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Circularity

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The Large Scale Structure of Inductive Inference

1. Fear of Circles

The non-hierarchical structure of relations of inductive support admits circularities. They are inevitable once we examine a large enough set of these relations. The circles may be small, when two propositions mutually support. The circles may be large, when extended chains of relations of support eventually connect back to their starting points. Some will find the mere presence of these circles, in itself, disturbing. They do so, apparently, with good reason. In debates, philosophical or not, defeat is assured if your opponent can expose your reasoning as circular. In formal structures, circularities are vicious and they must be eliminated, often by the most elaborate of novel theorizing. The damning verdict is automatic and unanswerable. You have found a circularity? There is no need to waste any more thought on the enterprise. It fatally flawed. The perpetrator of a circularity may be expected to resort to all manner of sophistry. But escape is impossible and the ultimate collapse of the enterprise is inevitable.

Such is the fear of circles, *horror circulorum*. This chapter is written for those in its grip. The goal is to provide them therapy. For the *horror* is based on an oversimplified view of circularities. It neglects the many forms that circularities can take. Some are as fatal as this dark view fears. Many are benign and, we shall see, others are even essential to a theoretical structure. To ban them unilaterally would restrict unnecessarily the scope of our theorizing. To show this, the chapter provides a small classification of circularities, according to how they affect the logic of the structure in which they appear. It will show that the circularities of relations of inductive support are benign and even essential.

There are three categories. First are the "vicious" circularities to be explored in Section 2. They lead to logical inconsistencies and underwrite the dark view of circularities as fatal defects.

When such circularities arise in inductive structures, they are transient and eliminated by suitable adjustments to the propositions in the structure. The second, explored in Section 3, are circularities in structures whose content is left indeterminate. These may merely be failed arguments or intermediate stages of development on the way to the third type. Or, if they are ineliminable, they may be the basis of a convention. In the third case, described in Section 4, the circularities are part of a well-behaved structure whose content is uniquely defined, without contradiction. This is the case of the relations of inductive support of a mature science. A final Section 5 summarizes how the circularities in relations of inductive support appear in the taxonomy. The mechanism identified in the next Chapter 4, "The Uniqueness of Domain-Specific Inductive Logics," leads to a convergence towards inductive structures with univocal import.

That benign circularities are possible is the tonic that can cure *horror circulorum*. It tells us that mere identification of a circularity in some system is a starting point, not an endpoint. If you want to take the next step and damn the system for the circularity, there is a positive obligation on you to establish that the specific form of circularity present is harmful. This cannot be done, I believe, for the circularities in a mature science. They are benign.

2. Vicious Circularity

A vicious circularity, as I shall use the term here, is a set of circular relations in some formal structure that leads to a contradiction.

2.1 The Idea

The term "vicious circle" has long been familiar in treatises on logic. Kirwan (1807) already found its usage established. Curiously, the formal definition he gave was merely of question begging, "*petitio principii*," which is described in more detail below in Section 3. Kirwan wrote of (pp. 441-42, his emphasis):

> … that mode of argumentation called the *vitious circle* [sic], in which one point is proved by another, and this other is proved *solely* by the first; so that the proofs are mutual and under the same point of view.

That what is described is really question begging is made quite clear by Munro's (1850) treatise whose exposition follows Kirwan's closely. Munro (1850, p. 231) illustrated the circle as:

> The whole of Dr. Brown's elaborate lectures on the nature of virtue amounts to nothing more than a vicious circle. We approve of actions, because they are right; and they are right, because we approve of them.

More curiously, Kirwan's own example was of a circle that produced a contradiction. His definition of "vitious circle" is immediately illustrated by the self-refutation of skeptics:

> … Thus the sceptics argue, that we ought to doubt of every thing, because human reason is fallible, and may deceive us. And since reason may deceive us, we should doubt of the validity of the reasons that induce us to doubt.

The idea of a vicious circle as essentially leading to a contradiction was cemented by Bertrand Russell's work in mathematical logic. In reflecting on Cantor's proof that there can be no greatest cardinal number, he arrived at what came to be known as Russell's paradox. It is given an early elaboration in Russell (1903, Ch. X), "The Contradiction." The paradox concerns sets and their members. Some sets may have other sets as their members. Naively, we easily accept that some may even be members of themselves. A set of sets can be a member of itself, for example. But what of those sets that are not members of themselves? What of the set of all such sets? The supposition that there is such a set immediately produces a contradiction. If it is a member of itself, then it is not a member of itself. But if it is not a member of itself, then it is a member of itself. The contradiction arises essentially through the circular relationship between the set and its members.

While the paradox looks at first like a minor annoyance that is easy to circumvent, it was immediately recognized as a deep problem for set theory and the foundations of mathematics. For it shows that sets could not be defined merely as the extension of any property. That is, we could not say "Consider the set of all things that have property P." where property P, expressed as some formula, could be freely chosen.[1] The most searching and elaborate investigations were needed to give set theory a non-contradictory foundation. One avenue was the development of the axioms of Zermelo-Fraenkel set theory. Russell's path led to the theory of types, found in his joint work with Alfred North Whitehead, *Principia Mathematica*. There Russell and Whitehead reinforced the odious character of vicious circles. The first named section in Volume I, Chapter 2

[1] This troublesome principle has been called "the intuitive principle of abstraction" in Stoll (1963, p. 6).

was entitled "The Vicious-Circle Principle" and one of its formulations was (Russell and Whitehead, 1910, p. 40) "Whatever involves *all* of a collection must not be one the collection." Breaches of this principle, they announced, were to be called "vicious-circle fallacies."

The vicious circle of Russell's paradox derived from its imprudent use of self-reference. Such imprudence proved to be a fertile source of analogous paradoxes. Russell (1908) provided a convenient compendium. It began with the now classic Epimenides (p. 222):

> Epimenides the Cretan said that all Cretans were liars, and all other statements made by Cretans were certainly lies. Was this a lie? The simplest form of this contradiction is afforded by the man who says "I am lying;" if he is lying, he is speaking the truth, and vice versa.

Its structure matches that of Kirwan's example of the self-refuting skeptics. The inventory continued with Russell's set paradox and a list of other related paradoxes familiar to readers of the literature, including Berry's paradox, Richard's paradox and the Burali-Forti contradiction. These paradoxes provided the impetus for a century of philosophical work on truth in the foundations of formal logic. It was designed to find ways of precluding paradoxical sentences like "This sentence is false." or finding unparadoxical ways of including them. The contradictions that follow from self-reference became one of the most powerful tools of formal logic. They are the basic device used in Gödel's famous demonstration of the incompleteness of arithmetic.

To philosophers who have any interest in formal matters, all this is so elementary as to have become part of "what everyone knows." At the same time, these foundational investigations have forged an automatic and enduring link between circularity and contradiction. And so the *horror circulorum* is established.

2.2 Vicious Circles in the Material Theory of Induction?

Are the circularities of inductive support vicious? Nothing compels it. As we shall see below, one can have circularities that are not vicious, that is, that produce no contradictions. Such are the circles arising among the relations of inductive support for mature sciences. The inductive support of these mature sciences is secure and even unassailable; and they would not be so if contradictions could be found within them.

This is the situation with the evidential support of a mature science. However, prior to this mature stabilization, contradictions can and do arise among the relations of support.

Developing sciences are commonly built upon hypotheses, whose evidential grounding has not been secured. Sometimes these hypotheses fail and that failure manifests in contradictions. In 1917, Einstein presented the first relativistic cosmology, using the assumption that the universe is static. His hypothesis was soon contradicted by Hubble's discovery of the recession of the galaxies.

These contradictions are not manifestations of an ineliminable, foundational flaw in the very idea of inductive support. They are unlike the vicious circularities of naïve set theory, whose circularities forced us to reconceive the very idea of a set and of the truth of propositions. Rather they are a natural part of the work of fallible investigators. The structures they produce are fallible but malleable, and it is a routine part of investigations to reform the structures to eliminate them. Einstein discarded his assumption of a static universe, while other theorists began to explore the dynamic, expanding universes compatible with general relativity. These contradictions and adaptations are of no more concern than an accounting error in a budget. Perhaps a receipt was mistyped, or an expense neglected. It is a simple but tedious exercise to find the error and correct it. There has been no fundamental breach of a principle of arithmetic that would forever preclude the use of budgets.

The radiocarbon dating of historical artifacts, described in Chapter 10, "Mutually Supporting Evidence in Radiocarbon Dating," shows how these contradictions arise within a circle and are remedied. Artifacts are dated by two means. The first derive from traditional historical analysis. The second derive from the measurement of the radioactive ^{14}C ("carbon 14") content of the artifact. What results are two sets of propositions, one historical and the other radiocarbon. Each should support the other. When radiocarbon dating methods were first explored, it soon became apparent that there were recalcitrant discrepancies in the dating provided by the two means. That is, there were contradictions within the circular relations of mutual support among the radiocarbon dates and historical dates.

The elimination of these contradictions became a major focus of research in radiocarbon dating methods. Radiocarbon dating depends essentially on knowing the original ^{14}C content of the artifact. That content is halved for each ^{14}C half-life of 5730 years. It was natural to suppose that these original levels match those of artifacts formed today. It soon became apparent that this assumption was the source of the contradictions. These levels have varied over historical times. Theoretically grounded reconstruction of these original levels proved unworkable. Instead, these levels were reconstructed by means of the historically known age of artifacts. The corrections needed were collected in a calibration curve, such as is shown in the later chapter. Using such curves, the radiocarbon and historical datings of artifacts were adapted to one another in the precise manner needed to eliminate the contradiction. After that adaptation, each set of datings could be used to check and affirm the other. The circularity among the two sets of propositions remained, but without contradictions.

In a similar vein, the structures of inductive support for a mature science can be disrupted by new, empirical discoveries. The disruption manifests as contradictions that can be removed by adjustments to the inductive structure. This is the common dynamic of scientific revolutions. Newton's seventeenth century mechanics prevailed for over two centuries. Its inductive support was, apparently, unassailable. One of its basic results was that the velocity of a uniform observer was to be added or subtracted from that of any propagation to recover the velocity the observer would find for it. The new evidence of Maxwell and Lorentz's nineteenth century electrodynamics destabilized Newton's mechanics. For, under Einstein's careful scrutiny, the electrodynamics revealed that light propagation violated this simple Newtonian result. The speed of propagating light was always the same, no matter the uniform motion of the observer.

The contradiction was resolved when Einstein realized that space and time themselves, at high speeds, do not behave as Newton had concluded. The evidence and relations of evidential support leading to Newton's theory were not discarded. Rather their limited scope was now recognized. They could be applied only to systems moving at much less than the speed of light. This restriction was readily implemented. Newton drew the evidence for his mechanics from the motions of ordinary falling bodies, moons and planets. These are all bodies whose speeds are much less than that of light. The evidential base of Einstein's special relativity embraced that of Newton's mechanics for small speeds and that of electrodynamics for higher speeds.

3. Indeterminate Circularities

A less troublesome form of circularity arises when the circles produce no contradictions but leave the structure indeterminate. The indeterminacy may not be obvious, since the analysis may be offered as determinate.

3.1 Begging the Question

A familiar example, known since Aristotle, is circular reasoning, "begging the question"[2] or the *petitio principii.* It is a form of reasoning that pretends to establish a conclusion, while only giving the illusion of doing so. Richard Whately's *Elements of Logic* gives what seems to be a standard definition for nineteenth century work.[3] Alerting us in a preface ("advertisement") that he uses square brackets "[…]" to indicate equivalent meanings, he tells us (1856, p. 184)

> … "*petitio principii*" ["begging the question,"] takes place when a premiss, whether true or false, is either plainly equivalent to the conclusion, or depends on it for its own reception.

He continues to note the delicacy of the identification. For unobjectionable deductive inferences will have this this character in case a premise entails the conclusion and conversely. Such is the case for inferences that demonstrate the equivalence of two physical conditions, such as the equivalence of the "Thomson" and "Clausius" forms of the second law of thermodynamics. To be worthy of the label *petitio principii*, there must be some sense that the inference is used deceptively, to pretend that more is gained than really is. Whately (1856, p. 222) notes "Obliquity and disguise being of course of most importance to the success of petitio principii…"

Examples are easy to find. One is a religious figure or tract for which infallibility is to be concluded, since the figure or tract themselves declare their infallibility. The more interesting cases of begging the question arise when the circularity is sufficiently hidden that its presence is easily overlooked. Mill (1882, p. 574; his emphasis) provides an example:

> Plato, in the *Sophistes*, attempts to prove that things may exist which are incorporeal, by the argument that justice and wisdom are incorporeal, and justice and wisdom must be something. Here, if by *something* be meant, as Plato did in fact mean, a thing capable of existing in and by itself, and not as a quality of some other thing, he begs the question in asserting that justice and wisdom must be something ; if he means any thing else, his conclusion is not proved.

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[2] This is the original sense, to which I adhere. A recent usage gives the expression the meaning "inviting the question."

[3] Mill (1882, p. 571) reports Whately's treatment extensively.

Another more extended example, I contend, arises in the many demonstrations of probabilism: Dutch book arguments, decision theoretic representations, the accuracy-based scoring rule argument, and so on. In *The Material Theory of Induction*, Chapters 10 and 11, I argue that all these proofs proceed by employing premises in which the basic assumptions of probabilism are already present in disguised form. When their presence is identified, the demonstration collapses. It then becomes easy to see that arbitrary adjustments to the rules of the betting scenario, to the properties of the preferences assumed or to the scoring rule used, can lead to variant, non-probabilistic calculi.

For present purposes, the essential fact is that circular reasoning simply fails to determine the conclusion sought. Nonetheless, the conclusion sought may be true, or it may be false.

3.2 Circularities that Produce Conventions

Similar indeterminacy-producing circularities arise among magnitudes in science. The indeterminacy is then often taken as evidence that the magnitude of some quantity can be set as a convention. Perhaps the best-known examples arise in relativity theory and in geometry. In his 1905 special relativity paper, Einstein argued that we could not affirm the simultaneity of spatially separated events, factually, by light signals (or, analogously, by any other means). For any scheme that uses light signals to ascertain the relative timing of such events requires that we know how fast light propagates in one direction. A natural scheme requires, for example, that we know that light propagates at the same speed from a place A to a place B as it does in the reverse direction. Yet to know this, we must be able to determine how quickly light propagates from one place to another. This determination requires that we can already compare the timing of events at these two places.

Einstein (1920, pp.22-23) summarized our predicament: "It would thus appear as though we were moving here in a logical circle."[4] The significance of this circle is that there are no independent facts separately for the simultaneity of spatially separated events and the speed of light propagating between them. Rather we can choose freely as a convention either the simultaneity relation or this speed. Then the other is determined. Here is how Einstein (p. 23, his emphasis) put it:

[4] "Man scheint sich also hier in einem logischen Zirkel zu bewegen."

> That light requires the same time to traverse [the forward path] as for [the reverse path] is in reality neither *a supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own free will in order to arrive at a definition of simultaneity.

Einstein's foremost expositor in this matter, Hans Reichenbach (1958, pp. 126-27), summarized a more extensive analysis of the same circularity as:

> Thus we are faced with a circular argument. To determine the simultaneity of distant events we need to know a velocity, and to measure a velocity we require knowledge of the simultaneity of distant events. The occurrence of this circularity proves that simultaneity is not a matter of knowledge, but of a coordinative definition, since the logical circle shows that a knowledge of simultaneity is impossible in principle.

Under Poincaré and Einstein's inspiration, Reichenbach (1958, §30) argued for a structurally analogous convention he called the "relativity of geometry." It depends on a similar logical circle. One can determine that the geometry of a space is Euclidean or otherwise by the expedient of surveying it with measuring rods. The essential condition is that the rods are rigid ones that measure distances truly. The complication, Reichenbach urged, is that rods may be acted upon by what he called "universal forces" that equally distort all bodies. This complication creates the circle. We cannot know which universal forces, if any, are acting on a rod unless we already know the true geometry of the space. The circle is resolved by declaring that we may select the geometry of space conventionally. We merely posit the universal forces needed so that our rod measurements give us that geometry.

For completeness, I should mention that both conventionality theses were hotly debated in the later part of the twentieth century, without any clear resolution. Those opposed to the conventionality claims urged that there were other non-conventional means to break the circles. We do not need to take sides in this debate for present concerns.[5] All we need to see is that these

[5] However, I incline towards the anti-conventionalist view. For an elaboration, see the chapter "The Conventionality of Simultaneity" and "Geometric Morals" in my online text, *Einstein for Everyone*, http://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/index.html Reichenbach's supposition of universal forces is troublesome since, if the mode of analysis is accepted,

circular dependencies among physical quantities can leave the quantities indeterminate. Even a fairly modest empiricism must be troubled by the idea of quantities whose values cannot be determined by any physical measurement or observation. If the indeterminacy is sustained, the comfortable resolution is to assert that there is no physical fact for these values. They may be chosen arbitrarily, that is, as a convention.

3.3 Indeterminate Circularities in Relations of Inductive Support

It is quite possible for this sort of circularity to arise among relations of inductive support. If they prove to be ineliminable, then we might expect an empirically-minded scientist to proceed as above. If we are sure that no evidence can break the circle, we have concluded that these are propositions whose truth is immune to evidential scrutiny. Such propositions are leading candidates for conventional stipulation. Indeed conventional stipulation will, by the supposition of the case, make no difference empirically.

The more common situation is the one that arises in the examples of circular dependencies recounted in the earlier chapters and explored in greater detail in later chapters. The circularities may initially be such as to leave the quantities of interest indeterminate. However further investigation brings new facts to bear that break the circularity. Indeed, a focus on exactly such investigations can become a major stimulus for further research. We have seen this dynamic repeatedly.

Dalton's original proposal of his atomic theory was trapped in a circle, as detailed in Chapter 11. To know the correct molecular formulae of substances, he needed to know the relative weights of the atoms combined in them. But he could only know those relative weights if he already knew the molecular formulae. This meant that his theory was compatible with water having a huge array of different molecular formulae: H_2O, HO, HO_2, and many more. He was free to stipulate any of them, without fear that the meager evidence at his disposal would contradict his choice. He chose HO. Indeed, had the circularity proved unbreakable, we might eventually have settled onto a curious sort of atomic theory in which the relative masses of the atoms could be set arbitrarily, much as we arbitrarily set the zero point for the potential of a Newtonian gravitational field. As we now know, this freedom was transient. It still took over half

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analogous suppositions can be used to establish the conventionality of any physical magnitude that is measured by some instrument.

a century of further work to bring enough additional facts to bear to break the circle and recover H_2O.

The determination of celestial distances involved similar indeterminacy-producing circularities. Our earliest efforts to determine the distance to the moon and sun were troubled by one. We could measure the angular sizes of these bodies, so that, if we knew their diameters, we could infer the distances to them. However, we needed to know just these distances to determine their diameters. Chapter 12, “The Use of Hypotheses in Determining Distances in Our Planetary System,” describes how diligent analysis by ancient astronomers was able to break the circularity and produce estimates of the diameters and distances.

Another circularity of a similar type appeared in Hubble’s classic 1929 paper on the recession of the nebulae. Hubble had apparent brightness measurements for 46 nebulae. To convert these to distances, he needed to know the absolute brightness of these nebulae. Then, using the fact that brightness diminishes with the inverse square of distance, the distances to the nebulae are recovered by comparing how bright the nebulae seem with how bright they really are. However, for 22 of them, Hubble lacked absolute brightness determinations. Indeed, absent other information, to know their absolute brightness, he needed first to know how distant they are. This closes the circle, leaving the distances to these 22 nebulae indeterminate. As recounted in Chapter 7, “The Recession of the Nebulae,” Hubble brought further statistical considerations to bear to break the circularity and recover determinate, if fallible, distances for these 22 nebulae.

In sum, this sort of indeterminacy-producing circularity can arise among relations of inductive support. It presents no foundational challenge to the very notion of inductive support. There are many possibilities, none foundationally troublesome. The circularities may be broken by further scientific investigations. If ineliminable, they may prove to arise from conventions. Or they may be ineliminable simply because of a paucity of evidence. Certain sorts of historical facts are obvious candidates. We might like to know many details of some ancient civilization. However, if sufficient archaeological evidence has not been preserved, we have no choice but to settle for indeterminacy, not of the facts but of what the evidence can determine about them. That is just how it should be.

4. Determinate Circularities

The most benign circularities are those that arise in determinate structures. Then none of the issues of contradiction or indeterminacy arise. This sort of circularity is widespread and so familiar that they rarely arouse complaints.

4.1 Elementary Examples

Simple computations of determinate magnitudes often involve circularities. An easy example is computation of the black area "B" and the white area "W" of the yin-yang symbol. From the symmetry of the figure we have

$$B = W$$

It is one half of a circular dependency. Assuming the total figure has unit area we also have

$$W = 1 - B$$

This is the second half of the circular dependency. There is nothing troublesome in the circularity. The two equations are solved uniquely to give

$$B = W = 1/2$$

A slightly fancier computation is the standard way that the following infinite sum is evaluated:

$$S = 1/2 + 1/4 + 1/8 + 1/16 + \ldots$$

In a familiar manipulation, the sum is doubled to yield

$$2S = 1 + 1/2 + 1/4 + 1/8 + \ldots = 1 + S$$

This last equation expresses a circular dependence, but is readily solved to give us the sum $S = 1$.

The only danger in this otherwise benign computation is that we must antecedently be assured that the infinite sum does have a definite, finite value. Even if we are not assured that the sum is finite, the circularity can still give us a determinate result. Consider

$$S = 1 + 2 + 4 + 8 + \ldots$$

It is doubled to yield

$$2S = 2 + 4 + 8 + 16 + \ldots = S - 1.$$

This circular equation in S has two solutions. S = -1 can be discarded if we preclude a negative sum. The applicable solution is S = infinity.

Finally, we might ask whether these circularities could mislead us when the sum sought is badly behaved. Such is the case with the Grandi's series, whose sum we might try to write as:

$$S = 1 - 1 + 1 - 1 + \ldots$$

Of course, there is no such sum. The partial sums oscillate indefinitely between 0 and 1. If we proceed formally, we might write

$$S = 1 - 1 + 1 - 1 + \ldots = 1 - (1 - 1 + 1 - \ldots) = 1 - S$$

This circular equation in S has a unique, finite solution, $S = 1/2$. This value cannot be the ordinary arithmetic sum of Grandi's series, for there is no such sum. However, if we consider generalized notions of summation that might be applied here, we could then take this circular dependency as part of the conditions of adequacy of the generalized notion. An example of such a generalized notion is the Cesàro sum. It proceeds by taking the *arithmetic average* of the first n terms in the series. The sum of the entire series is just the limit of this average as n goes to infinity. The Cesàro sum for the Grandi series is 1/2.[6]

4.2 An Extreme Example

These examples of benign circularity have been elementary. They serve to show, however, that circularities within well-defined structures are common and unremarkable. At the other extreme, we can have similarly benign circularities in quite exotic structures. Most striking of these is one that directly challenges the historical stimulus of *horror circulorum*, Russell's Vicious-Circle Principle. The principle prohibits circularities, such as sets that are members of themselves. In response, the edifice of modern set theory, as exemplified in the Zermelo-Fraenkel system, was built precisely to preclude such circularities.

All this changed with the appearance of Peter Aczel's non-wellfounded set theory or hyperset theory. It provides an account of sets that allows for just the sort of circularities prohibited by Russell's principle, but without inducing his paradoxes. The details of the theory go well beyond what can be reviewed here. Most briefly, the approach drops the Foundation Axiom of the Zermelo-Fraenkel system and replaces it with the Anti-Foundation Axiom. The import of the transition can be seen in the case of the simplest circularity in set membership. Following Barwise and Etchemendy (1987, pp. 37-41), it is the set Ω, defined circularly by the fact of its self-membership:

$$\Omega = \{\Omega\}$$

[6] In another approach, we consider $S(a) = 1 - a + a^2 - a^3 + a^4 - \ldots = 1/(1+a)$ for $0<a<1$. We *define* $S(1) = \mathrm{Lim}_{a \to 1} S(a)$ and it does have the value 1/2.

That is, the set Ω is defined as that set that has itself as its sole member. If we substitute for Ω, we can rewrite the set as $\Omega = \{\{\Omega\}\}$. Continuing, we have $\Omega = \{\{\Omega\}\} = \{\{\{\Omega\}\}\} = \{\{\{\{\Omega\}\}\}\}$. A full substitution leads to an infinite nestling of set memberships:

$$\Omega = \{\{\{\{\{\{\{ \dots \}\}\}\}\}\}\}$$

Precisely this infinite nestling of set memberships is prohibited by Zermelo and Fraenkel's Axiom of Foundation. All such nestlings, according to it, must terminate finitely. Aczel's Anti-Foundation Axiom allows it because it can be given a definite graph theoretic representation[7] and, moreover the axiom asserts its uniqueness.

In this set Ω, we have just the sort of circularity that should trigger *horror circulorum*, a set that is its own member. However, that very circularity defines a determinate, unique structure in non-wellfounded set theory.

4.3 Intermediate Examples

Between these elementary and exotic instances of benign circularities, there are many more instances, all of them part of unremarkable, routine science. A great achievement of nineteenth century physics was Maxwell's electrodynamics. Its basis, in modern formulation, are the four vector differential equations knows as "Maxwell's equations." In the simplest case of electric and magnetic fields in vacuo, these equations fix the electric field strength vector **E** and the magnetic field strength vector **H**. Using the older Gaussian system of units (in which the equations are simpler) and standard notational conventions, the first two equations are just

$$\nabla.\mathbf{E} = 0 \text{ and } \nabla.\mathbf{H} = 0$$

These equations do not govern how the fields evolve in time, such as when electromagnetic waves propagate. Their time evolution is recovered from the next two equations, which exhibit a tight circular dependence. The third is

$$\nabla \mathrm{x} \mathbf{H} = (1/\mathrm{c})\, \partial \mathbf{E}/\partial t$$

It asserts that a strengthening electric field produces a rotational magnetic field, whose lines of force form circles around those of the electric field. The fourth equation is

$$\nabla \mathrm{x} \mathbf{E} = - (1/\mathrm{c})\, \partial \mathbf{H}/\partial t$$

[7] It is just $\Omega \rightarrow \Omega \rightarrow \Omega \rightarrow \Omega \rightarrow \dots$

It asserts an analogous process: a strengthening magnetic field produces a rotational electric field, whose lines of force form circles around those of the electric field.

This circular dependence among quantities like **E** and **H** is common. A second and much more elaborate set of circularities arises in Einstein's gravitational field equations for his general theory of relativity. They are used to determine the basic quantity of the theory, the metric tensor. It is, expressed in coordinate based components, a matrix of 10 quantities: $g_{ik} = (g_{00}, g_{01} = g_{10}, g_{02} = g_{20}, \ldots, g_{33})$. These ten quantities are fixed by Einstein's ten, second order, coupled, non-linear partial differential equations. Through their coupling, they harbor an elaborate set of circular interdependencies among the components g_{ik}.

While circularity is inherent in both Maxwell's equations and the Einstein's equations, they produce quite determinate structures. That is, allowing for standard gauge freedoms, they both admit well-posed initial value problems. Loosely speaking, that means that if we determine the configuration of fields for the present moment, then their evolution into the future is uniquely determined. We have no trouble using these equations to determine precisely how radiowaves propagate and how black holes form.

4.4 Determinate Circularities among Relations of Inductive Support

Circularities among physical quantities arise routinely as a benign feature of determinate structures in physical theories. Similarly, it is routine for the structures of inductive relations in a science to harbor circularities, even as the bearing of those relations is univocal. Such is the most common case among the examples of circularities in inductive structures seen in this chapter and elsewhere in this book. For example, Dalton's original atomic theory was beset with a circularity. Subsequent research removed it and gave us determine molecular formulae and atomic weights. The ancient circularities that troubled the determination of distances to celestial bodies were resolved, so that we now have very precise determinations of them. Hubble's 1929 analysis was hampered by a circularity that precluded the direct determination of distances to 22 of the 46 nebulae in his data set. After further investigations, the distances to these closer nebulae are no longer in any doubt. While radiocarbon and historical dating of artifacts enter into circular dependencies, we now have sufficient cross-checking of the methods that the original uncertainties have been eliminated.

5. Conclusion

Circularities arise routinely among rich structures of evidential support. They are no mere accident. Rather they are part of what enables a mature science to establish the familiar solidity of its evidential support. For those in the grip of *horror circulorum*, their presence is a source of concern and doubt. In this chapter, I have sought to demonstrate that this fear is unfounded.

Some circularities are worrisome. Such are the vicious circularities whose contradictions forced us to abandon the naïve notion of a set and to develop elaborate theories of truth. We saw in Section 2 that there can be circularities that produce contradictions in relations of inductive support. However, they are not of the same type that would force us to abandon the very idea of inductive support. They are transient difficulties that are resolved by further investigations.

Other circularities do not produce contradictions but leave their structures underdetermined. That is troublesome only if it is pretended otherwise. It is this deception that renders begging the question objectionable. Otherwise, these circularities can be employed usefully to establish the conventionality of a physical magnitude. In the case of relations of inductive support, these indeterminacies can arise in intermediate stages of investigation. If they prove ineliminable, it may be that we have found a hidden convention; or it may be just that insufficient evidence exists for us to learn definitively about the target system.

Most commonly, the indeterminancies are eliminated by further investigations. They lead to an inductive structure with univocal import that is characteristic of a mature science. As Section 4 recounts, in this they are like many of the circularities among physical quantities in science that are untroubled by indeterminacies. That they arise commonly in mature sciences is not happenstance. In the next chapter, "The Uniqueness of Domain-Specific Inductive Logics," I will argue that this uniqueness results from a definite mechanism. If there are competing systems, the competition is unstable. If one system gains an advantage by learning facts favorable to it but weakening its competitor, it follows from the material conception of inductive inference that this strengthens the inductive reach of the first, while diminishing that of the competitor. As long as further evidence is available and investigators pursue it, this instability is self-reinforcing and leads to the unique admissibility of the first system.

References

Barwise, Jon and Etchemendy, John (1987) *The Liar: An Essay on Truth and Circularity*. New York: Oxford University Press.

Einstein, Albert (1920) *Relativity: The Special and the General Theory*. 3rd ed. Trans. R. W. Lawson. London: Methuen.

Kirwan, Richard (1807) *Logick*. Vol. 1. London: Payne and MacKinlay in the Strand.

Mill, John Stuart (1882) *A System of Logic*. 8th ed. New York: Harper & Bros.

Munro, H. H. (1850) *A Manual Of Logic, Deductive and Inductive*. Glasgow: Maurice Ogle and Son.

Reichenbach, Hans (1958) *The Philosophy of Space and Time*. Trans. M. Reichenbach and J. Freund. New York: Dover.

Russell, Bertrand (1903) *The Principles of Mathematics*. Vol. 1. Cambridge: At the University Press.

Russell, Bertrand(1908) “Mathematical Logic as Based on the Theory of Types,” *American Journal of Mathematics*, **30**, pp. 222-262

Russell, Bertrand and Whitehead, Alfred North (1910) *Principia Mathematica*. Cambridge: at the University Press.

Stoll, Robert R. (1963) *Set Theory and Logic*. New York: Dover, 1979.

Whately, Richard (1856) *Elements of Logic*. 8th ed. Rev. New York: Harper & Bros.