality of inertial and gravitational mass. Most damaging was his conclusion that this equality would fail in the type of cases dealt with in the second thought experiment above. But now his analysis of the choice of T as source density showed how a Lorentz covariant, scalar theory of gravitation could escape Einstein's objection in exactly those most damaging cases.

Einstein was in no mood for retraction, and with good reason. Having presented T as the only viable choice of gravitational source density, he proceeded to argue that the choice was a disaster. A theory that employed T as the gravitational source density must violate the law of conservation of energy. Einstein's argument was presented within a thought experiment—our fifth thought experiment—and it was beguilingly simple. See Figure 5. He imagined electromagnetic radiation trapped in a mirrored, massless box. We shall assume it cubic in shape for simplicity. The system is lowered into a gravitational field. Since it has gravitational mass, an amount of energy proportional to this mass is extracted.

Einstein now introduced another apparatus to raise the radiation. He imagined a mirrored shaft extending out of the gravitational field. Within the shaft are two mirrored, massless baffles, firmly fixed together. The radiation is introduced into the space between the baffles and is raised out of the gravitational field as the baffles are raised. We shall again assume for simplicity that the space between the baffles is cubic.

We have already seen that the gravitational mass of the mirrored box used to lower the radiation is due entirely to the stresses in its walls. It now follows immediately that the system of radiation and baffles has only one-third the gravitational mass of the radiation/box system, for in elevating the radiation trapped between the baffles, one need move only one-third as many stressed members.[18] Only one-third as much energy need therefore be supplied to raise the radiation in the baffle apparatus as is released when the radiation is lowered in the box. Since no energy is involved in raising and lowering the massless box and baffles when devoid of radiation, a complete cycle of raising and lowering the radiation yields a net gain of energy. This violates the law of conservation of energy.

Einstein must have been very pleased with this outcome. In a single blow, it ruled out not just Lorentz covariant, scalar theories of gravitation, but any relativistic gravitation theory that employed a scalar potential. Thus the "undeniable complexity" (Einstein and Grossmann 1913, part 1, section 7) of Einstein's second-rank tensor theory seemed unavoidable.

6. Sixth Thought Experiment: Lowering and Raising a Stressed Rod

Einstein's triumph was short lived. In July 1913, Nordström (1913b) sub-

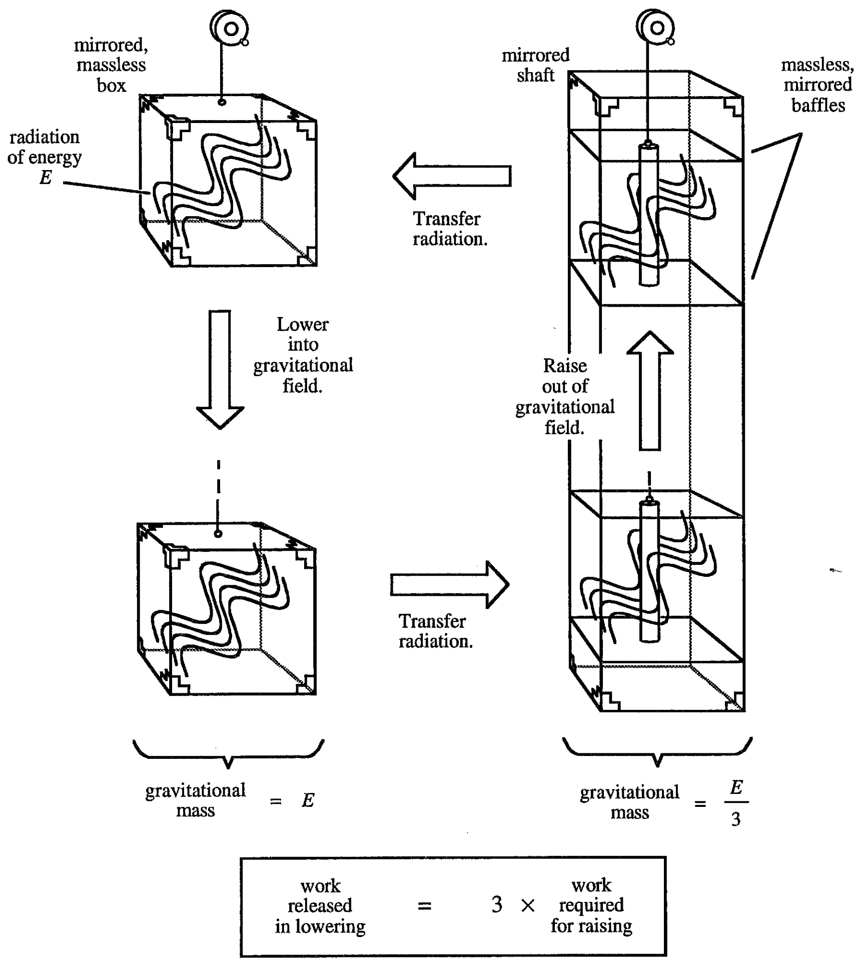

Figure 5. Trace T as source density violates energy conservation.

mitted his so-called "second" theory to *Annalen der Physik*. This theory used the trace T as its gravitational source density and fully exploited the opportunities it provided for enabling the equality of inertial and gravitational mass. Moreover, it was able to incorporate an escape from Einstein's attack on all relativistic scalar theories of gravitation.

The basic equations of the theory remained (3) and (4), except that the four-force F_μ was replaced by a four-force density K_μ:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{\partial^2\phi}{\partial u^2}=g(\phi)\nu,$$

$$K_\mu=-g(\phi)\nu\frac{\partial\phi}{\partial x_\mu},$$

where $u=ict$.

The major alteration was the inclusion of the gravitation factor $g(\phi)$. Its purpose was to allow for the fact that the total inertial mass and energy of a system must vary with the gravitational potential, whereas the gravitational mass of the system will be independent of the potential. If a system had inertial mass m when in an external gravitational field of potential ϕ, then its gravitational mass M_g was given by

$$M_g=g(\phi)m. \tag{12}$$

If we now considered a matter distribution whose parts lay in regions of differing gravitational potential, the gravitational mass of the whole distribution would be given by a g-weighted integral over its volume

$$M_g=\int g(\phi)\nu\,\mathrm{d}V.$$

At this point, the expressions for both $g(\phi)$ and the source density ν remained undetermined. Nordström now reversed the direction of Einstein's reasoning. Einstein had shown that choosing T as source density enabled the equality of inertial and gravitational mass for complete static systems. Nordström postulated this equality and from it derived Einstein's choice for source density

$$\nu=-\frac{1}{c^2}T$$

and an expression for g

$$g(\phi)=\frac{c^2}{A+\phi}.$$

The constant A could be set arbitrarily as a gauge freedom. Under the natural choice $A=0$, which yielded the potential ϕ', Nordström's second theory now provided a very simple relationship between the energy E, inertial mass m, and gravitational mass M_g of a complete stationary system

$$E=mc^2=M_g\phi'.$$

This dependence of the energy and mass of a system on the gravitational potential ϕ' was closer to familiar classical expressions than the corresponding result (6) of Nordström's first theory.

Satisfactory as these results were, they did not yet provide an escape from Einstein's objection to all relativistic scalar theories of gravitation. It is odd that this objection is mentioned nowhere in Nordström's paper, even though a major part of the paper is devoted to developing effects that were able to defeat that objection. These effects emerged from a long series of analyses of different gravitational systems, including Nordström's model of the electron, stressed rods, light clocks, gravitation clocks, and harmonic oscillators. Nordström found that a very wide range of physical quantities would depend upon gravitational potential. These included the lengths of bodies, times of processes, masses, energies, and stresses. When these dependencies were taken into account, it turned out that Einstein's violation of the law of conservation of energy no longer arose.

A simple thought experiment illustrates most simply how the dependence arises in the case of the lengths of bodies and how this dependence defeats Einstein's objection. This is our sixth thought experiment. Nordström attributed the thought experiment to Einstein although Einstein published it nowhere himself. Since Nordström (1913b) was submitted from Zurich, the home of both Einstein and von Laue, this raises the question of precisely who developed the ideas that enable escape from Einstein's objection.

Einstein's thought experiment cuts directly to the heart of the mechanism that allowed a violation of energy conservation in the fifth thought experiment. A body gains gravitational mass upon being stressed. This additional gravitational mass generates energy when the body is lowered into a gravitational field. That gravitational mass disappears when the body is unstressed. If we raise the unstressed body, we create a cycle that yields a net gain in energy. The radiation in the fifth thought experiment actually only plays an incidental role in providing a mechanism for stressing bodies that were to be raised and lowered.

The escape Nordström and Einstein now offered is ingenious. If a stressed body expanded upon being lowered into a gravitational field, then energy would be absorbed as the work required to expand the body against the stresses. Could the expansion be so adjusted that it absorbed exactly all the energy released in the fall of the gravitational mass of the stresses themselves? If so, the construction of an energy-generating cycle would be blocked. Nordström's (1913b, pp. 545–545) account of Einstein's thought experiment shows us that this adjustment is easily achieved (see Figure 6). He wrote:

> Herr Einstein has proved that the dependence in the theory developed here of the length dimensions of a body on the gravitational potential must be a general property of matter. He has shown that otherwise

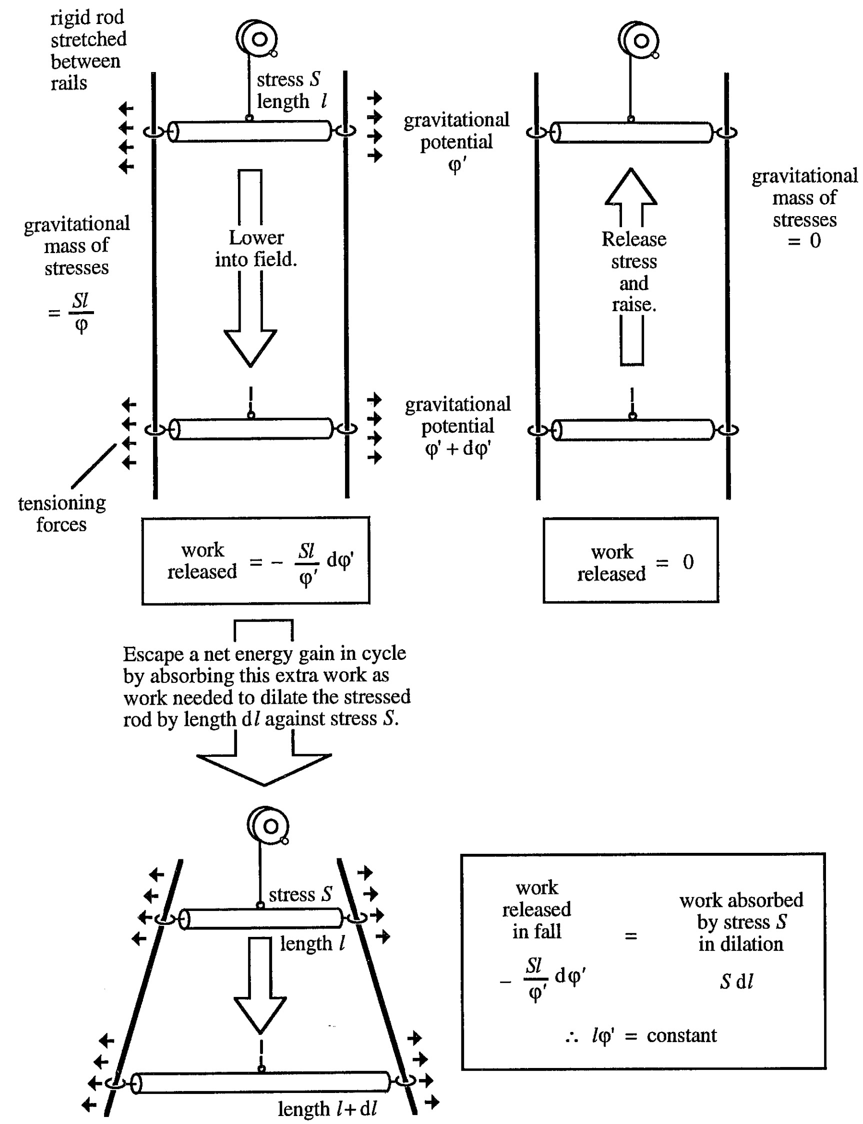

Figure 6. Gravitational potential dependence of length restores energy conservation.

> it would be possible to construct an apparatus with which one could pump energy out of the gravitational field. In Einstein's example, one considers a non-deformable rod that can be tensioned movably between two vertical rails. One could let the rod fall stressed, then relax it and raise it again. The rod has a greater weight when stressed than unstressed, and therefore it would provide greater work than would be consumed in

raising the unstressed rod. However because of the lengthening of the rod in falling, the rails must diverge and the excess work in falling will be consumed again as the work of the tensioning forces on the ends of the rod.

Let S be the total stress (stress times cross-sectional area) of the rod and l its length. Because of the stress, the gravitational mass of the rod is increased by

$$\frac{g(\phi)}{c^2} Sl = \frac{1}{\phi'} Sl.$$

In falling [an infinitesimal distance in which the potential changes by $\mathrm{d}\phi'$ and the length of the rod by $\mathrm{d}l$], this gravitational mass provides the extra work

$$-\frac{1}{\phi'} Sl\,\mathrm{d}\phi'.$$

However, at the same time at the ends of the rod the work

$$S\,\mathrm{d}l$$

is lost [to forces stressing the rod]. Setting equal these two expressions provides

$$-\frac{1}{\phi'}\,\mathrm{d}\phi' = \frac{1}{l}\,\mathrm{d}l,$$

which yields on integration

$$l\phi' = \text{const}.$$

Thus simply requiring that the length of a body vary inversely with the gravitational potential ϕ' is sufficient to preserve the conservation of energy against the threat of Einstein's earlier thought experiment. Einstein clearly accepted this escape, as he acknowledged within his exposition of Nordström's theory (Einstein 1913, p. 1253) and again more briefly in his addendum to the journal printing of Einstein and Grossmann (1913).

7. Conclusion

With the intrusion of these kinematical effects into Nordström's theory, it ceased to be a conservative, Lorentz covariant theory of gravitation and became more akin to Einstein's own theory, in which gravitation, space, and time were intimately intermingled. Just how close it had come to Einstein's theory was revealed by Einstein and Adriaan D. Fokker in a paper the following February (Einstein and Fokker 1914). Since the times of all processes and the lengths of all bodies were affected equally by the gravitational potential ϕ, the times and spaces of the background Minkowski space-time had ceased to be directly measurable by real rods and clocks. Instead they revealed a non-Minkowskian space-time with the characteristic

property that there exist preferred coordinate systems (x, y, z, t) in which the invariant interval is given by

$$\mathrm{d}s^2 = \phi^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - c^2\,\mathrm{d}t^2). \tag{13}$$

After postulation of this basic property for space-time, the theory developed in a remarkably similar way to Einstein's theory. The trajectory of a body in free fall in the gravitational field was a geodesic of the space-time. The law of conservation of gravitational and non-gravitational energy-momentum was given by the vanishing of the covariant divergence of the stress-energy tensor. Finally, the field equation of Nordström's second theory proved to be just

$$R = kT,$$

where R is the curvature scalar and k a constant. Einstein was not able to introduce generally covariant field equations based on the Riemann curvature tensor into his own gravitation theory until November 1915.

In 1914, Einstein could not offer decisive grounds for picking between his and this final version of Nordström's theory. The strongest argument he could muster against Nordström's theory was that it failed to satisfy the requirement of the relativity of inertia, a requirement whose essential content would be transformed into Mach's principle. The presence of the preferred coordinate systems (x, y, z, t) in (13) was judged by Einstein as a residual, absolute element that had to be jettisoned if the principle of relativity were to be generalized to accelerated motion.

The three soon-to-be classic tests of general relativity could offer no help in deciding between the two theories. Both Einstein's and Nordström's theory predicted a red shift in light from the sun and of equal magnitude. Unlike Einstein's theory, Nordström's theory predicted no deflection in a beam of starlight grazing the sun. However, the world would still wait five years for Eddington's celebrated expeditions. Finally, accounting for the anomalous motion of Mercury had not yet emerged as a *sine qua non* of any new gravitation theory. Einstein's theory of 1913 actually failed to account for this anomalous motion, a shortcoming that was oddly never mentioned in Einstein's publications of this period. Nordström (1914) analyzed planetary motions according to his theory. He found that it predicted changes in planetary orbits that were very small in comparison with the perturbations due to other planets and thus felt justified in concluding that this theory was "in the best agreement with experience" (p. 1109).

What decisively changed the standards for evaluation of gravitation theories was a result communicated by Einstein (1915) to the Prussian Academy on November 15, 1915. He showed that his gravitation theory, now equipped with generally covariant field equations, was able to account almost exactly for the anomalous advance of Mercury's perihelion. Overnight, the margin of error in astronomical prediction allowed a gravitation theory dropped by at least an order of magnitude. As von Laue noted in his sympathetic review (1917, p. 305), Nordström's theory was no match for Einstein's when it came to Mercury, for Nordström's theory predicted a slight retardation of the planet's perihelion. The failure was now deemed so complete that von Laue did not even bother to report the magnitude of the retardation.

After the excitement of Eddington's eclipse expedition and the public acclaim of Einstein and his theory, the fate of Nordström's theory was sealed. It could offer little competition to the seductive charms of Einstein's theory. By the time of Pauli's authoritative survey (1921, section 50), in less than a paragraph Nordström's theory was dismissed briefly and decisively as a viable gravitation theory.

NOTES

[1] M. von Laue to A. Einstein, December 27, 1911, EA 16-008. For further discussion, see Norton (1985, section 4.1).

[2] For philosophical analyses of thought experiments from various perspectives, see Horowitz and Massey (1991), which contains Norton (1986), and see also Brown (1991) and Sorensen (1992).

[3] Einstein to J. Stark, September 25, 1907, EA 22-333.

[4] One of the most informative is Einstein (1933, pp. 286–287).

[5] Here and henceforth, Greek indices will vary over 1, 2, 3, 4 and Latin indices over 1, 2, 3. I will employ the coordinate system $(x_1, x_2, x_3, x_4) = (x, y, z, u = ict)$ as was common in four-dimensional physics in the early 1910s. Summation over repeated indices will be implied.

[6] From the orthogonality of four-velocity U_μ and four-acceleration $dU_\mu/d\tau$, we infer from the contraction of (4) with U_μ that

$$0 = F_\mu U_\mu = -m\frac{\partial\phi}{\partial x_\mu}\frac{dx_\mu}{d\tau} = -m\frac{d\phi}{d\tau},$$

so that $d\phi/d\tau = 0$.

[7] In a lecture given on April 14, 1954, according to notes taken by Wheeler (1979, p. 188).

[8] p^0_{ik} is the (three-dimensional) stress tensor.

[9] $\gamma = 1/\sqrt{1 - v^2/c^2}$.

[10] See Norton (1992, section 9), and Janssen (manuscript).

[11] Einstein's analysis did not consider the corresponding exchange of momentum associated with the temporary imbalance of external forces, which would lead to the momentum expression in (8). I add this to my analysis below since it is a trivial and obvious extension of Einstein's original thought experiment.

[12] I follow Einstein in assuming that we are treating a case in which the forces between the charges on the body are small compared with the external forces and can be neglected.

[13] As usual, we have $t = \gamma(t' + (v/c^2)x')$ and $x = \gamma(x' + vt')$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$.

[14] One obvious problem with (9) that Einstein did not mention is that it is ill-defined for source matter that, unlike dust, has no natural rest frame.

[15] Von Laue's (1911a, section 5) definition was unnecessarily restrictive and did not include bodies rotating uniformly about their axes of symmetry. Nordström (1913b, pp. 534–535) quietly extended the analysis to "complete *stationary*" systems, which did include such rotating bodies.

[16] Under Nordström's choice of coordinate system, with $x_4 = ict$, $T_{44} = -$(energy density), whereas under Einstein and Grossmann's (1913) choice of metrical signature $(-, -, -, +)$, $T_{44} = +$(energy density). I have also followed Einstein in simplifying the analysis by ignoring the fact that the total energy of a system must vary with gravitational potential, whereas its gravitational mass will not. Thus the expression for the proportionality of the inertial and gravitational mass of a system must contain a factor that is a function of the gravitational potential. This effect is explicitly incorporated into Nordström's (1913b) second theory through the factor $g(\phi)$, and the proportionality is expressed as relation (12) of Section 6 below. For the analysis of this section and the following, this g factor can be taken as approximately constant and its effect absorbed into other constants in the equations.

[17] This conclusion holds for free radiation, and for this reason there is no gravitational bending of light in Nordström's (1913b) second theory, since it employs T as its source density.

[18] To see this most clearly, imagine that each pair of opposing walls of the box are held together by a slender rod that carries all the stresses needed to hold the walls against radiation pressure. One set of opposing walls and rods forms the set of baffles. Three identical sets can be fitted together to form the cubical box.

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