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DISCUSSION

Comments on Laraudogoitia's 'Classical Particle Dynamics, Indeterminism and a Supertask'

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ABSTRACT

We discuss two supertasks invented recently by Laraudogoitia [1996, 1997]. Both involve an infinite number of particle collisions within a finite amount of time and both compromise determinism. We point out that the sources of the indeterminism are rather different in the two cases—one involves unbounded particle velocities, the other involves particles with no lower bound to their sizes—and consequently that the implications for determinism are rather different—one form of indeterminism affects Newtonian but not relativistic physics, while the other form is insensitive to the classical vs relativistic distinction. We also note some interesting linkages among supertasks, indeterminism and foundations problems in the general theory of relativity.

1 Classifying supertasks that lead to indeterminism

Laraudogoitia [1996, 1997] has described two very simple supertasks in Newtonian dynamics that are both intriguing and revealing. In both determinism fails. Laraudogoitia simply attributes this failure of determinism to the known indeterminism of classical dynamics. We urge that one can be more precise about the origin of the indeterministic behaviour and, if one is interested in how these supertasks may be realized in various physical theories, one must be more precise since the two examples bring about indeterminism by quite different means.

Both of Laraudogoitia's examples involve the interaction of infinitely many bodies and to this extent they are similar. In each case, the interaction is a sequence in time of infinitely many collisions of pairs of bodies. Each individual collision is well behaved. The composition of finitely many is also well behaved.

Pathologies arise, however, when we proceed to the limit of the composition of infinitely many of these collisions. For, as we have recalled elsewhere in the context of supertasks (Earman and Norton [1996], Section 8), an infinite composition can have quite different properties from the finite composition, and this is essentially responsible for the paradoxical character of many supertasks. For example, while energy or particle number may be conserved in finitely many collisions, neither need be conserved in infinitely many—as these supertasks show.

In order for these infinitely many collisions to realize a supertask, the rate at which they occur has to be accelerated so that infinitely many are completed in a finite time. The mechanism of this acceleration is essential to the conversion of a task into a supertask and it differs essentially in Laraudogoitia's two cases. In the example of Laraudogoitia [1997], the rate of these collisions is accelerated by allowing there to be no upper bound on the velocities of the bodies. Thus the time between successive collisions can decrease without limit. In the case of Laraudogoitia [1996], the rate of collision is accelerated by employing bodies with no lower limit to their size so that infinitely many can be enclosed within a finite volume of space. Thus the distances to be covered by the bodies between successive collisions can decrease without limit and the infinity of collisions be completed in a finite time—and this without arbitrarily great speeds.

Because of their different means of accelerating the rate of collision, the two supertasks relate quite differently to classical and relativistic physics. The type of Laraudogoitia [1997] is blocked by the transition from classical to relativistic kinematics since the latter prohibits acceleration to speeds greater than light. The type of Laraudogoitia [1996] can be implemented in both classical and relativistic kinematics since it requires no speeds greater than light. Whether a physical theory, classical or relativistic, admits these latter supertasks depends on whether it admits bodies with no lower limit to their size[1] and for which there is no lower limit to the time of collision interactions. This is a quite different type of physical problem and, as we shall see below, one with its own peculiar pathologies.

2 Supertasks that depend on there being no upper limit to velocities

The supertask of Laraudogoitia [1997] achieves indeterminism by boosting particles to infinite velocity in finite time so that they disappear at spatial infinity. Since the relevant dynamics is time-reversible, the time-reversed process is also admissible. It portrays particles appearing from spatial infinity with nothing in the past determining the time of appearance. This strategy for realizing an indeterministic process is well known in the literature. Mather and

[1] Equivalently, one could use point mass particles interacting via short range repulsive force for which the range can be made smaller and smaller with no finite lower limit.

McGhee [1975] used it in an example with finitely many particles. They derive the unlimited energy needed to achieve unbounded velocities by drawing on the infinite potential well of point mass particles interacting via Newton's $1/r^2$ law except when they collide, in which case the collisions are regularized as elastic bounces. Only recently was it rigorously demonstrated by Xia [1992] that five point mass particles interacting via Newton's $1/r^2$ law can escape to spatial infinity in a finite time without colliding. Once again the reversibility of the dynamics means that the appearance of five particles from spatial infinity is admissible. Lanford ([1975], pp. 50–3) describes an infinite collection of hard spheres, initially at rest. They are spontaneously perturbed by a violent excitation to arbitrarily high velocities that propagates into the system of particles from spatial infinity.

In all these examples, spatial infinity plays a special role. The indeterminism arises from bodies, particles or excitations spontaneously materializing from spatial infinity. Thus one may be tempted to believe that it is the communion with spatial infinity, made possible by the lack of upper bound on velocities, that is responsible for the loss of determinism. In this section we intend to show that, whatever may be the role of spatial infinity in these examples, spatial infinity is inessential in that once there is no upper bound to velocities, indeterminism can arise without any communion with spatial infinity.

The supertask that shows this is merely a small modification of Laraudogoitia's [1997] ingenious supertask. To see this how this new supertask works, we need to present a simplified and slightly modified version of Laraudogoitia's [1997] supertask as shown in Figure 1. The supertask arises in a collection of hard spheres in classical dynamics. All the spheres of the supertask have the same mass. The figure shows a black target sphere moving in an (x, y) Cartesian plane with unit velocity in the +y direction. In the time t = 0 to t = 1 the black sphere will pass from y = 0 to y = 1. Approaching the black sphere from the +x direction is a countable infinity of white spheres numbered 1, 2, 3, …, all

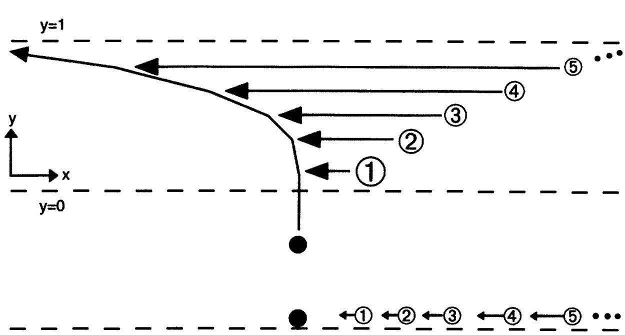

Fig 1. Boosting a body to unbounded velocities.

moving in the $-x$ direction only. The white spheres are confined to the region $y = 0$ to $y = 1$. We assume that their size decreases without limit so that they can fit in this space and not interfere with each other's motion. Each white sphere has been carefully positioned so that it will strike the black sphere and, moreover, when it collides with the black sphere it will leave the y-velocity of the black sphere unchanged. It is a well-known result of classical dynamics that in such hard sphere collisions, the two colliding spheres simply exchange their x-velocities. So, if the black sphere has initial x-velocity v_x and the white sphere initial x-velocity u_x, their x-velocities after collision will be u_x (black sphere) and v_x (white sphere).

To arrive at the variant of Laraudogoitia's supertask, we set up the white balls so that their initial velocities increase without bound as we proceed through spheres 1, 2, 3, ... With each collision, the black sphere adopts the x-velocity of the white sphere that strikes it. Thus the net effect of all the collisions is that the velocity of the black sphere increases without bound in the $-x$ direction. Since the y-velocity of the black sphere remains constant at unity, these unbounded boosts are completed before the black sphere arrives at $y = 1$. Thus the black sphere's velocity has been boosted without bound in the time interval $t = 0$ to $t = 1$. The black sphere disappears to spatial infinity in the $-x$ direction before it reaches $y = 1$.

With these unbounded boosts in velocity achieved, indeterminism is just a few steps away. To complete them Laraudogoitia effects a modification of the supertask of Figure 1 that seems impossible at first glance but wonderfully simple at the second. In Figure 1, we have kept all the white spheres separately partitioned in their own x-layers of space. Laraudogoitia collapses all the spheres to point particles and projects the entire supertask into a one-dimensional space (shown in the bottom portion of Figure 1). All the particles retain their initial x-positions and initial x-velocities. But their y-coordinates are set to 0 and they are constrained to the one-dimensional x-continuum. A brief reflection shows that this does not compromise the boosting of the black particle to unlimited velocities. Moreover it also follows that, in this one-dimensional version of the supertask, all the white balls as well will be boosted to unlimited velocity and disappear in finite time to spatial infinity.[2] The time

[2] To see this note that the black particle will still receive the impact from each of the white particles. The nth white particle ($n > 1$) can no longer strike the black particle directly. But in colliding with the $(n-1)$th particle, it exchanges its velocity with the $(n-1)$th particle. This exchange continues through particles $(n-2)$, $(n-3)$, ..., 1 until the velocity of the nth particle is transmitted by white particle 1 to the black particle. We now see immediately that white particle 1 must also achieve unbounded velocities prior to $t = 1$. White particle 1 is the sole mechanism for accelerating the black particle. For any velocity V achieved by the black particle, particle 1 must eventually achieve a velocity greater than V so that a collision from the white particle can boost the black particle past V. Since the black particle is eventually boosted without limit, it follows that white particle number 1 must also be so boosted and within the same time. A similar argument applies to the remaining white particles, all of which are boosted without limit prior to $t = 1$.

reversal of this process now consists in an empty space spontaneously filling with particles that pour out at unbounded speed from spatial infinity.

A quite small modification of this supertask leads to an instance of indeterminism which does not arise through the spontaneous pouring out of particles or bodies from spatial infinity. In the supertask of Figure 1, the black sphere is boosted to infinite velocity through infinitely many collisions with white spheres approaching from the +x direction. In the modification, we simply allow the white spheres to approach from alternate directions, as shown in Figure 2. They strike the black sphere at $x = 0$ if approaching from the $-x$ direction and at $x = 1$ if approaching from the +x direction. As before, the black sphere has an initial y-velocity of unity. The initial velocities and positions of the white spheres are set so that this y-velocity of the black sphere is unaltered but its x-velocity rises according to the following schedule:

to +2 after the collision with white sphere 1
to −4 after the collision with white sphere 2
to +8 after the collision with white sphere 3
to −16 after the collision with white sphere 4
etc.

Since the black sphere must cover an x-distance of unity between collisions, it follows that the time to complete all collisions is just $1 = 1/2 + 1/4 + 1/8 + 1/16 + \ldots$. If the collision with white sphere 1 is at $y = 0$, $t = 0$, then the x-coordinate of the black sphere oscillates increasingly rapidly between 0 and 1 during the course of the time $t = 0$ to $t = 1$ with its value becoming indeterminate at $t = 1$.

Indeterminism arises in this supertask. The positions and velocities of all the spheres prior to $t = 1$ fail to specify the position of the black sphere at $t = 1$.

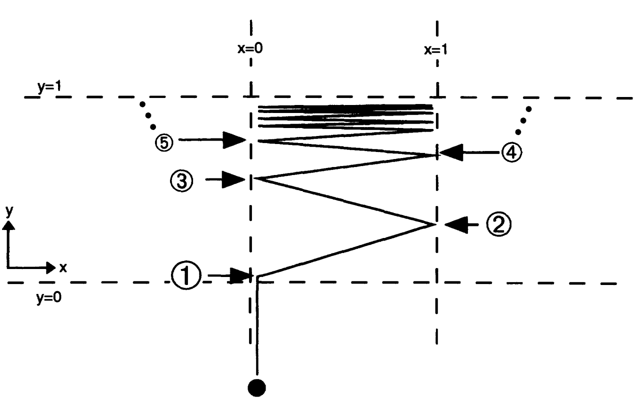

Fig 2. A body boosted to unbounded velocities in a confined spatial region.

However, the indeterminism does not arise from the spontaneous appearance of particles or disturbances at spatial infinity. The pathology arises through the infinite oscillations of the black sphere confined within the unit square $0 < x < 1$, $0 < y < 1$. At time $t = 0$, the positions and velocities of all spheres that will be involved in the supertask are finite and determined. It is interesting to notice that this supertask is a mechanical instantiation of the Black transfer device supertask described in Earman and Norton ([1996], Section 7).

The failure of determinism in the example just given is admittedly less psychologically disturbing than in cases where excitations appear from spatial infinity. For in the former case we can predict with certainty not only that determinism will fail but we can also forecast when and how it will break down, whereas in the latter cases the breakdown comes with no hint of forewarning.

3 Supertasks that depend on there being no lower bound to the size of bodies

Laraudogoitia's [1996] supertask is both beautiful and beautifully simple. In it infinitely many point particles $P_1, P_2, P_3, \ldots$ of equal mass are arrayed in a one-dimensional space so that they occupy positions $x = 1$, $x = 1/2$, $x = 1/4$, ... respectively. Another particle of equal mass approaches from the $+x$ direction at velocity v in the $-x$ direction. It collides with the particle P_1 which recoils with velocity v in the $-x$ direction, leaving the incoming particle at rest. P_1 then collides with P_2, leaving the P_1 at rest and P_2 moving at velocity v in the $-x$ direction. The process continues to completion in a finite time, since the time between successive collisions is $1/2v$, $1/4v$, $1/8v$, ... and these intervals sum to $1/v$. At the end of the process all the particles have come to rest. One's first reaction, of course, is to expect that the velocity v and its corresponding energy has been transmitted to the last particle in the sequence of collisions. After finitely many collisions, there is a last particle to carry this velocity and energy. But in the infinite case there is no such last particle. The velocity and energy cease to be. The time reversal of the process is an indeterministic process. In it, infinitely many particles are arrayed at rest in the interval $0 < x \leq 1$. A spontaneous disturbance propagates from $x = 0$ causing the particle at $x = 1$ to be ejected with velocity v.

As Laraudogoitia points out, the supertask is not limited to point particles only. It may also be instantiated by hard spheres laid out in a straight line as long as the size of the spheres diminishes sufficiently rapidly for infinitely many to fit into a finite interval.

Laraudogoitia contends that this supertask illustrates the indeterminism of classical Newtonian dynamics. In so far as it does, we contend, it is not through any feature peculiar to classical dynamics. Indeed the same supertask is compatible with a relativistic kinematics in so far as it does not require

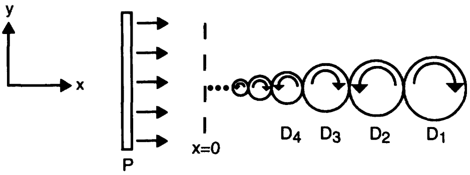

Fig 3. Will the rotating disks fling the plate up or down?

unbounded velocities. What is essential to the supertask is the admissibility of bodies of indefinitely small size, so that infinitely many can be enclosed within a finite volume, and that there be no lower bound on the time of their collision interactions.[3] Any matter theory that admits such bodies will admit pathologies. The locus of the difficulty is that such a theory will admit infinite sequences of bodies enclosed in finite volumes for which there is no last member. Thus Laraudogoitia's [1996] result depends on there being no last particle or body to carry off the velocity or energy. Here is another pathological case.

Consider an action-by-contact dynamics.[4] When a moving body approaches a collection of bodies, the interaction that ensues is fixed by determining which body or bodies are the first to be encountered by the moving body and then applying the rules that govern the contact dynamics. In the case of a finite collection of bodies, there will always be a first body or bodies encountered. In the present case, however, this need not be the case. Imagine cylindrical disks D_1, D_2, D_3, ... arrayed along a line as in Figure 3 and firmly fixed in their positions. The disks are in contact and diminish in size as one proceeds in the $-x$ direction so that infinitely many fit into a finite length whose greatest lower bound is $x = 0$. A flat plate P approaches the disks from the $-x$ direction. It cannot just pass through the disks; it must eventually collide with some of them. But which is the first disk that will be encountered? There is no last disk in the sequence D_1, D_2, D_3, ... and therefore no closest disk to be encountered first. This may seem only mildly disconcerting, but worse is to follow. Imagine

[3] The supertask in Figure 1 does also employ infinitely many spheres whose size reduces without limit. However, this feature is inessential to the indeterminism that arises. For example, the supertask remains intact if the process is projected onto a one-dimensional space as shown in Figure 1 but with each sphere replaced by a sphere of a single constant size. We conjecture that a similar modification can be effected for the supertask of Figure 2.

[4] Although obvious, there is an unremarked conceptual problem with the idea of action by contact. 'Contact' can mean either that (a) there is a zero spatial distance between two bodies and/or (b) the bodies share one or more points in common. Bodies with sharp boundaries (i.e. bodies that occupy closed regions of space) cannot satisfy (a) without also fulfilling (b), but the latter condition contradicts the notion that bodies are impenetrable. A body with a sharp boundary and a fuzzy body (i.e. a body that occupies an open set of points) can satisfy (a) without also satisfying (b). But the contact between a sharp body and a fuzzy body composed of an infinite sequence of ever smaller sharp bodies can involve awkward consequences, as we will now illustrate.

that each disk rotates on its axis, with the odd numbered disks D_1, D_3,... rotating clockwise and the even numbered disks D_2, D_4,... rotating anti-clockwise. We assume that friction at their points of contact keeps all rotations geared together. Thus if the rim velocity of D_1 is 1 foot per second, then all disks will share this same rim velocity. If the plate P would contact any disk, it would be accelerated in the direction of the rim velocity at the point of contact. This if the plate is brought to a halt at $x = 0$ by contact with a disk, then, since all disks are rotating, it must be set into motion in either the $+y$ or the $-y$ direction. But any such motion would reveal the sense of rotation of the last disk. A motion in the $+y$ direction would tell us the disk is in the set D_1, D_3,... and a motion in the $-y$ direction would tell us the disk is in the set D_2, D_4,.... That would be tantamount to telling us whether the 'last number' of the infinite sequence 1, 2, 3,... is odd or even!

We may read this example as displaying another indeterministic process; the rules of the contact dynamics are unable to determine how the process will develop when the plate meets the disks. One can also characterize the pathology as displaying an inconsistency when such systems of bodies are coupled with a contact dynamics—but that characterization requires the assumption that all interactions have outcomes delimited by the rules. Similarly we may convert the indeterminism that arises from unbounded velocities into inconsistencies. For example, we may embed the supertask of Figure 2 in a theory that insists that bodies have a position at all times given by the limit of positions at earlier times. Then we have an inconsistency, since the position of the black sphere has no limit at $t = 1$.

4 Supertasks, determinism and the special and general theories of relativity

Many of the more problematic supertasks discussed in the literature implicate some form of indeterminism, a feature that may help to explain why many philosophers, who may have a prejudice in favour of determinism, have held that these supertasks are impossible. The notorious Thomson lamp (Thomson [1954/55]) involves a kind of logical indeterminism: the instructions for switching the lamp on and off infinitely many times between $t = 0$ and $t = 1$ do not by themselves determine the state of the lamp for $t \geq 1$ (see Benacerraf [1962]). Black's transfer device (Black [1950/51]), which involves shuttling a marble back and forth over a minimum finite distance an infinite number of times between $t = 0$ and $t = 1$, involves a physical indeterminism, at least if it is demanded that the world line of the marble be continuous; for then the marble cannot be at any determinate finite spatial location at $t = 1$. Such indeterminism does not render the supertasks physically impossible according to the laws of classical physics, at least if the systems in question are of the idealized variety (e.g. perfectly elastic billiard balls interacting via collisions, point mass particles interacting according to Newton's $1/r^2$ law). As shown by Laraudogoitia [1996, 1997] and our examples above and elsewhere (Earman and Norton [1996]) such idealized systems can be used to implement a number of the examples of allegedly impossible supertasks discussed in the literature.

Of course, the real world does not contain such idealized systems. More importantly, the real world is relativistic rather than Newtonian. This makes a world of difference since, as we noted above, the special theory of relativity rules out all those examples of supertasks and of indeterminism involving unbounded velocities. Determinism is still threatened by Laraudogoitia's example employing bodies of unboundedly small size. But within a special-relativistic theory of matter that prohibits such bodies, clean results about determinism can be demonstrated without the need to appeal to question-begging boundary conditions (see Earman [1986]).

This is not the end of the story, however, since the world is not special-relativistic but general-relativistic. Once again this makes all the difference in the world as regards both determinism and supertasks. General-relativistic spacetimes open up the possibility of performing bifurcated supertasks in which one observer carries out an infinite number of operations in an infinite amount of proper time and communicates the results to a second observer who receives them in a finite amount of proper time. The general-relativistic spacetimes that allow bifurcated supertasks have been characterized (Earman and Norton [1993]). Not surprisingly these spacetimes implicate indeterminism because they cannot contain a Cauchy surface. (A Cauchy surface for a spacetime is a spacelike hypersurface ('time slice') that meets every causal curve without endpoint exactly once. The existence of a Cauchy surface is a necessary condition for the global version of Laplacian determinism.) Such spacetimes will violate the strong form of Penrose's cosmic censorship hypothesis which conjectures that, consistent with Einstein's field equations for gravitation, naked singularities will not form under physically reasonable conditions.[5] An example of a general-relativistic spacetime which lacks a Cauchy surface and which admits a bifurcated supertask is anti-de Sitter spacetime (see Figure 4). The flattening out of the light cones permits particles to escape to and appear from spatial infinity, wrecking determinism. An analogue of the supertask in Figure 1 can be performed by accelerating the target particle not to an unbounded velocity but towards the velocity of light. A particle accelerated as shown in Figure 4 will never reach the time slice $t = 1$; by that time the particle has disappeared from spacetime.

[5] See Earman [1995] for a detailed discussion of the cosmic censorship hypothesis.

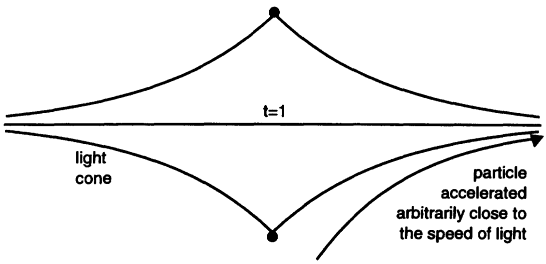

Fig 4. Anti-de Sitter spacetime.

To our way of thinking, it is the connection of supertasks to interesting and in some cases unsettled questions in the foundations of physics that makes supertasks more than the mere playthings of philosophers. We urge that future discussions of supertasks exploit this connection.

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