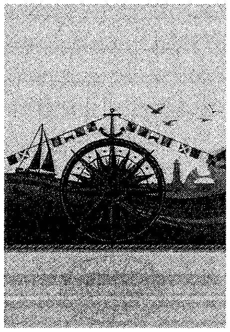
JOHN D. NORTON

CHAPTER 13

PARADOXES OF SAILING

The Physics of Sailing and the Import of Thought Experiments



Paradoxes have long been a driving force in philosophy. They compel us to think more clearly about what we otherwise take for granted. In antiquity, Zeno insisted that a runner could never complete the course because he'd first need to go half way, and then half way again, and so on indefinitely. Zeno also argued that matter could not be infinitely divisible, else it would be made of parts of no size at all. Even infinitely many nothings combined still measure nothing. These simple thoughts have forced us to develop ever more careful and sophisticated accounts of space, time, motion, continuity, and measure. And modern versions of these paradoxes continue to vex us.

This engine of paradox has continued to power us to this day. Relatively recently, Einstein fretted over a puzzle. How was it possible that all inertially moving observers would find the same speed for light? Surely if one of them was chasing rapidly after the light that observer would find the light slowed. But Einstein's investigations into electricity and magnetism assured him that the light would not slow. He resolved this paradox with one of the most influential conceptual analyses of the twentieth century. He imagined clocks, synchronized by light signals, and concluded that whether two events are judged simultaneous will depend upon the motion of the observer judging.

What is distinctive about these philosophical paradoxes is that they are not mere expressions of practical limitations. The difficulties they expose lie within the very ideas themselves. Zeno's worry was not that a real runner might fail to complete a long race because of tiredness. His concern related to the very idea of any runner, no matter how accomplished or idealized, completing any race, no matter how short. The difficulty lies in the ideas of space, time, and motion. In his paradox of measure, Zeno was not concerned that we might never find a real knife capable of slicing matter indefinitely finely. His concern was that matter must be such that infinite division lies beyond even the sharpest knife, whose edge has been honed to the perfection of an ideal mathematical point.

Now let us consider sailing. There are many difficult technical problems associated with sailing. If a sailboat is to be even minimally serviceable, its design must conform to an engineering lore that has grown through the centuries. In general, the problems this tradition solves do not rise to the level of paradox. However, there are some puzzles attached to sailing that are more fundamental than a particular engineering challenge. In this essay I will consider three. They do not have the importance of the paradoxes of Zeno and Einstein. Indeed, as I shall try to show, their diagnosis and resolution is a short and, I hope, entertaining diversion. However, they are foundational paradoxes, for they challenge no particular sailboat but the very idea of sailboats powered by the wind. They are:

- If a sailboat is powered by the wind, how can it sail into the wind?
- If a sailboat is powered by the wind, how can it sail faster than the wind?
- If a sailboat "makes its own wind" when it moves, why does it need any other wind?

The first two will be familiar to sailors, most of whom will have made their peace with them. The third is less straightforward. That sailboats "make their own wind" is commonly said by sailors, but few explicitly pursue the thought to its paradoxical end. We shall do so now.

While profound philosophical morals will not be found in these paradoxes, I will suggest that they connect nicely to two issues in recent philosophy. The first two paradoxes will lead us directly into a conundrum concerning causal metaphysics. The third will lead us to ponder an intriguing mode of investigation of nature, the thought experiment.

Sailing – Philosophy for Everyone: Catching the Drift of Why We Sail, First Edition.
Edited by Patrick Goold.
© 2012 John Wiley & Sons, Inc. Published 2012 by John Wiley & Sons, Inc.

Here are the first two paradoxes again, spelled out in greater detail. They are treated together since they involve essentially the same issues.

1. The cause of a sailboat's motion is the motion of the wind.
2. The effect of a cause cannot be greater than or contrary to the cause.
3. Therefore, a wind-powered sailboat cannot sail faster than the wind or into the wind.

But this conclusion is contradicted by the reality:

4. Real sailboats routinely sail into the wind, and sailboats designed for speed can sail faster than the wind.

Those who are not sailors will likely find the argumentation leading to the conclusion (point three) convincing, at least initially. Its plausibility depends upon a limited experience of what the wind can do. It calls to mind dry leaves blown about by wind. The wind may lift them, but it will not move them faster than or contrary to its own motion. The conclusion is also correct for some cases of sailing, such as old-fashioned square-rigged sailing ships running before the wind. Then their sails function like big bags catching the wind. The boat will be blown in the direction of the wind, near enough, and, as long as it sails in that direction, the ship will never move faster than the wind.

However, the conclusion is incorrect for almost any sailboat that can align its sails in a fore–aft direction. This is especially so for the most common type of small sailboat now used recreationally. A Bermuda-rigged sailboat has a single mast with two triangular sails, a jib and a mainsail, oriented in the fore–aft direction. Such sailboats routinely sail into the wind, and, if designed for speed, easily sail faster than the wind when sailing across the wind.

Where the above analysis (points one to three) fails for such boats is that it mischaracterizes the causal processes. The motion of the wind is not the immediate cause of the motion of the boat. A more immediate cause is the force with which the wind presses on the sails. For even light winds, this force can be considerable. In what are called "moderate breezes" on the Beaufort scale of wind (thirteen to seventeen miles per hour), the wind generates pressures of around one pound per square foot on the sails. Small modern sailboats, under twenty feet in length, can carry two hundred square feet of sail, and older designs often carried significantly more.[1] So, the wind exerts a considerable force of many hundreds of pounds on the sails. This force now acts independently of the motion of the wind that produced it. A few hundred pounds of force pressing on the sails will lead the sailboat to heel over, just as if someone were to attach a rope to the center of effort of the sail and pull it.

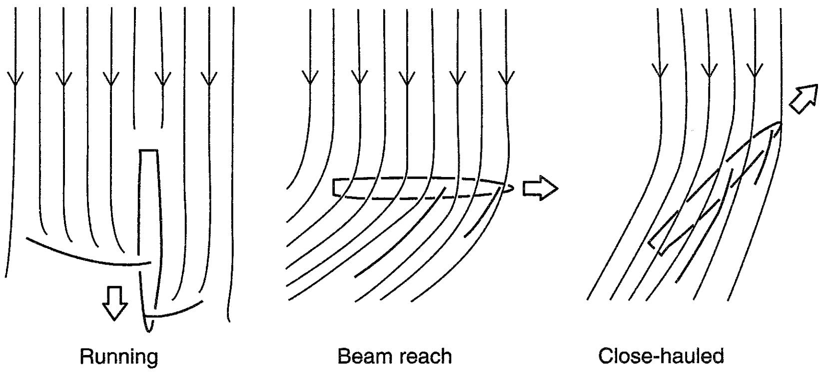
Some of this force can be directed toward the bow of the boat and drives it over the water. How much boat speed results from a given force depends almost entirely on the design of the boat's hull and, as a result, the resistance the water provides to its motion. Once that forward-directed force is fixed, so is the motion of the hull. It makes no difference if the force comes from the press of the wind, oars, and paddles or a motor-powered propeller. The force contains no coded record of the speed of the wind that produced it for the sailboat to read covertly and respect!

A small boat of the familiar monohull design can easily be driven up to a maximum speed that cannot be passed by greater forces generated by sails. For small boats this maximum speed is commonly less than the speed of the wind. But that is purely an accident of the hull design. If the hull is designed for speed, nothing prevents the boat achieving speeds greater than the wind. Two-hulled catamarans present considerably less resistance that monohull boats. If sailing across the wind, they do not lose the press of the wind when they move fast. Then, well-designed catamarans are easily able to sail faster than the wind. The wind can provide considerable force; their hulls provide little resistance, so off they go!

To see how a sailboat can gain against the wind, we need to consider the different "points of sail" of a sailboat. These are the different ways in which a sailboat can proceed in relation to the wind. They are shown in Figure 13.1.

When a sailboat is on a run, the wind blows directly from its stern. Then the sails function like bags just catching the wind. On this point of sail the fastest the boat can move is the speed of the wind. As the boat approaches the speed of the wind, the boat's motion cancels out the speed of the wind, so that the wind felt on the boat by the sails diminishes. When the boat is close to the speed of the wind, the air on deck becomes calm. The experience is not unlike being carried by the wind in a balloon. One's speed over the ground may be quite high, but in the balloon's basket the air will be still.

All this changes when the boat sails across the wind on a beam reach. On this point of sail, the sails are let out so that they deflect the wind toward the rear of the boat. The resulting pressure on the sails yields a force, "F_{wind}," pointed diagonally forward, as shown in the first diagram of Figure 13.2.


Running

Beam reach

Close-hauled

FIGURE 13.1 Points of sail. Author's copyright.

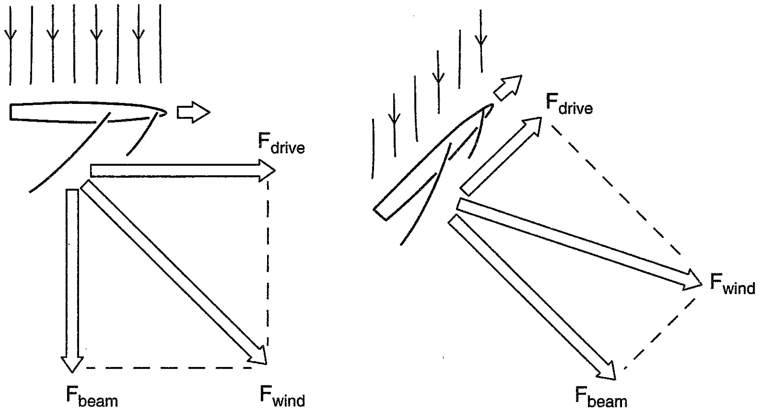


FIGURE 13.2 Resolution of forces on the sail. Author's copyright.

If the boat's hull was simply a tub, then this force would move the boat in that diagonal direction. However, an essential part of hull design is to make it as resistant as possible to sideways motion. This is usually effected with a centerboard in small boats and a broad, flat keel in bigger boats. The force on the sails, F_{wind}, can be divided into two components, as shown in Figure 13.2. One, "F_{drive}," is parallel to the boat's motion, and the other, "F_{beam}," is perpendicular to it. The high resistance to sideways motion means that the sideways force, F_{beam}, produces little or no motion, whereas the low resistance to forward motion means the forward force, F_{drive}, produces motion forward. Hence the boat is driven across the wind.

Only a small modification to the above analysis shows how sailboats can sail into the wind. When a sailboat is close-hauled, as shown in the second diagram of Figure 13.2, the wind still produces a force on the sails. That force, F_{wind}, can once again be decomposed into two parts, F_{drive} and F_{beam}. Since the sails are now pulled in closer to the centerline of the boat, the component F_{drive} is smaller in relation to F_{beam}. However, the hull will still prevent F_{beam} producing sideways motion, so that F_{drive} will drive the boat forward.

This forward motion will now gain against the wind. It is common for Bermuda-rigged sailboats to be able to sail at 45 degrees to the wind. As a result, if a close-hauled sailboat tacks repeatedly – that is, zig-zags across the wind – it can follow a track whose average course points directly into the wind.

In sum, the first two paradoxes are resolved by denying the second premise, that the motion of the wind, as a cause of the motion of the boat, cannot have an effect greater than or contrary to itself. When powering a sailboat, the motion of the wind can produce faster motions in the sailboat and motions directed against the wind.

To a philosopher, what is important in this last analysis is the centrality of causal notions. In the abstract, it seemed entirely unremarkable to expect that the effect of a cause cannot be greater than or contrary to the cause. Yet this simple causal truism was wrong and generated the first two paradoxes.

We see here in miniature one of the dominant and, in my view, most important facts about our investigations into causation. At any moment in history, we have held to a repertoire of facts about causation that we believed to be necessities. They are assertions that, shielded from deeper reflection and a broader exposure to experience, seem unassailable. However, when we think more and learn more about the world, we find we must abandon them.

Until the seventeenth century, it was widely accepted on Aristotle's authority that a final cause, the goal toward which a process moved, was as important as the efficient cause, that which initiated the process. In that century, the advent of mechanical philosophy was premised on the denunciation of final causes. However, we had by no means then "got it right." The century's hero, Isaac Newton, felt he had such an unassailable grasp on causation that he could, in 1692, denounce causal action at a distance as "so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it."[2] Yet, by the nineteenth century, Newton's gravitation was widely accepted to be precisely this, unmediated action at a distance. In that century, the notion of causation was stripped down to its barest essentials.

It came to be equated with determinism, the simple fact that the present state fixes the future. This pure and apparently secure notion of causation fell. It was overturned with the advent of modern quantum theory in the 1920s. According to that theory, the present cannot fix the future. The best we can have are probabilities for a range of different futures.

These are just a few episodes in the history of our failure to grasp what causality demands. It is important that we see just what this failure has been. It has not been our failure to discern what has always concerned causal thinking: how it is that things in the world are connected. The little history just unfolded is a story of our coming to understand better and better how things are connected. The failures of the story were our efforts to discern ahead of science what sorts of connections new science must reveal.

How are we to interpret this long history of failure? There are causality optimists who think that the best response to failure is to try and try again. Eventually, they hope, we will hit upon the true causal principles that govern the world and all possible sciences. My own view is rather different and represents a minority view among theorists of causation.[3] It is that we need to learn that efforts to legislate causal principles ahead of experience are doomed to failure at the hands of new investigations.

As a result, I believe that the familiar causal talk is very different from what it seems. One could be forgiven for imagining that science is exploring a realm governed by some general law of causality that rules from the metaphysical heights above all sciences and to which all sciences must defer. In my view, something like the reverse is correct. Science is revealing to us deeper truths about the interconnectedness of things in the world than we could have ever imagined. In order to facilitate our understanding of it, we graft causal talk onto those discoveries. The repeated cycle of the failure and revival of causal talk is really a history of the elasticity of causal terms and our eagerness to apply them to whatever science may deliver. We do not have and will never have a factual principle of causality to which all sciences, known and as yet unknown, must conform.

Sailors commonly remark that sailboats create their own wind. The effect is a familiar one. If you pedal a bicycle at 10 mph on a calm day, you will find yourself pedaling into a 10 mph headwind created by your motion. Exactly the same thing happens with a sailboat. A sailboat travelling at 10 mph is sailing into a 10 mph wind it has created. Of course, sailors never see this headwind in isolation. The wind they see, the apparent wind, is always the vector sum of the created wind and the true wind. So, if the sailboat is on a beam reach in 10 mph winds, the two winds combine to yield a 14 mph wind coming at an angle of 45 degrees to the bow. The two velocities are at right angles and so must be summed by Pythagoras' theorem:

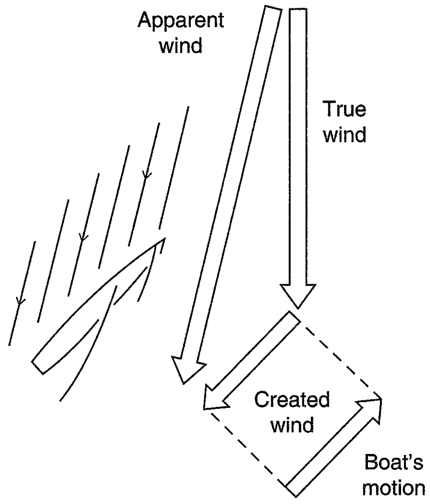


FIGURE 13.3 True and apparent wind for a close-hauled sailboat. Author's copyright.

$$(\text{apparent wind speed})^2 = (\text{true wind speed})^2 + (\text{created wind speed})^2$$

Figure 13.3 shows how the true wind and apparent wind are combined when a sailboat is close-hauled. It will be important for later discussion to note that the effect of the created wind is to move the direction of the apparent wind closer to the bow.

Thus far we have no paradox. By their motions, sailboats create wind. Our quest is for paradoxes and there does seem to be an intriguing paradox lurking in shadows. It arises from the essential difference between the cases of a bicycle and a sailboat. A bicycle is powered by your muscles; a sailboat is powered by the very thing created, the wind. Here is the paradox:

5. A moving sailboat creates its own wind.
6. A moving sailboat is powered by the wind.
7. Therefore, a moving sailboat is in part self-powered and is thus, in part, a perpetual-motion machine.

But this conclusion is contradicted by the following:

8. Perpetual-motion machines (self-powered devices) are impossible.

The concern is that a sailboat is, in part, realizing a device whose impossibility underlies one of the most important laws of physics, the conservation of energy. For it appears to be achieving just what perpetual-motion-machine makers have long sought. Their goal is a device that derives the power to run from its own internal operations. They have tried many designs. For example, they have equipped an electric car with a generator so that as the car moves the generator is turned. The generator produces electricity that, supposedly, now fully powers the car's electric motor. This simple design and its thousand and one variants have all failed. Is a sailboat the thousand and second variant that has finally succeeded?

It is not too hard to see that the traditional design of a sailboat acquires no added motive force from the created wind. Qualitatively, the result comes from combining effects that work in opposing directions. The force that drives the sailboat comes from the speed of the wind over the sails. So, an increase in the speed of the wind over the sails will increase the force on the sails. It doesn't matter whether the wind is the true wind or the apparent wind. Sails cannot distinguish the two. The force on the sails is determined by the speed of the wind at the sails, however it arises.

If that were the only effect, then we would be well on our way to realizing the paradox just sketched. However, there is a counteracting effect. As Figure 13.3 shows, the effect of adding the created wind to the true wind is to move the direction of the wind closer to the bow. As a result, the angle between the wind direction and the sails decreases; the wind now comes closer to blowing parallel to the sails' surfaces. This diminished angle reduces the wind-generated force on the sails in two ways. First, the volume of air scooped up by the sails diminishes since the profile of the sails facing the wind is smaller. Second, the force-generating deflection of the wind is now passes through a smaller angle. Figuratively, the wind strikes a more glancing blow onto the sails and thus exerts a weaker force on them.

These effects have been described only qualitatively. However, when they are combined, the effects that diminish the force overwhelm the one that increases it, so that the net effect is a loss of motive force. To see quantitatively that this is so, one needs to construct a careful mathematical model of the interaction of the wind with the sails. The result follows after some elaborate juggling of trigonometric functions. I will not reproduce them here, since the details of the calculations are tedious and not any more illuminating than the reciting of the qualitative effects above.

One can, however, get a sense that the apparent wind cannot drive a sailboat merely by recalling an experience familiar to every sailor. Imagine the sailboat sitting becalmed in completely dead air. If the boat is given a small push, perhaps from a paddle or a hand on a dock, it will move forward. That motion will create wind. However, the wind will blow straight down the centerline of the boat and, therefore, will be unusable by the sails as a way of generating any forward-directed motive force. The boat will gently slow to a halt, just as the generator–dynamo self-powered car cannot sustain an initial push.

What this last analysis shows is that a particular design of sailboat, the common Bermuda rig, is unable to realize the perpetual-motion machine of the paradox. Does that settle the matter? Might another design fare better? Might an improved design of sailboat be able to extract energy from the created wind and thus realize a perpetual-motion machine? Here the decision is not so straightforward. The normal response to a proposal for a perpetual-motion machine is that it is impossible because it would violate the law of conservation of energy. However, in addition it is customary to complete the refutation by pinpointing where the design fails. The generator–electric motor car, for example, fails because the slightest loss of energy due to friction means that the generator cannot supply as much energy as the electric motor demands.

What complicates the question is that a sailboat has an external source of energy, the kinetic energy of the true wind, as well as the possibility of the internally created energy of the created wind. Any analysis must disentangle the two. If a sailboat generates more energy when it is moving faster and thus experiences a greater apparent wind, which is the source of the extra energy? Is it merely more energy harvested licitly from the kinetic energy of the true wind? Or are we generating more energy from the created wind in violation of the law of the conservation of energy?

What we should like to develop is a general sense that the created-wind perpetual-motion machine will always be defeated by internally counteracting effects. The greater apparent wind will deliver greater energy, but all gains will be lost by some other effect that essentially arises in connection with the created wind. To see that things will always work out this way is hard if we examine the functioning of any real sailboat or even any real wind-powered device. For all such devices are beset by many inefficiencies, such as frictional energy losses or incomplete extractions of wind energy. If a boat functions better when sailing into the wind, is

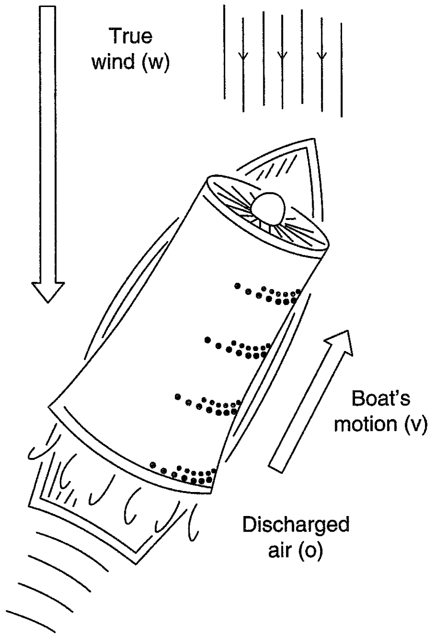


FIGURE 13.4 Wind-turbine-powered boat. Velocities with respect to water. Author's copyright.

it truly because of some sort of perpetual-motion effect, or is it simply the result of the reduction of inefficiencies?

The way to escape this problem is to consider an imaginary, wind-powered boat in which all the inefficiencies are idealized away. In this thought experiment, we consider a device that is perfectly efficient in extracting energy from the wind and is beset by no dissipative processes. For concreteness, we will imagine that our boat extracts energy from the wind with a large system of wind turbines and that this energy then powers its propeller. Any idealized system capable of extracting all the energy from the wind could be used; the turbine system is used simply because it is easy to visualize and compute. Its operation is shown in Figure 13.4.

The boat sails at vector velocity $\mathbf{v}$ into a true wind with vector velocity $\mathbf{w}$. The wind turbines are perfectly efficient, so that the wind turbine extracts all the kinetic energy of the wind that enters its throat. This means that the wind enters the turbine throat at velocity $\mathbf{w}$ and, as the boat moves off, it discharges a wake of entirely quiescent air; that is, air with zero velocity.

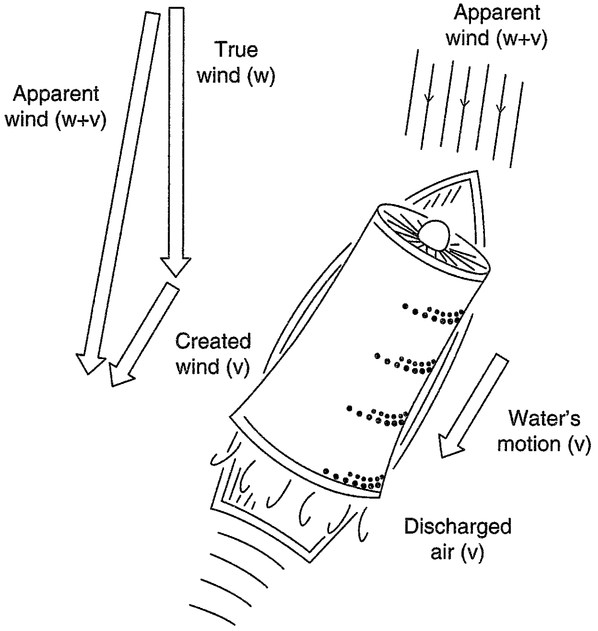


FIGURE 13.5 Wind-turbine-powered boat. Velocities with respect to boat. Author's copyright.

How does this moving boat appear to a sailor on its deck? We merely add a velocity $\mathbf{v}$ to each of the velocities, with the result shown in Figure 13.5. The boat is now at rest and the water beneath the boat moves at velocity $\mathbf{v}$ toward the stern. The air discharged by the wind turbine is at rest with respect to the water, so it also moves at $\mathbf{v}$ toward the stern. Finally, the air entering the turbine moves at an apparent velocity of $\mathbf{w}+\mathbf{v}$. This added velocity $\mathbf{v}$ is the wind created by the boat's motion.

We use these velocities to compute the energy the turbine extracts from the wind, for the turbine has no way of distinguishing true from apparent wind. All it knows is that it scoops up air at velocity $\mathbf{w}+\mathbf{v}$ and discharges it at $\mathbf{v}$. It turns out that the resulting energy extraction is greater than the kinetic energy of the true wind passing through the turbine (see the Appendix for the calculations). We interpret the extra energy as supplied by the created wind and write:

$$\begin{aligned}\text{Total energy extracted} = {}& \text{Kinetic energy from true wind} + \\ & \text{Kinetic energy from created wind}\end{aligned} \tag{1}$$

If this were the entirety of the analysis, then we would have achieved a device that generates energy from nothing. However, it is not. The total energy of (1) is not available to power the boat. There is a consumption of energy that arises inescapably as part of the operation of the wind turbine. In order to extract energy from the wind, the turbine must take rapidly moving air and slow it down. That means that the turbine must apply a force to the wind. This is an ineliminable resistance force against which the boat must work. Moving against this force consumes energy. It turns out that this energy consumption matches exactly the extra, created energy:

$$\begin{matrix}\text{Energy consumed in moving} \\ \text{against resistance force}\end{matrix} = \begin{matrix}\text{Kinetic energy} \\ \text{from created wind}\end{matrix} \qquad (2)$$

Combining (1) and (2), we recover

$$\text{Net energy extracted} = \text{Kinetic energy from true wind} \qquad (3)$$

Hence, the extra energy we thought we had gained from the created wind is exactly consumed as the energy needed move the boat against the wind. That is, the net energy extracted is just the kinetic energy extracted from the true wind. The boat is not a perpetual-motion machine that is powered even in part by its own self-created energy.

In sum, we learn for the highly idealized wind-powered boat of the thought experiment that it can extract energy from the wind created by its own motion. However, exactly that extra energy is consumed by an inescapable counteracting effect. The result seems quite general. There is nothing in the thought experiment that specifically requires a wind turbine to extract the energy. Any device will be subject to essentially the same analysis. Making the boat more realistic by removing the idealization of perfect efficiency and no dissipative frictional effects will not help. It will carry us further from the possibility of a perpetual-motion machine. We now develop the sense that extracting net energy from the created wind is an appealing but impossible illusion.

For a philosopher interested in epistemology – the study of how we get to know things – this last conclusion is fascinating. The thought experiment has taught us something important about the operation of wind-powered vehicles like sailboats that is much harder to recover from experiment. We could have conducted a series of tests on a variety of sailboats to see whether we could gain net energy from the created wind. Presumably each test would have told us that we could not, in that case. However we would always have been left wondering whether our failure to extract net energy from the created wind merely resulted from our lack of ingenuity in finding the clever design of boat that could do it. The thought experiment, however, indicates that our failure is a matter of principle. The quest for a better design can end.

Merely thinking about examples so idealized as to be unrealizable gives us a more secure and more general understanding of physical possibility than real experiments. How is that possible? This is the central problem of the epistemology of thought experiments. This problem has attracted a flourishing philosophical literature. I'll mention two extreme views in this literature. One is defended by my colleague Jim Brown of the University of Toronto[4] and the other by me.

Brown is a Platonist and he urges that something in the right sort of thought experiment enables us to tap into a Platonic realm in which the laws of nature reside. The thought experiment lets us "see" the laws in a way that mere material experiments cannot. If this seems far-fetched, it might be helpful to recall the case that is the model for Platonic thought, mathematics. Draw an equilateral triangle – one with three equal sides – on a piece of paper and measure its angles. To within the accuracy of measurement, you will find that the angles are the same. Repeat the exercise for several more triangles. The results will be the same. That is no surprise. You fully expected it to be so, to the extent that any slight differences in your measurements would be dismissed as errors. But how did you get this knowledge that trumps actual experience? It is because thought affords you a deeper understanding of triangles than mere measurement can bring. Your mind can grasp the ideal triangles of the Platonic realm of which the triangles you drew are but poor imitations.

My view is the opposite of Brown's. It is deflationary and finds nothing epistemically remarkable in thought experiments. While they certainly have great rhetorical powers, epistemically they can do nothing more than ordinary argumentation. They are, I maintain, merely picturesque argumentation. As a result, you get nothing more out of a thought experiment than what you put into it as assumptions and what can be wrestled from those assumptions by deductive or inductive argumentation. In the thought experiment concerning wind-powered boats, what was assumed was the Newtonian mechanics of frictionless fluids. That theory conforms to the conservation of energy. As a result, it was a foregone conclusion that it would not allow the creation of energy from nothing. The only novelty was to see precisely how the theory blocked its creation. We did not learn anything that transcended the assumptions made. Had we made different assumptions, such as some concocted mechanics that did not respect energy conservation, we could have arrived at a thought experiment that vindicates the free creation of energy.

To see how Brown and I have sought to settle our differences and for an entry into the literature on thought experiments, see James Brown, "Why thought experiments transcend empiricism" and James Norton, "Why thought experiments do not transcend empiricism."[5]

Appendix: Analysis of the Wind-Powered Boat

Air will enter the inlet of the turbine with cross-sectional area[6] $\mathbf{A}_{in}$ at density r_{in} and velocity $\mathbf{w}+\mathbf{v}$. It is discharged at the outlet with cross-sectional area $\mathbf{A}_{out}$ at density r_{out} and velocity $\mathbf{v}$. Conservation of mass requires

$$r_{in}A_{in}\cdot(w+v) = r_{out}A_{out}\cdot v \qquad \text{(A0)}$$

Considering velocities in the boat frame of reference, the turbine scoops up air with energy density $(1/2)\ r_{in}\ |\mathbf{w}+\mathbf{v}|^2$ at a volumetric rate $\mathbf{A}_{in}\cdot(\mathbf{w}+\mathbf{v})$, and discharges air with energy density $(1/2)\ r_{out}\ |\mathbf{v}|^2$ at a volumetric rate $\mathbf{A}_{out}\cdot\mathbf{v}$. Hence, the total power – that is, the total rate at which energy is delivered by the turbine – is

$$P_{total} = (1/2)r_{in}|w+v|^2 A_{in}\cdot(w+v) - (1/2)r_{out}|v|^2 A_{out}\cdot v$$

Applying equation (A0), this becomes[7]

$$P_{total} = (1/2)r_{in}|w|^2\big(A_{in}\cdot(w+v)\big) + r_{in}(w\cdot v)\big(A_{in}\cdot(w+v)\big) \qquad \text{(A1)}$$

This equation corresponds to equation (1) of the main text. The first term represents the rate of delivery of kinetic energy by the true wind. The true wind, moving at speed $|\mathbf{w}|$, has kinetic energy density at the inlet of $(1/2)\ r_{in}\ |\mathbf{w}|^2$ and arrives at a volumetric rate $(\mathbf{A}_{in}\cdot(\mathbf{w}+\mathbf{v}))$. The second term is the energy delivered by the created wind, which has an apparent energy density at the inlet of $r_{in}\ (\mathbf{w}\cdot\mathbf{v})$.

To operate, the turbine scoops up air with a momentum density r_{in} $(\mathbf{w}+\mathbf{v})$ and discharges air with the reduced momentum density $r_{out}\mathbf{v}$. To slow the air, the turbine must apply a force to the air equal to the rate of change of momentum:

$$F_{resistance} = r_{in}\,(w+v)\big(A_{in}\cdot(w+v)\big) - r_{out}v(A_{out}\cdot v)$$

Applying (A0), this expression reduces to

$$F_{resistance} = r_{in}w\big(A_{in}\cdot(w+v)\big)$$

Since the boat moves at velocity $\mathbf{v}$, energy is consumed in working against this force at the rate

$$P_{resistance} = F_{resistance}\cdot v = r_{in}w\cdot v\big(A_{in}\cdot(w+v)\big) \qquad \text{(A2)}$$

This corresponds to equation (2) of the main text. The net power available is just the difference

$$P_{net} = P_{total} - P_{resistance} = (1/2)r_{in}|w|^2\big(A_{in}\cdot(w+v)\big) \qquad \text{(A3)}$$

This equation corresponds to equation (3) of the main text.

NOTES

1. Norman L. Skene, *Elements of Yacht Design* (Dobbs Ferry, NY: Sheridan House, 2001), p. 92.
2. Isaac Newton, "Four letters to Richard Bentley." In Milton K. Munitz (Ed.), *Theories of the Universe* (New York: Free Press, 1957), pp. 211–219.
3. See John D. Norton, "Causation as folk science," *Philosophers' Imprint* 3:4 (2003, http://quod.lib.umich.edu/p/phimp/3521354.0003.004). Reprinted in Huw Price and Richard Corry (Eds.), *Causation and the Constitution of Reality: Russell's Republic Revisited* (Oxford: Oxford University Press, 2007), pp. 11–44. See also John D. Norton, "Do the causal principles of modern physics contradict causal anti-fundamentalism?" In Peter K. Machamer and Gereon Wolters (Eds.), *Thinking about Causes: From Greek Philosophy to Modern Physics* (Pittsburgh, PA: University of Pittsburgh Press, 2007).
4. James R. Brown, "Why thought experiments transcend empiricism." In Christopher Hitchcock (Ed.), *Contemporary Debates in the Philosophy of Science* (Malden, MA: Blackwell, 2004), pp. 23–43.
5. Brown, "Why thought experiments transcend empiricism," pp. 23–43 and John D. Norton, "Why thought experiments do not transcend empiricism." In Hitchcock (Ed.), *Contemporary Debates in the Philosophy of Science*, pp. 44–66.
6. The vector $\mathbf{A}_{in}$ has a magnitude equal to the cross-sectional area of the inlet and a direction normal to the cross-section. The same is true for $\mathbf{A}_{out}$.
7. Using $(1/2)\ [|\mathbf{w}+\mathbf{v}|^2 - |\mathbf{v}|^2] = (1/2)\ [|\mathbf{w}|^2 + 2\ \mathbf{w}\cdot\mathbf{v} + |\mathbf{v}|^2 - |\mathbf{v}|^2] = (1/2)\ |\mathbf{w}|^2 + \mathbf{w}\cdot\mathbf{v}$.

PHILOSOPHY FOR EVERYONE

Series editor: Fritz Allhoff

Not so much a subject matter, philosophy is a way of thinking. Thinking not just about the Big Questions, but about little ones too. This series invites everyone to ponder things they care about, big or small, significant, serious... or just curious.

Running & Philosophy: A Marathon for the Mind
Edited by Michael W. Austin

Wine & Philosophy: A Symposium on Thinking and Drinking
Edited by Fritz Allhoff

Food & Philosophy: Eat, Think and Be Merry
Edited by Fritz Allhoff and Dave Monroe

Beer & Philosophy: The Unexamined Beer Isn't Worth Drinking
Edited by Steven D. Hales

Whiskey & Philosophy: A Small Batch of Spirited Ideas
Edited by Fritz Allhoff and Marcus P. Adams

College Sex – Philosophy for Everyone: Philosophers With Benefits
Edited by Michael Bruce and
Robert M. Stewart

Cycling – Philosophy for Everyone: A Philosophical Tour de Force
Edited by Jesús Ilundáin-Agurruza
and Michael W. Austin

Climbing – Philosophy for Everyone: Because It's There
Edited by Stephen E. Schmid

Hunting – Philosophy for Everyone: In Search of the Wild Life
Edited by Nathan Kowalsky

Christmas – Philosophy for Everyone: Better Than a Lump of Coal
Edited by Scott C. Lowe

Cannabis – Philosophy for Everyone: What Were We Just Talking About?
Edited by Dale Jacquette

Porn – Philosophy for Everyone: How to Think With Kink
Edited by Dave Monroe

Serial Killers – Philosophy for Everyone : Being and Killing
Edited by S. Waller

Dating – Philosophy for Everyone: Flirting With Big Ideas
Edited by Kristie Miller
and Marlene Clark

Gardening – Philosophy for Everyone: Cultivating Wisdom
Edited by Dan O'Brien

Motherhood – Philosophy for Everyone: The Birth of Wisdom
Edited by Sheila Lintott

Fatherhood – Philosophy for Everyone: The Dao of Daddy
Edited by Lon S. Nease and
Michael W. Austin

Coffee – Philosophy for Everyone: Grounds for Debate
Edited by Scott F. Parker
and Michael W. Austin

Fashion – Philosophy for Everyone: Thinking with Style
Edited by Jessica Wolfendale and Jeanette Kennett

Yoga – Philosophy for Everyone: Bending Mind and Body
Edited by Liz Stillwaggon Swan

Blues – Philosophy for Everyone: Thinking Deep About Feeling Low
Edited by Abrol Fairweather
and Jesse Steinberg

Tattoos – Philosophy for Everyone: I Ink, Therefore I Am
Edited by Robert Arp

Sailing – Philosophy for Everyone: Catching the Drift of Why We Sail
Edited by Patrick Goold

Edited by Patrick Goold

SAILING
PHILOSOPHY FOR EVERYONE
Catching the Drift of Why We Sail

Foreword by John Rousmaniere

WILEY-BLACKWELL
A John Wiley & Sons, Inc., Publication