ECE 2795 Rectifiers and inverters

Note Title ~~11/2/2008~~

(v. 2/2015)

Rectifiers ⟶ ac to dc

Inverters ⟶ dc to ac

With rectifiers and inverters it's all about managing harmonics ⟶ creating harmonics thanks to nonlinear nature of power electronics circuits

⟶ Filtering out unwanted harmonics.

Good references: Dr Mohan's power electronics book

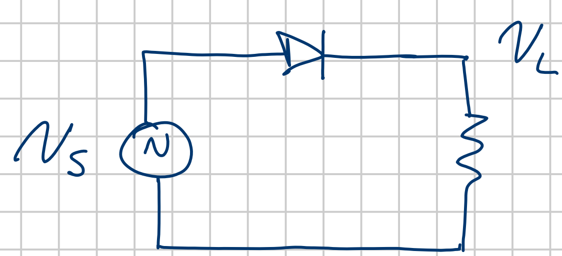
Dr Krein's " " "

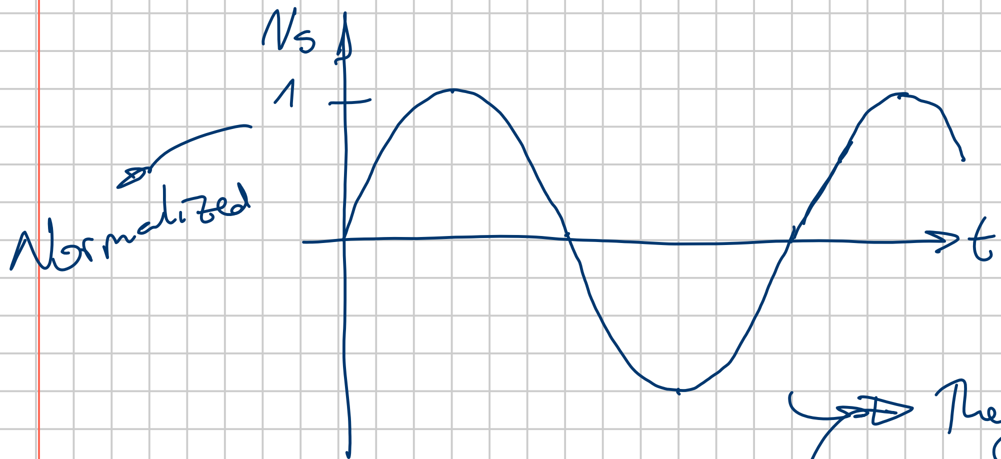
Rectifiers

- Typically they can be single-phase or 3-phase
- the most simple topologies use diodes.
- The most common rectifier circuits convert ac-voltage to dc current

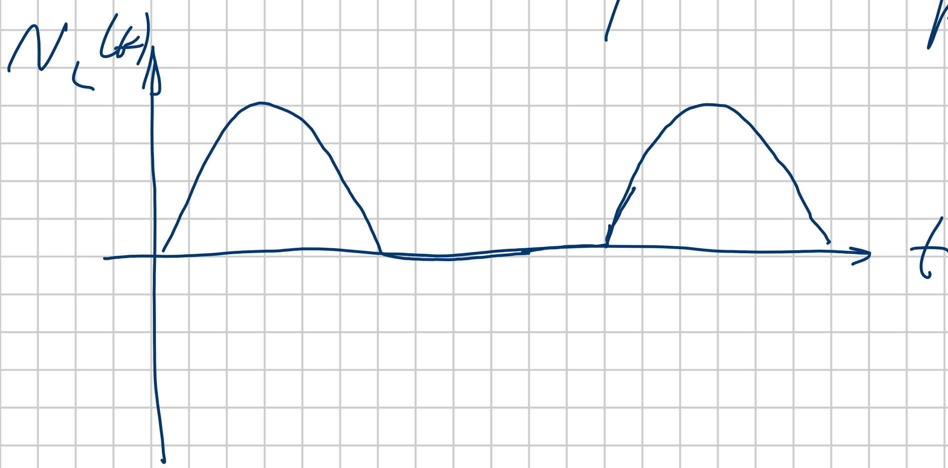
Let's start with the single-phase circuits.

Half wave:





They have the same period

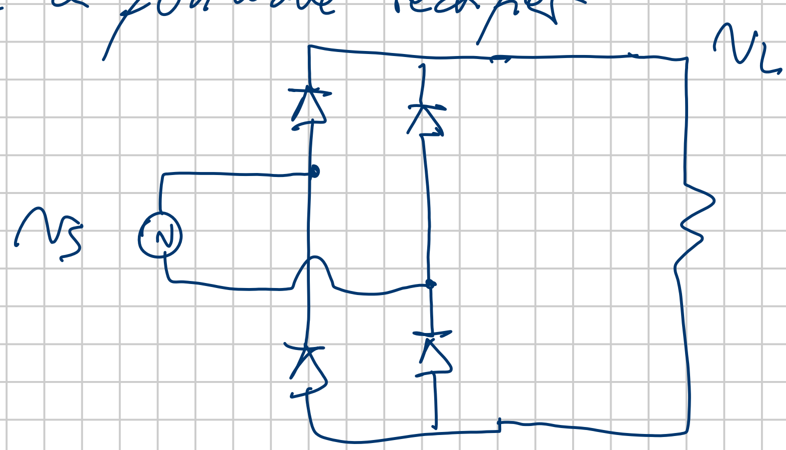


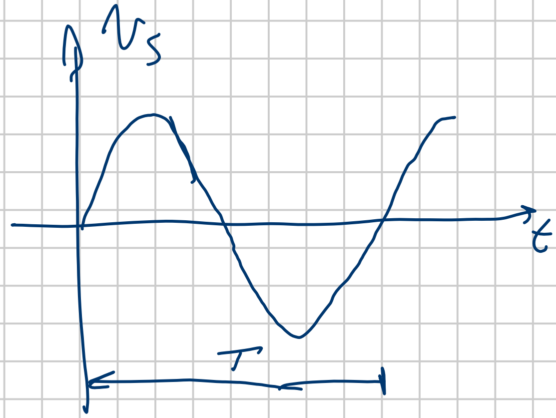
Since I want dc output, what's my dc component V_{dc} in $v_L(t)$?

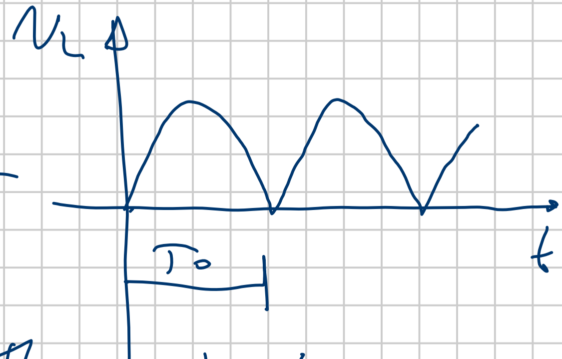
$$V_{dc} = \frac{1}{T}\int_0^{T/2} \sin\frac{2\pi}{T}t \, dt = -\frac{T}{2\pi}\,\frac{1}{T}\cos\frac{2\pi}{T}t\Big|_0^{T/2} =$$

$$= -\frac{1}{2\pi}\left(\cos\pi - \cos 0\right) = \frac{1}{\pi}$$

Let's see a fullwave rectifier







Doubles the input frequency

dc component:

$$V_{dc} = \frac{1}{T_0}\int_0^{T_0} \sin\left(\frac{2\pi}{T}t\right) dt = \frac{T}{2\pi T_0}\left(-\cos\left(\frac{2\pi}{T}t\right)\right)\Bigg|_0^{T_0} =$$

$$= -\frac{1}{\pi}\left(\cos\left(\frac{2\pi}{2T_0}T_0\right) - 1\right) = \frac{2}{\pi}$$

So the dc component is double than in the previous case

What about the harmonic content:

Remember that $V_{RMS}^2 = V_{dc}^2 + V_{h\,RMS}^2$

In the 1/2 bridge case

$$V_{RMS}^2 = \frac{1}{T}\int_0^{T/2} \sin^2\left(\frac{2\pi}{T}t\right) dt = \frac{1}{T}\left(\frac{t}{2} - \frac{T}{4\cdot 2\pi}\sin\frac{4\pi}{T}t\right)\Bigg|_0^{T/2} =$$

$$= \frac{1}{T}\left(\frac{T}{4} - 0 - \frac{T}{8\pi}\sin 2\pi + 0\right) = \frac{1}{4}$$

So $$V_{h\,RMS}^2 = \frac{1}{4} - \left(\frac{1}{\pi}\right)^2 \cong 0.15$$

For the full bridge case:

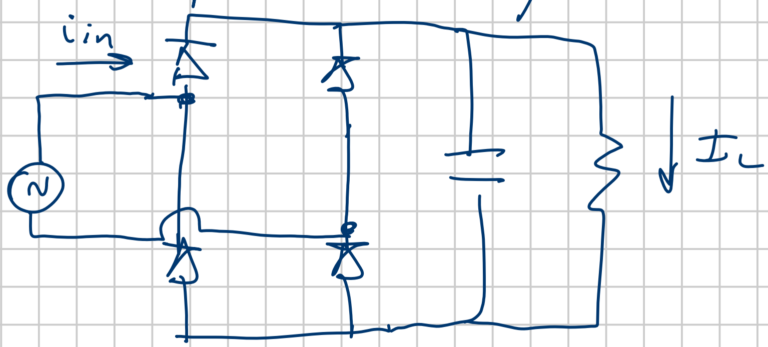
$$V_{RMS_h}^2 = \frac{1}{T_0}\int_0^{T_0} \sin^2\left(\frac{2\pi}{T}t\right)dt = \frac{1}{T_0}\left(\frac{t}{2} - \frac{T}{4 \cdot 2\pi}\sin\frac{4\pi}{T}t\right)\Big|_0^{T_0} =$$

$$= \frac{1}{T_0}\left(\frac{T_0}{2} - 0 - \frac{T}{8\pi}\sin\left(\frac{4\pi}{2T_0}T_0\right) - 0\right) = \frac{1}{2}$$

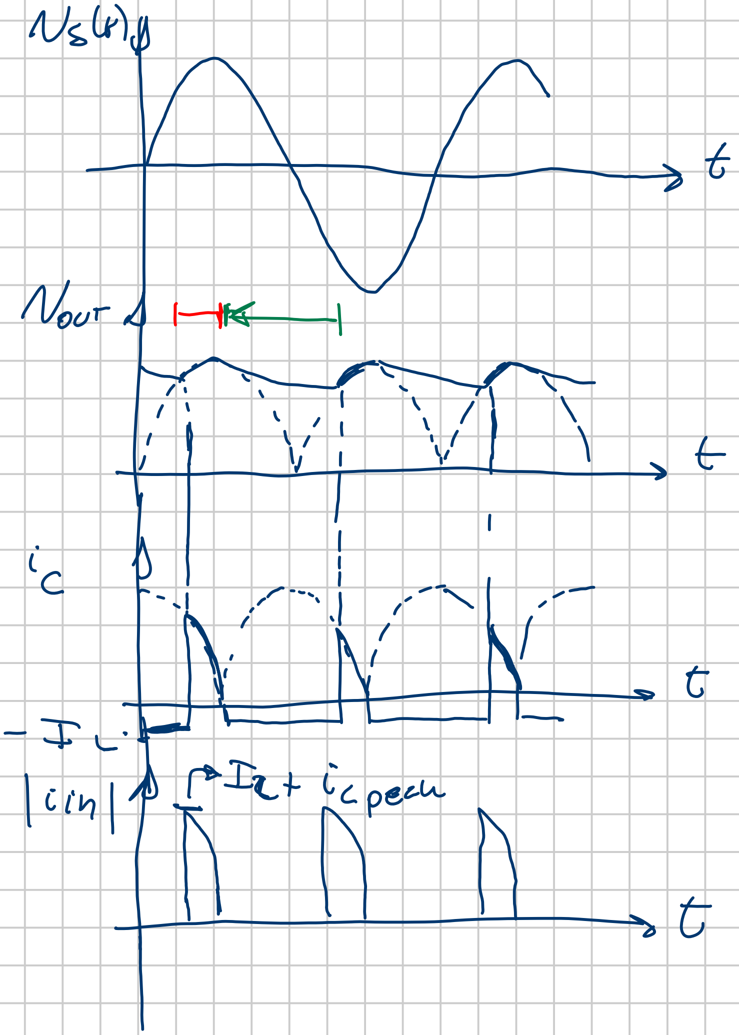
$$V_{h\,RMS}^2 = \frac{1}{2} - \left(\frac{2}{\pi}\right)^2 \cong 0.095$$

So in the full bridge not only there is a smallest harmonic content but it is at a double frequency. Hence, it is easier (but not easy) to filter than in the 1/2-bridge case.

How do we filter it? Well, if we want to keep the dc component only then we need a low-pass filter. The simplest one is just one capacitor:



But, we still need a "large" capacitor. Let's see the waveforms.



The capacitor needs to hold the load for a long time

— capacitor discharges

— capacitor charges

$i_{in} = I_L + i_c$

$i_c = C\frac{dV}{dt} = \left|C\frac{d(\sin\omega t)}{dt}\right|$

$i_c = |\omega C \cos\omega t|$

What is the output voltage ripple?

Well, from

$$i_c = C\frac{dV_c}{dt}$$

$$\downarrow$$

$$I_c \cong C\frac{\Delta V_c}{\Delta t}$$

Hence,

$$\Delta V_c \simeq \frac{I_L \, \Delta t}{C}$$

So $$\Delta V_c \simeq \frac{I_L \, (t_d - t_c)}{C}$$

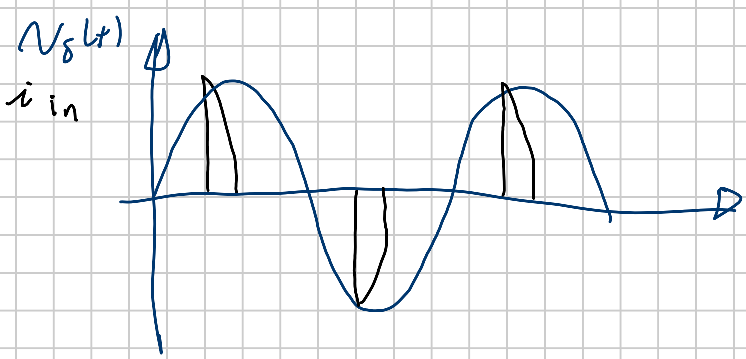
If C is large enough $t_c \simeq 0$ and $t_d \simeq T/2$

So
$$\Delta V_c \simeq \frac{I_L \, T/2}{C}$$

$$\boxed{\Delta V_c \simeq \frac{I_L}{2 f C}}$$

Since f is the line frequency (at most a few hundred hertz in returbines), I usually need a "large" capacitor for "small" ripples

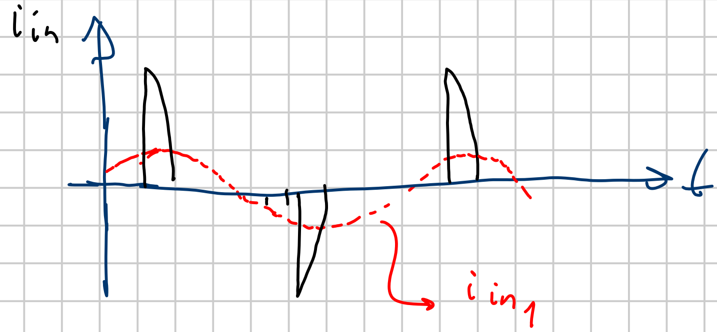
The current and voltage at the source are:



What is the power factor as seen by the source

Remember
$$pf = \frac{P}{V_{rms} \, I_{rms}}$$

For $P = V_{s\,rms} I_{1\,rms} \cos\varphi_1$



$$I_{in\,rms} = I_{in_{1\,rms}}^2 + \underbrace{I_{in_h}^2}$$

→ Tends to be high when compared to $I_{in_1}^2$ because it is formed by pulses (higher content of higher harmonics)

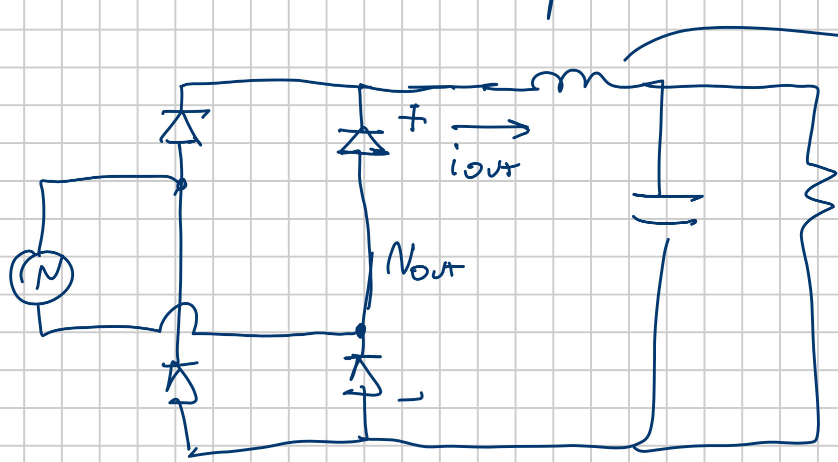
So $$pf = \frac{\cancel{V_{s\,rms}} I_{1\,rms} \cos\varphi_1}{\cancel{V_{s\,rms}} \left(I_{in_{1\,rms}}^2 + I_{in_h}^2 \right)} < 1$$ → And in reality is quite low

This is not good for the source

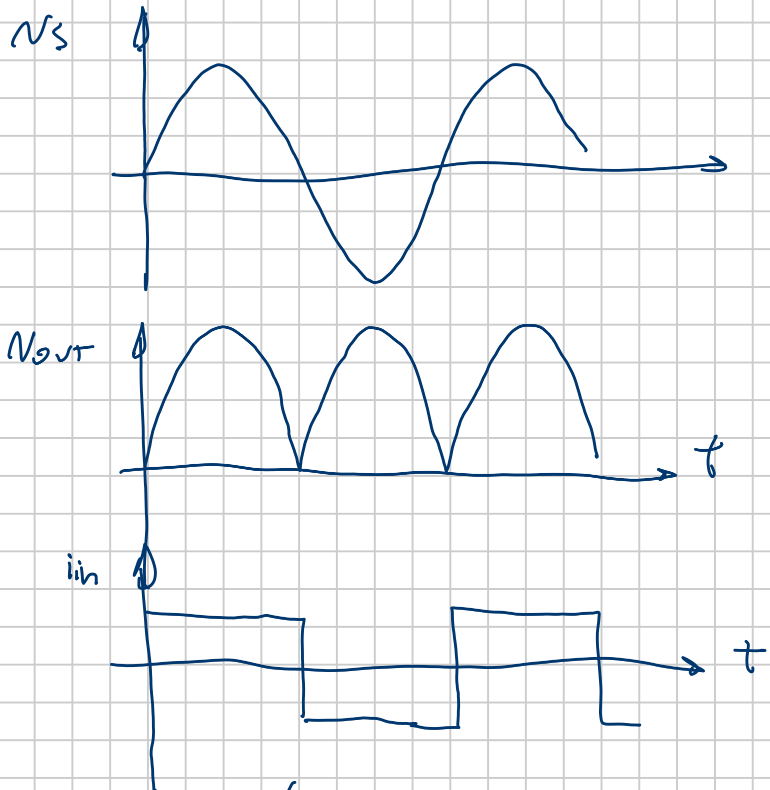
What can we do to improve this? → We can add this inductor in here.

↓

it provides a current interface

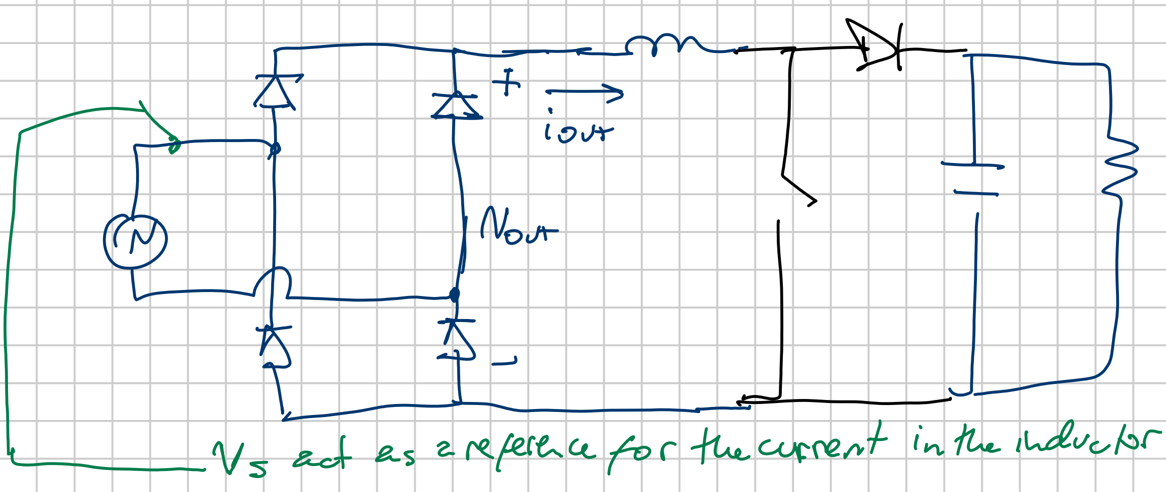


The inductor combined with the capacitor form a 2nd order low-pass filter. So the output is more constant with a smaller capacitor.
With a "large" inductor i_{OUT} is approximately constant an equal to I_L. Then

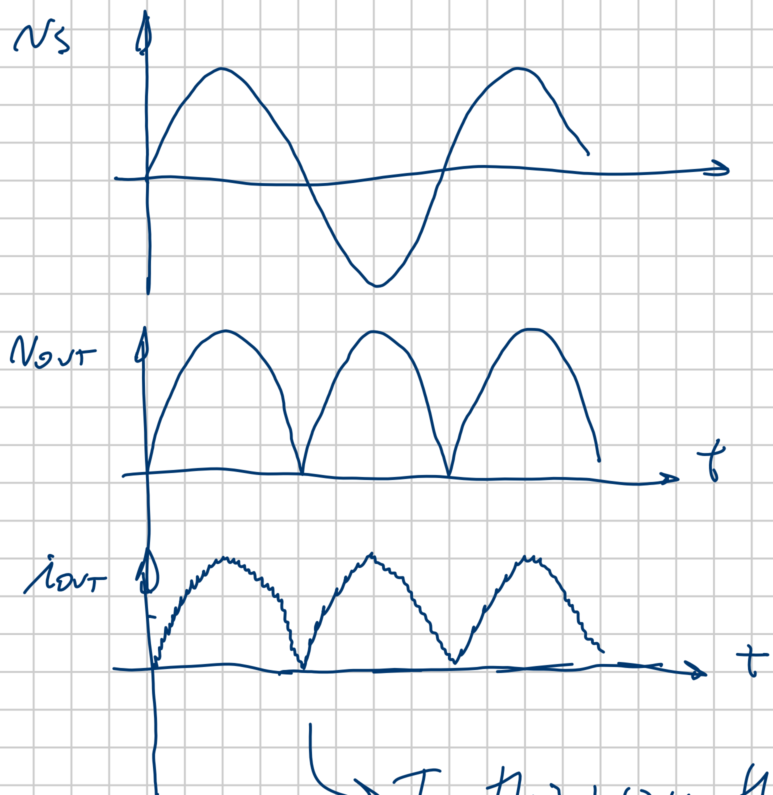


↳ Now i_{in} has a higher fundamental with respect to the harmonic content so the power factor is better but it can be still low.

One other solution to use a boost converter after the rectifier

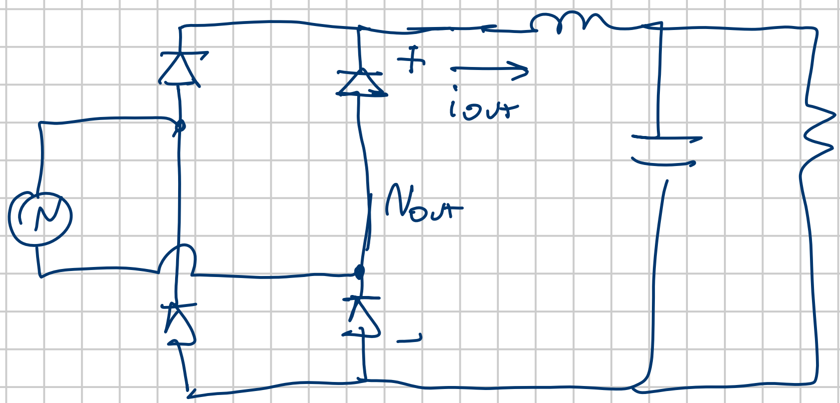


The current in the inductor is controlled to follow the voltage in the source. The output capacitor takes care of the last filter stage.

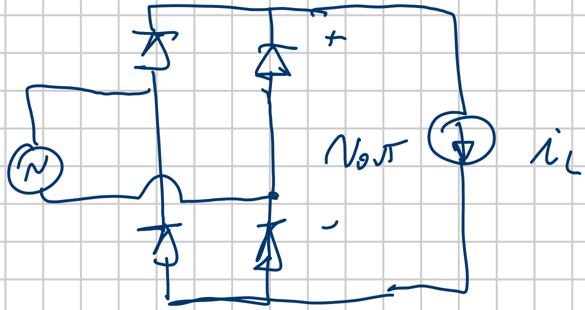


In this way the pf ≃ 1.

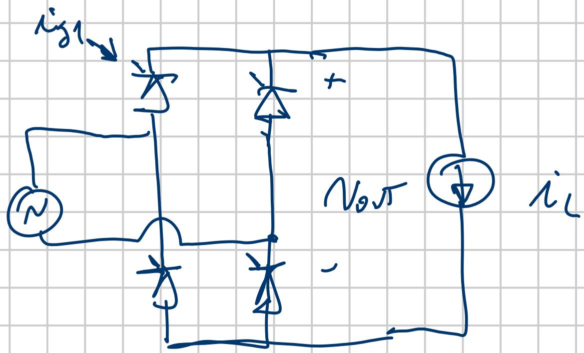
Let's see some additional issues in rectifiers

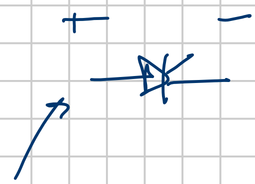


I can draw it as:

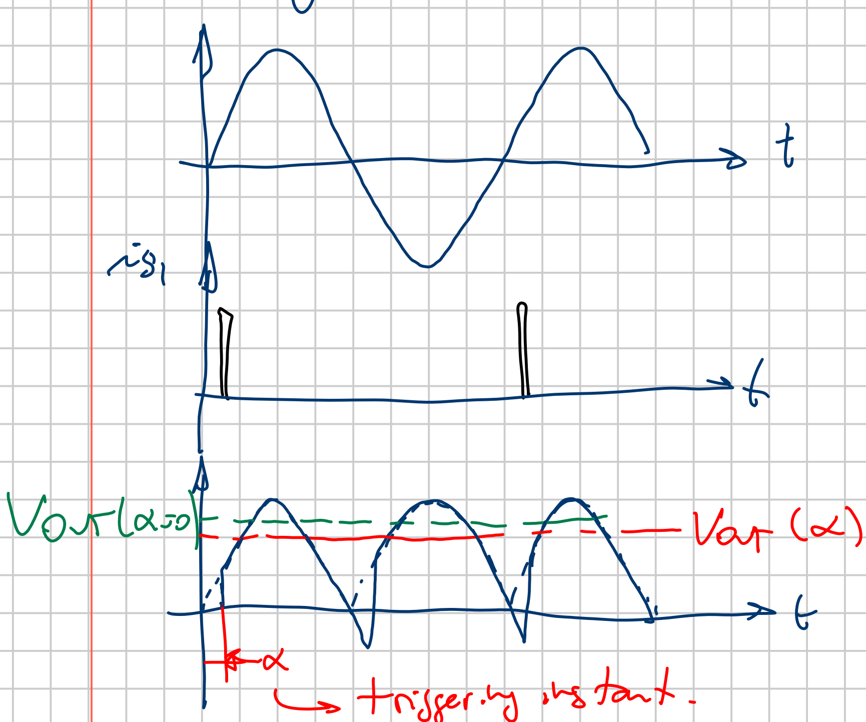


One issue is that I cannot regulate my output (i.e., if Vs changes, Vout changes). One solution is to use SCRs instead of diodes:





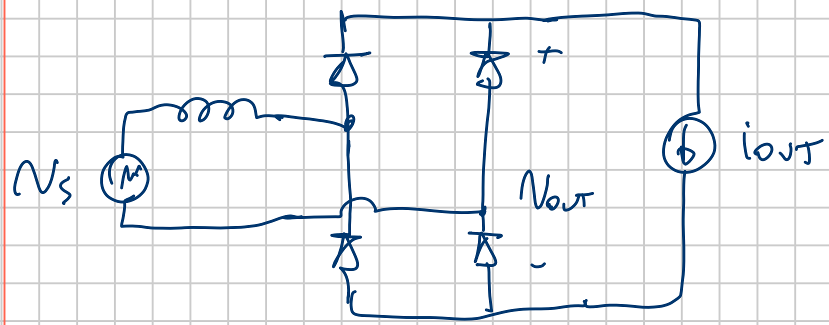
An SCR conducts when it is forward biassed <u>and</u> it has been triggered by injecting an adequate current pulse at the gate.

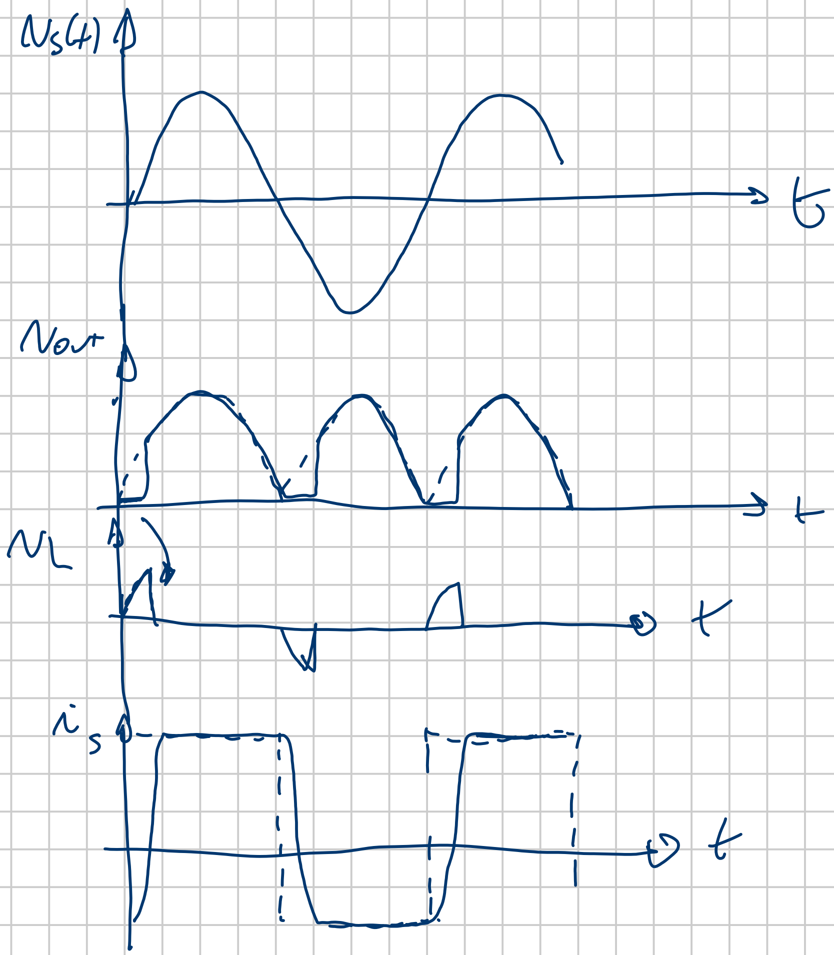


→ So I can now change Vout by changing α

Another issue is that most DG sources that require a rectifier (e.g. microturbine) have a current interface

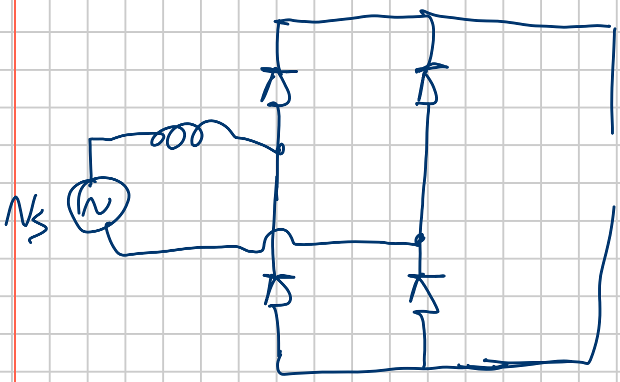
So I actually have



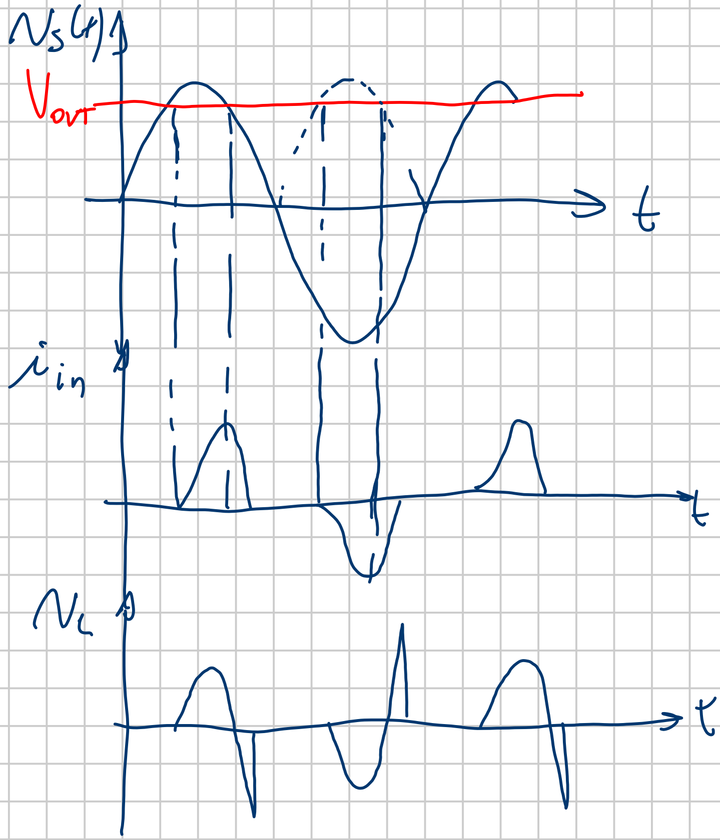


So how do the curves look with a current input and voltage output?

↳ This is the most likely scenario



V_{out} = constant

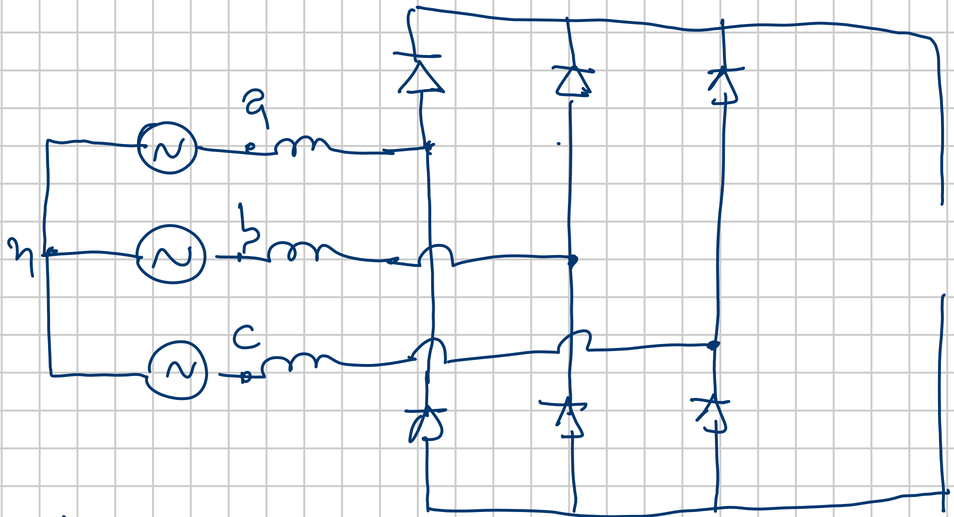


→ it yield a bad power factor

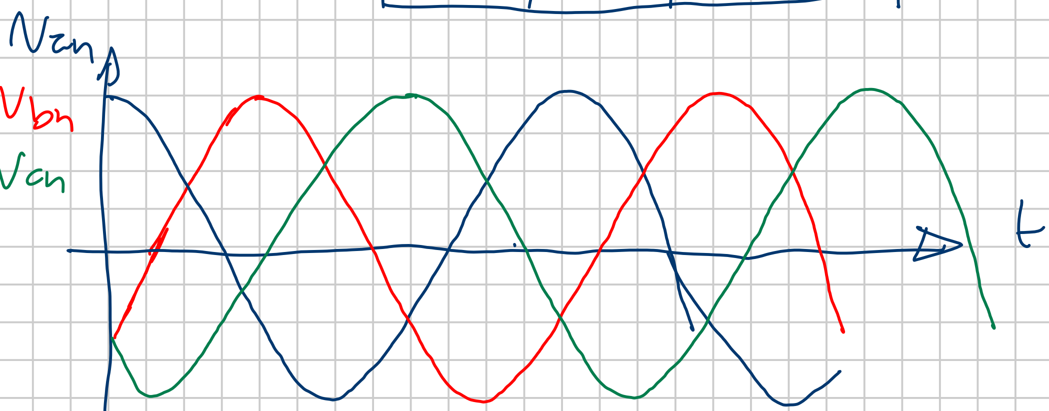
Needs to be compensated

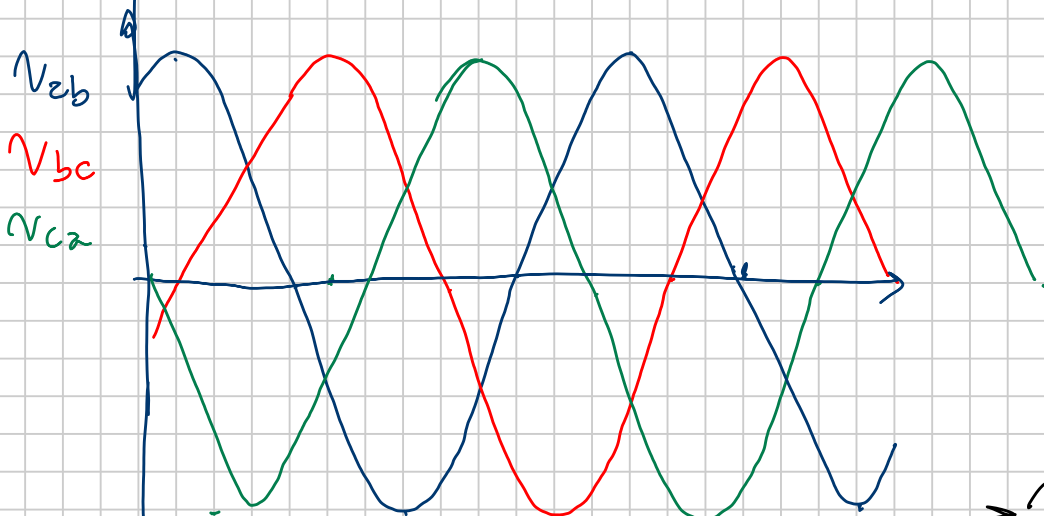
e.g. dc link capacitor and a boost converter

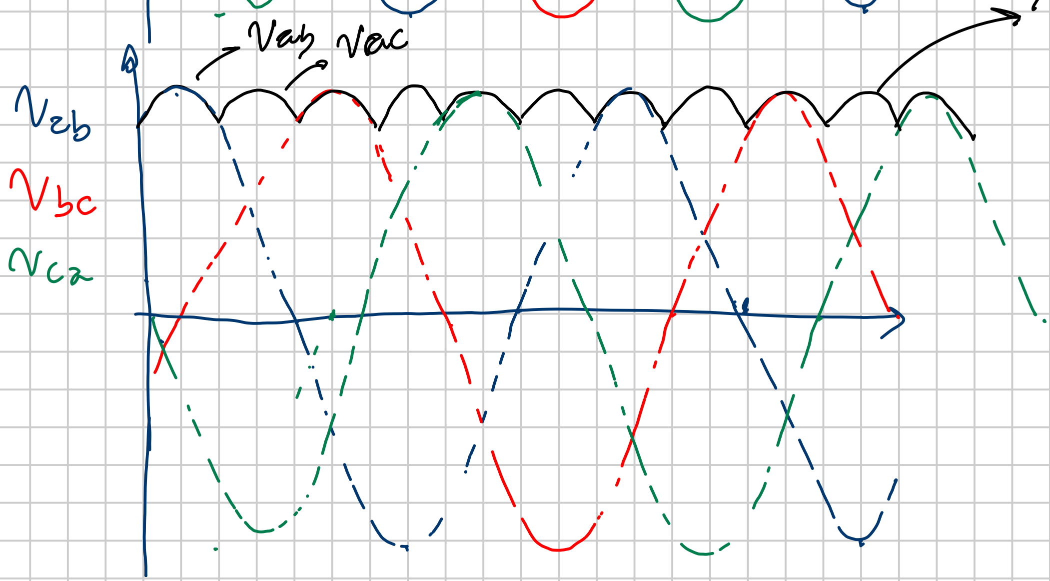
3-phase rectifier



$V_{out} = \text{constant}$

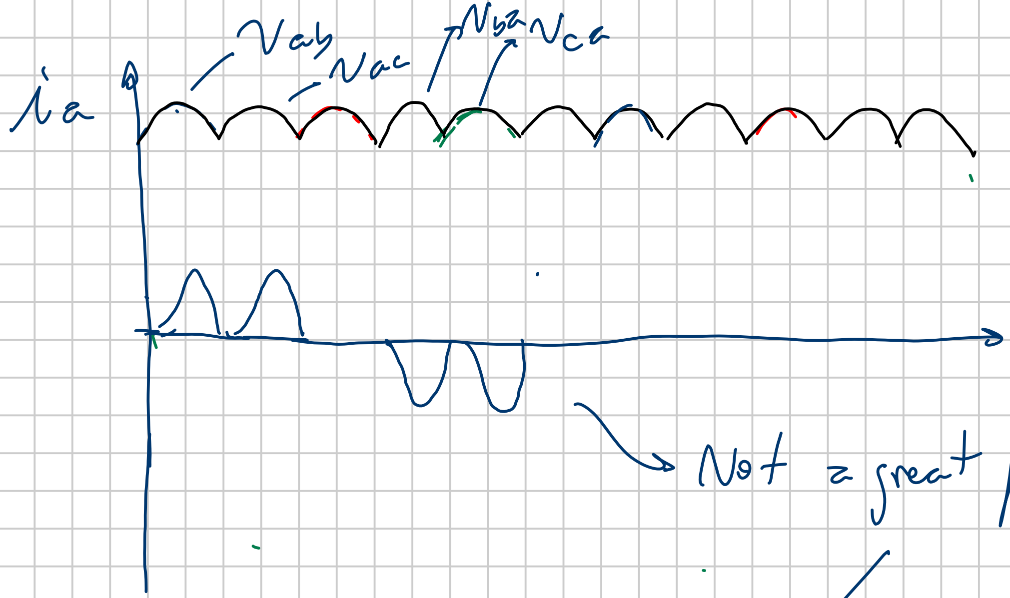






v_{out} unfiltered

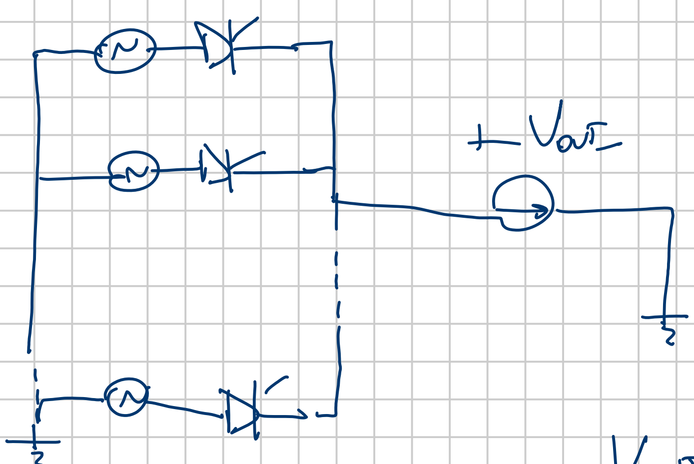
its frequency is 6 times the line frequency (easier to filter harmonics)



Not a great power factor either

it is also not as simple to compensate than in the single phase case

Generalized m-phase rectifiers

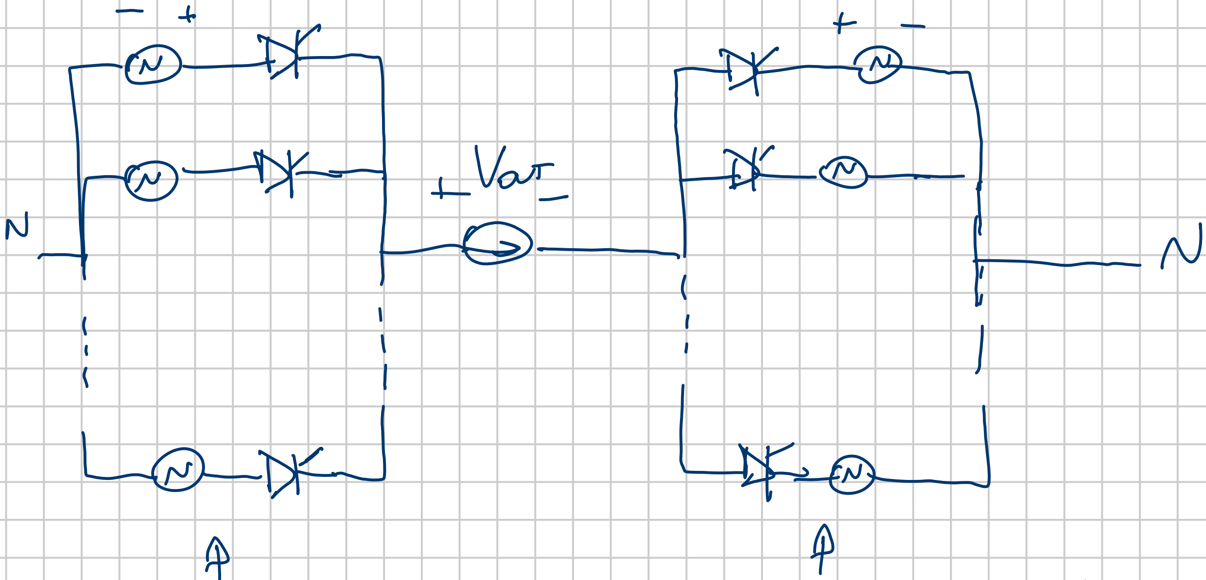


midpoint rectifier

$$V_{out} = \frac{m\sqrt{2}\,V_{s\,L-N\,RMS}}{\pi} \sin\left(\frac{\pi}{m}\right)\cos\alpha$$

for $m \geq 2$

Full bridge rectifier



Midpoint (M) Complementary (C)

$$V_{out} = \frac{\sqrt{2}}{\pi} V_{S\,L\text{-}L\,RMS}\; m \sin\left(\frac{\pi}{m}\right)\left[\cos\alpha_M - \cos\alpha_C\right]$$

for $m \geq 2$

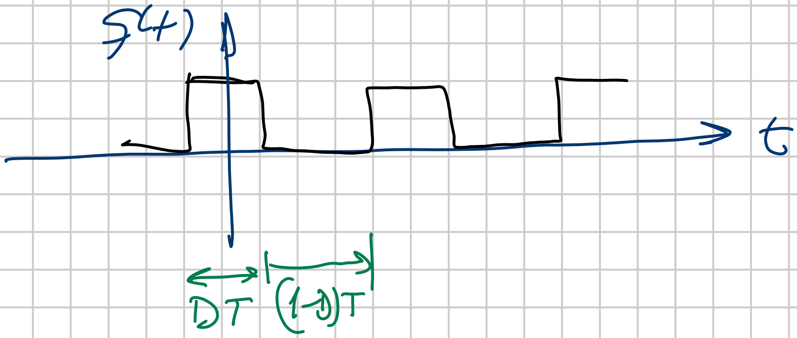
Special case → With diodes $\alpha_M = 0$, $\alpha_C = 180°$ then

$$V_{out} = \frac{2\sqrt{2}}{\pi} V_{S\,L\text{-}L\,RMS}\; m \sin\left(\frac{\pi}{m}\right)$$

Inverters

Understanding the switching function is essential for analyzing inverters behavior.

For $f(t)$ as:

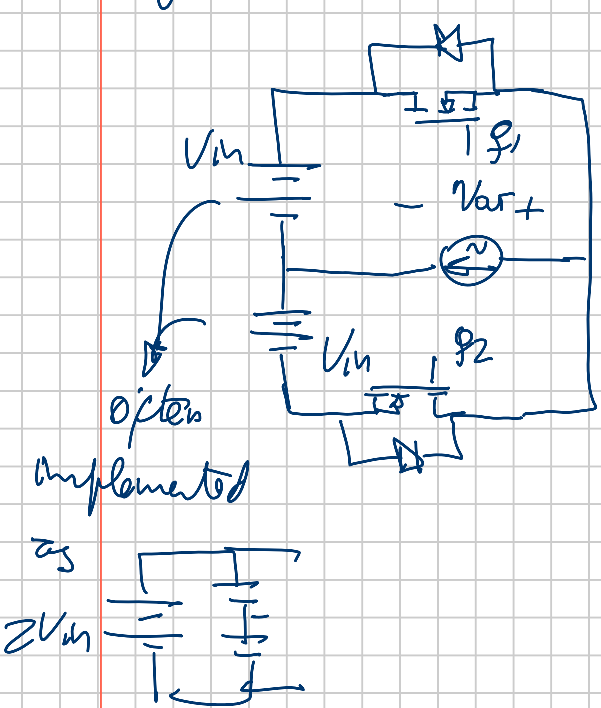


$$f(t) = D + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi D)}{n}\cos(n\omega_0 t - n\phi_0)$$

$D \rightarrow$ average value

$$f(t)|_{rms} = \sqrt{D}$$

Like we did with the rectifiers let's assume first a single-phase output half bridge inverter



From KVL $q_1 + q_2$ cannot be more than 1

From KCL $q_1 + q_2$ cannot be less than 1

Hence, $q_1 + q_2 = 1$

When $q_1 = 1$, $V_{OUT} = V_{in}$

When $q_2 = 1$, $V_{OUT} = -V_{in}$

$$V_{OUT} = q_1(t)\,V_{in} - q_2(t)\,V_{in}$$

$$V_{OUT} = (2q_1 - 1)\,V_{in}$$

$$\underbrace{\langle V_{OUT} \rangle}_{\text{average}} = (2D_1 - 1)\,V_{in}$$

So if the load is inductive which requires $\langle V_L \rangle = 0$,
then $\langle V_{OUT} \rangle = 0 \Rightarrow 2D_1 = 1$

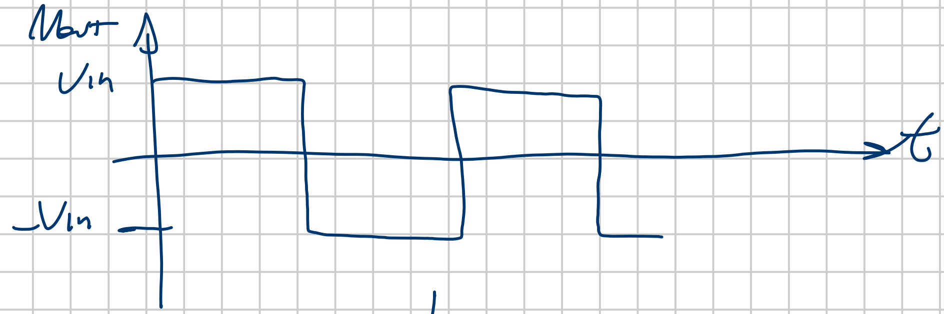
$$D_1 = \frac{1}{2}$$

and since $q_1 + q_2 = 1 \rightarrow D_1 + D_2 = 1$

$$D_2 = D_1 = \frac{1}{2}$$

Since we want an ac output, then let's assume that
$D_1 = D_2$ regardless of whether or not the load is
inductive.

then



2-level output   square wave

$$V_{out} = \frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi/2)}{n}\cos(n\omega_{sw}t - n\phi)$$

it is obtained from $V_{out} = (2q_1 - 1)V_{in}$

and $q_1(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\frac{1}{2}\pi n)}{n}\cos(n\omega_0 t - n\phi_0)$

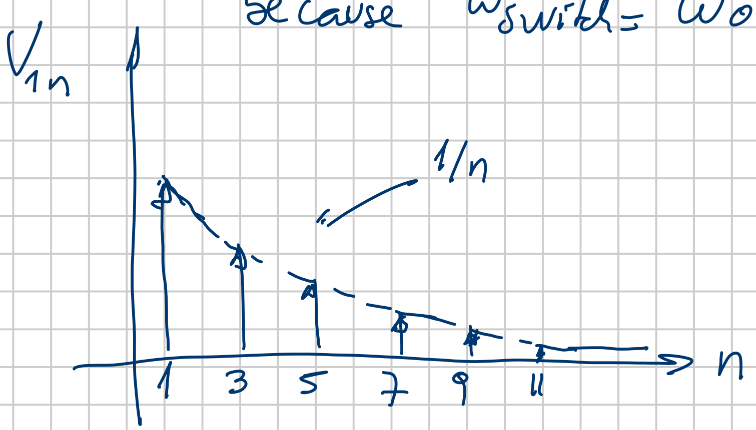
$D = \frac{1}{2}$

Usually we are mostly concerned with the fundamental

$$V_{1out} = \frac{4V_{in}}{\pi}$$

Issues: 1) Output is fixed $\rightarrow$ No voltage regulation

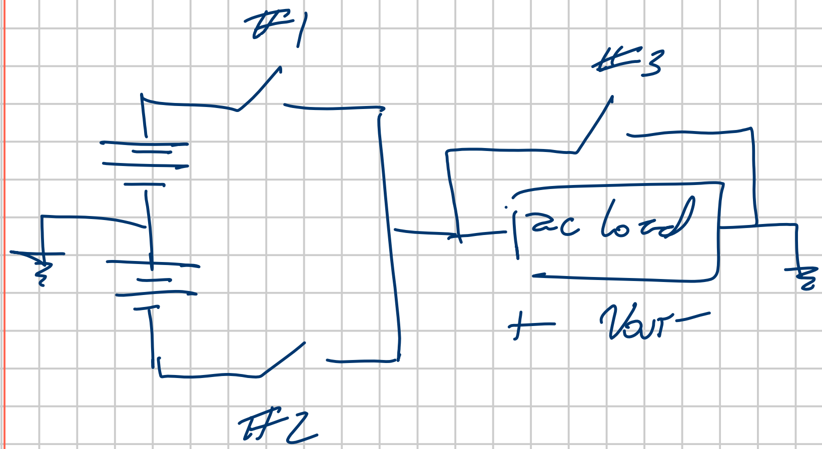
2) Harmonics too close to fundamental. And fundamental frequency is usually low because $\omega_{switch} = \omega_{out}$

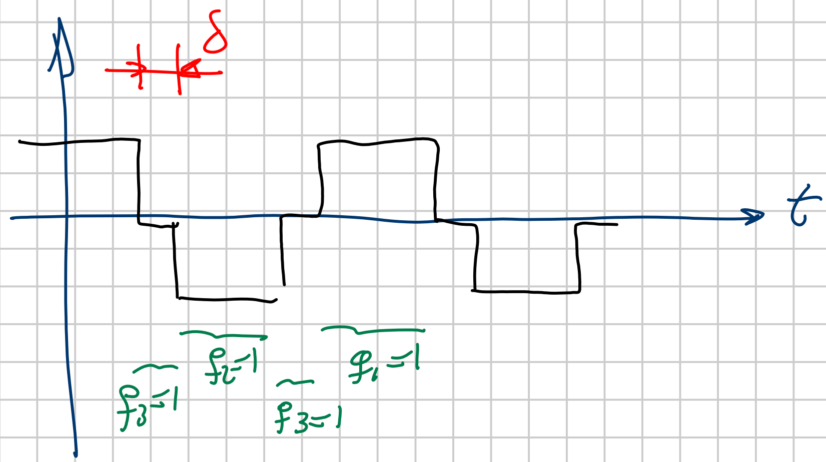


$\leftarrow$ Spectrum for V_{out}

How can we have output voltage regulation?

One solution is the following one:



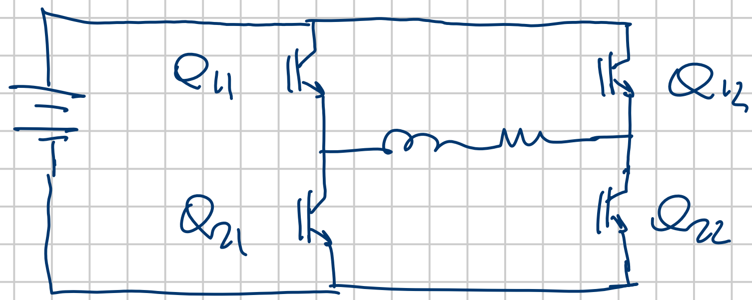


$$V_{OUT} = \frac{2V_{in}}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n} \left[\cos(n\omega_{sw}t) + \cos(n\omega_{sw}t - n\delta)\right]$$

The fundamental component is

$$V_{OUT_1} = \underbrace{\frac{4V_{in}}{\pi}\cos\frac{\delta}{2}}_{V_{OUT_1}} \cos\left(\omega_{sw}t - \frac{\delta}{2}\right)$$

I can achieve the same behavior with a full bridge inverter:



From KVL & KCL:

$$\begin{cases} f_{11} + f_{21} = 1 \\ f_{12} + f_{22} = 1 \end{cases}$$

If Q_{11} and Q_{21} or Q_{12} and Q_{22} are simultaneously on I have shoot-through

$$\begin{cases} Q_{11},\ Q_{22} \text{ ON} \longrightarrow V_{out} = V_{in} \\ Q_{12},\ Q_{21} \text{ ON} \longrightarrow V_{out} = -V_{in} \\ Q_{11},\ Q_{12} \text{ ON} \longrightarrow V_{out} = 0 \\ Q_{21},\ Q_{22} \text{ ON} \longrightarrow V_{out} = 0 \end{cases}$$

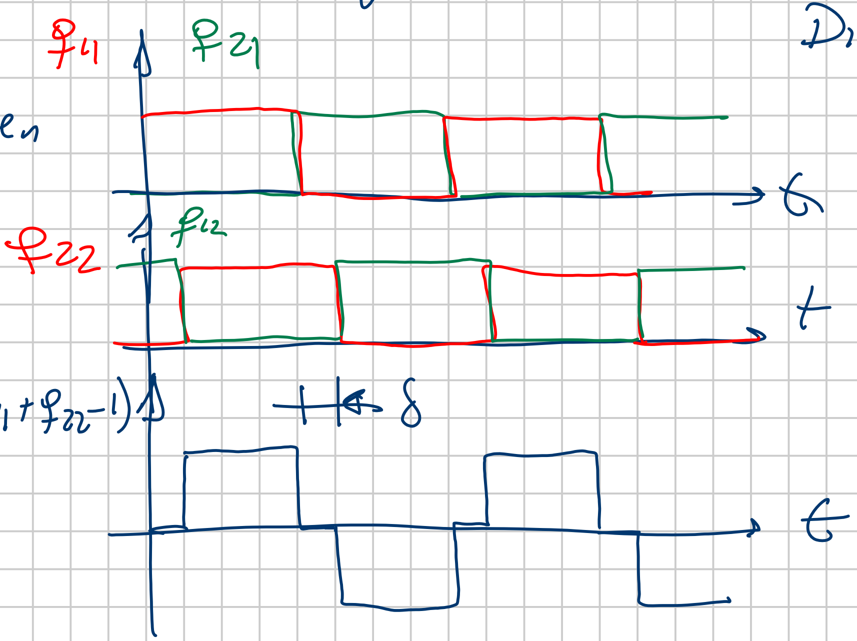
$$V_{out} = f_{11} V_{in} - f_{12} V_{in} = (f_{11} - f_{12}) V_{in} = (f_{11} + f_{22} - 1) V_{in}$$

To avoid having dc we need $D_{11} = D_{21} = 1/2$

$D_{12} = D_{22} = 1/2$

Then

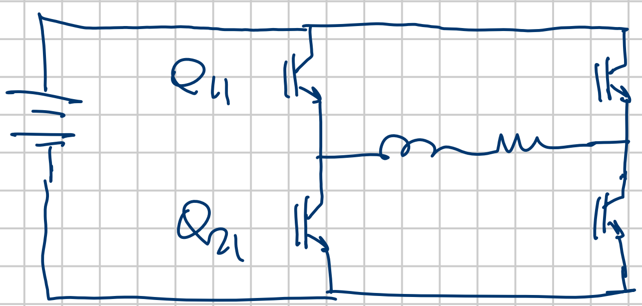
$V_{out} = V_{in}(f_{11} + f_{22} - 1)$



So I have addressed issue #1 above. But this approach does not address issue #2. Can I address both simultaneously? Yes, with pulse-width modulation (PWM):

PWM

Let's consider the inverter seen before



and let's operate it only with the following 2 states:

$$\begin{cases} \text{State 1: } Q_{11} \text{ ON and } Q_{22} \text{ ON} \longrightarrow V_{OUT} = V_{in} \\ \text{State 2: } Q_{12} \text{ ON and } Q_{21} \text{ ON} \longrightarrow V_{OUT} = -V_{in} \end{cases}$$

S_1 last for $d \cdot T_S$ and S_2 lasts for $(1-d)T_S$ where d is the duty cycle for a given switching period and T_S is the switching period.

Although d stays fixed during each switching period it can change from switching period to switching period

My goal is to obtain an ac output in which the fundamental has a frequency of $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

So lets define k as

$$k = \frac{T_o}{T_s} \longrightarrow k \gg 1$$

Let's consider now one switching interval $\hat{T_s}$.
During that particular switching interval d is fixed and equals: $d_{11} = D|_{\hat{T_s}}$

Then, the average output voltage for that switching interval $\hat{T_s}$ is:

$$\langle V_{out} \rangle|_{\hat{T_s}} = D|_{\hat{T_s}} V_{in} + (1 - D|_{\hat{T_s}})(-V_{in}) =$$
$$= V_{in}(2D|_{\hat{T_s}} - 1)$$

This expression comes from $V_{out} = (\varsigma_{11} + \varsigma_{22} - 1) V_{in}$. If $\varsigma_{11} = \varsigma_{22}$ then

$$\boxed{V_{out} = (2\varsigma_{11} - 1) V_{in}} \quad (1)$$

average $\downarrow$ $\downarrow$ average

$$\langle V_{out} \rangle = (2D_{11} - 1) V_{in}.$$

So from $\langle V_{out} \rangle|_{\hat{T_s}} = V_{in}(2D|_{\hat{T_s}} - 1)$ Let's assume, as I said before, that the duty cycle changes from switching cycle to switching cycle in the following particular way:

$$d_{4}(t) = \frac{1}{2} + \frac{1}{2} m(t)$$

where $m(t) = m \cos(\omega_0 t)$

modulation signal

modulation index $\rightarrow$ $\boxed{m \triangleq \frac{V_{OUT1}}{V_{in}}}$ $\rightarrow$ desired fundamental amplitude

where $m \leq 1$ $\forall t$ so $|m(t)| \leq 1$ and

$$|d_{4}(t)| \leq \frac{1}{2} \quad \forall t$$

So

$$d_{4}(t) = \frac{1}{2} + \frac{1}{2} m \cos(\omega_0 t)$$

and

$$\langle V_{OUT} \rangle (t) = V_{in} m \cos(\omega_0 t)$$

The problem here is that I said

"Although d stays fixed during each switching period it can change from switching period to switching period"

And in the above equation d changes continuously with time. So the correct form is

$$D|_{T_S} = \frac{1}{2} + \frac{1}{2} m \cos(\omega_0 t(nT_S))$$

As $T_S \rightarrow 0$ ($f_{SW} \rightarrow \infty$) then $D|_{T_S} \rightarrow d(t)$

and $t(nT_S) \rightarrow t$.

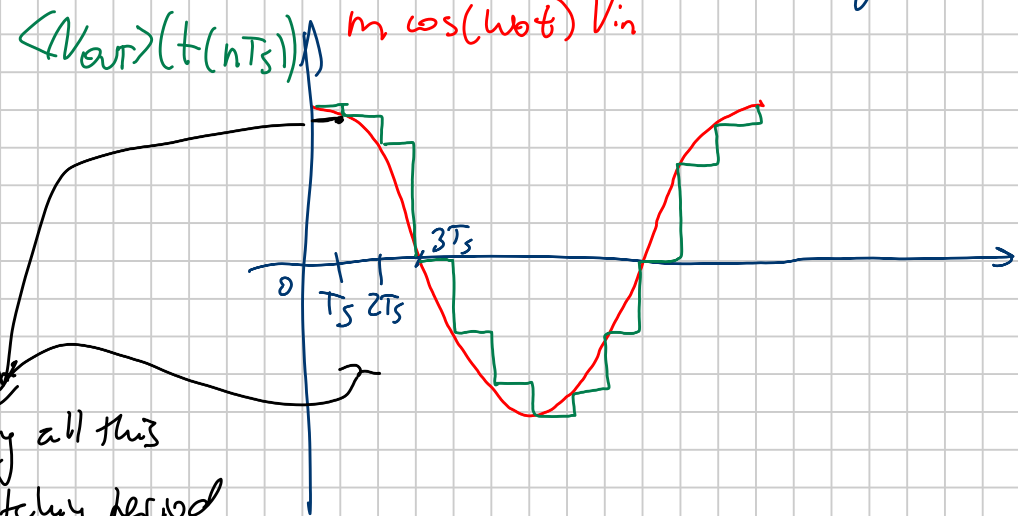
So, how do I take $t(nT_S)$. In other words, how do I sample the modulation signal?

There are 2 main approaches for sampling m(t).

1) Uniform PWM (UPWM)

I sample every T_s seconds, usually at the start of each switching period.

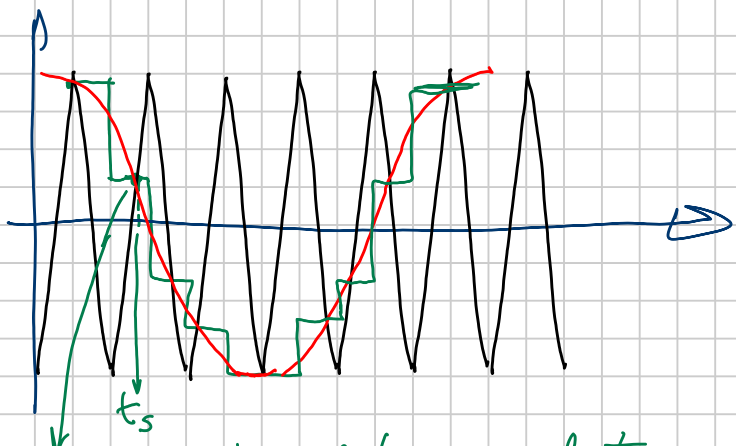
This is easy to do with digital implementation



During all this switching period

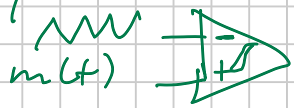
$$D|_{T_S} = \frac{1}{2} + \frac{1}{2} m\cos(\omega_0 T_s)$$

2) Natural PWM (NPWM) → Sample is given by a triangle waveform



→ to find t_s I need to solve the trascendental equation $\ell_c(t_s) = m(t_s)$

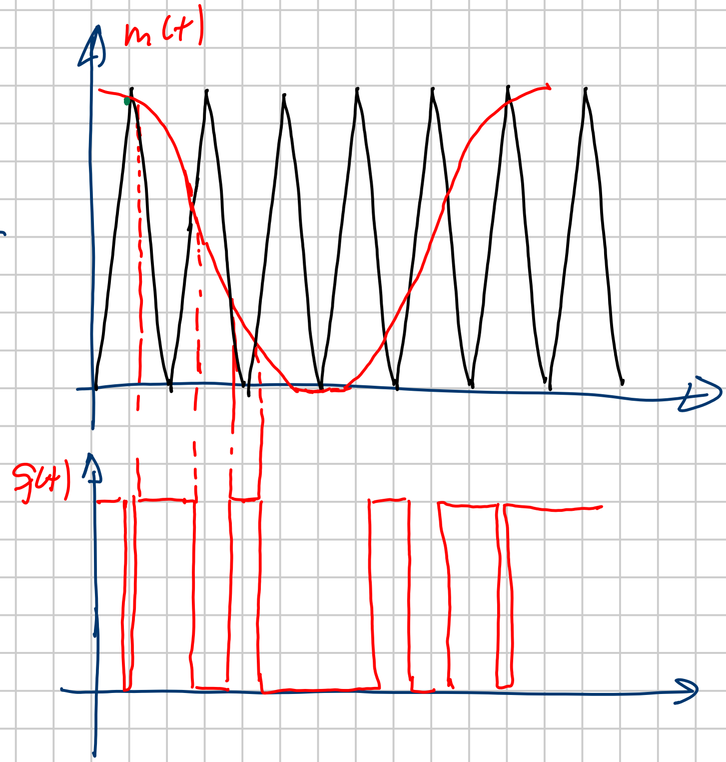
This is difficult when it is implemented digitally but it is easy when it is implemented analogically



$m(t)$ has, in fact, an offset of 1/2 with respect to the red curve above. So $q(t)$ looks something like the following.

If $m >$ triangle $q=1$

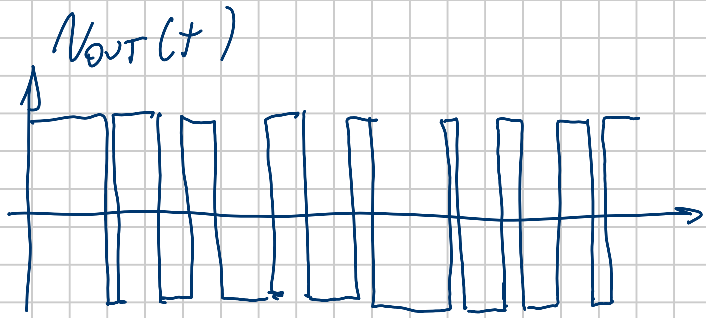
If $m <$ triangle $q=0$



Typically $T_s << T_0$ so $q(t)$ looks closer to the following

And since $V_{out} = \left(2\, q_{11}(t) - 1\right) V_{in}$

if $d_{11}(t) = \frac{1}{2} + \frac{1}{2} m(t)$



$$v_{OUT}(t) = \underbrace{\left(2\,D|_{\hat{T}_S} - 1\right)V_{in}}_{\langle v_{OUT}\rangle|_{\hat{T}_S}(t)} + \frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\pi D|_{\hat{T}_S}\right)}{n}\cos\left(n\,\omega_{sw}t\right)$$

If $T_S \to 0$ then

$$v_{out}(t) = \left(2d_{11}(t) - 1\right)V_{in} + \frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\pi d_{11}\right)}{n}\cos(n\omega_{sw}t)$$

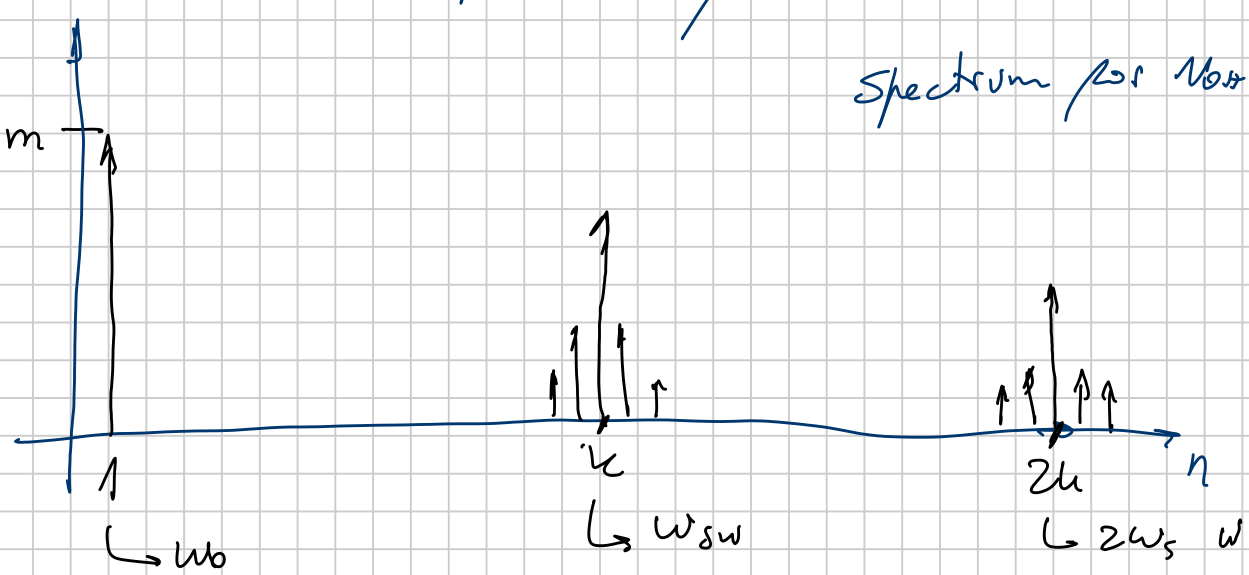
$$v_{OUT}(t) = \underbrace{m(t)\,V_{in}}_{\text{Fundamental}} + \underbrace{\frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\pi\left(\frac{1}{2}+\frac{m(t)}{2}\right)\right)}{n}\cos(n\omega_{sw}t)}_{\text{harmonic content}}$$

Fundamental

$v_{OUT_1}(t) = m\,V_{in}\cos\omega_0 t$

For T_S small (k large) both NPWM and UPWM yields the same spectrum for v_{OUT}

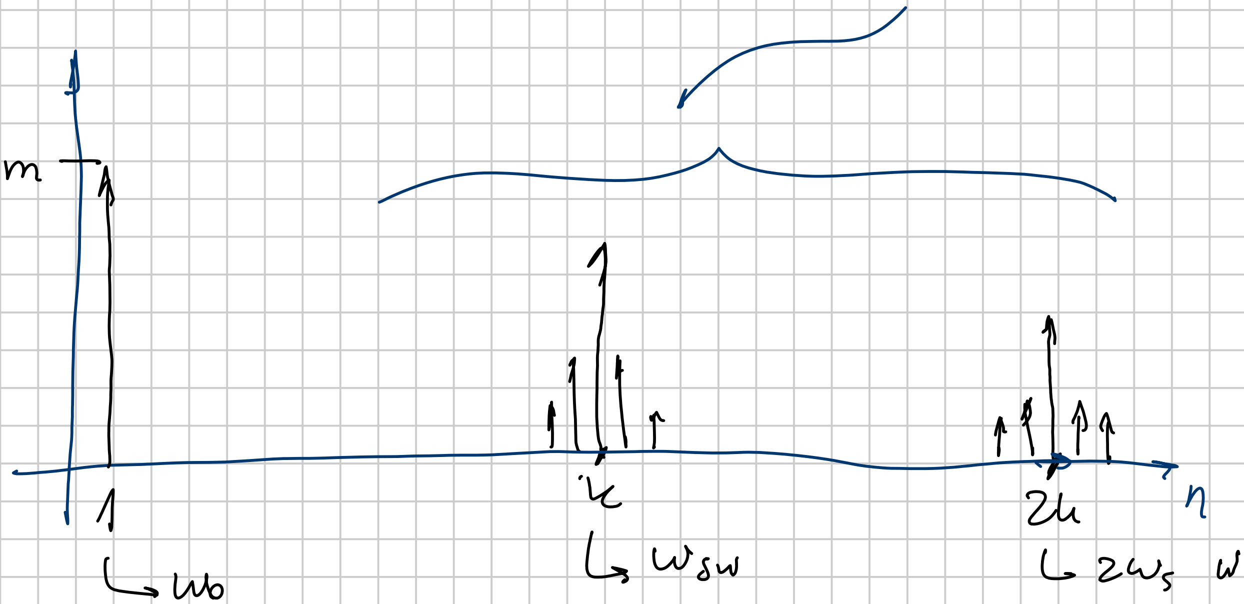
Spectrum for v_{OUT}



In reality → NPWM → it is more difficult to implement but doesn't have harmonics around the fundamental (w_0)

→ UPWM → it's easier to implement but for low k it yields harmonics around w_0

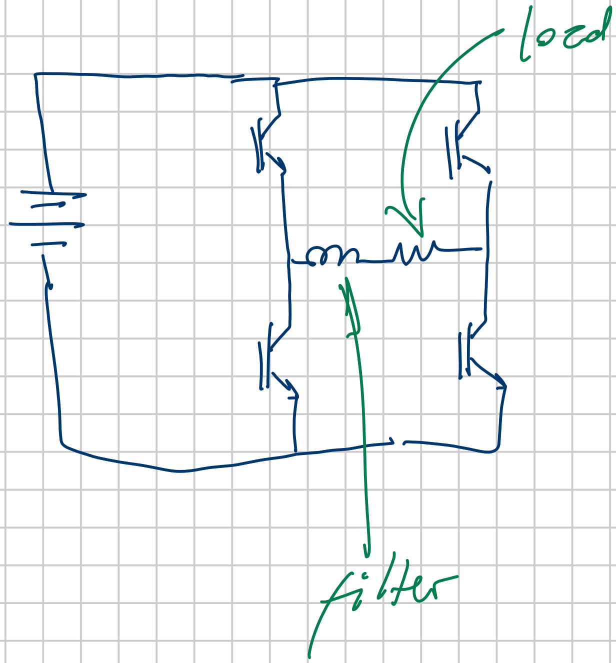
How do I filter the unwanted harmonics?



Answer: I use a low-pass filter:

The advantage is that wsw is away from wo, so I can reduce the size of my filter components by choosing a higher corner frequency

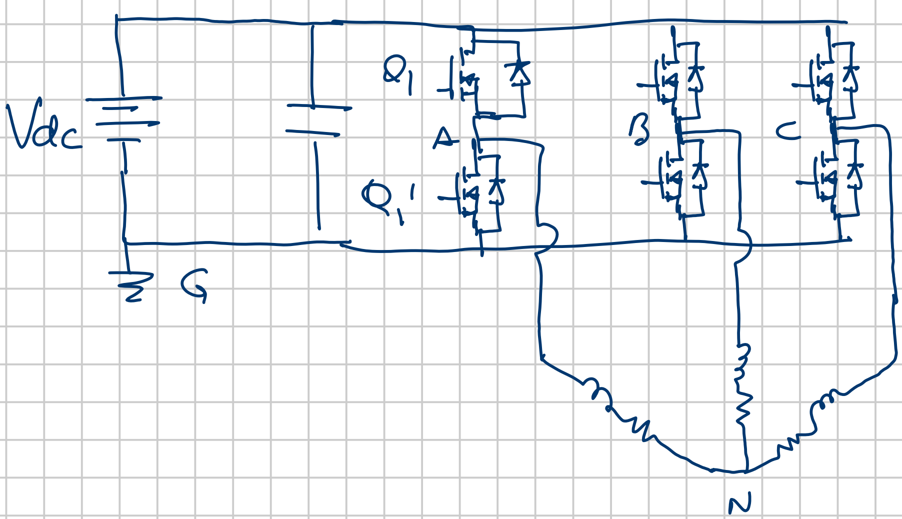
So the filter looks something like this.



→ Voltage input
current output

Voltage-source inverter
VSI

3-phase VSI:



If Q1 and Q1' are simultaneously on I have shoot-through
↓
I need to add standby time

Possible voltages $V_{AG}, V_{BG}, V_{CG} = \begin{cases} V_{dc} \\ 0 \end{cases}$

$$V_{AB}, V_{BC}, V_{CB} = \begin{cases} +V_{dc} \\ -V_{dc} \\ 0 \end{cases}$$

$$V_{AN}, V_{BN}, V_{CN} = \begin{cases} 2/3\, V_{dc} \\ +1/3\, V_{dc} \\ -1/3\, V_{dc} \\ 2/3\, V_{dc} \end{cases}$$

$$V_{AG} = \begin{cases} V_{dc} & \text{with } Q_1 = \text{on} \\ 0 & \text{with } Q_1 = \text{off} \end{cases}$$

$$V_{AG\,fund}(t) = q_{11}(t)\, V_{dc} = \left(\frac{1}{2} + \frac{1}{2} m_a(t)\right) V_{dc}$$

↳ fundamental

In 3 phase systems:

$$m_a(t) = m \cos \omega_0 t$$
$$m_b(t) = m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$
$$m_c(t) = m \cos\left(\omega_0 t + \frac{2\pi}{3}\right)$$

$$V_{AB\,fund} = V_{AG\,fund} - V_{BG\,fund} = \left\{ \frac{1}{2}\, m_a(t) - m_b(t) \right\} V_{dc} =$$

↑ line voltage

$$= \left\{ \frac{1}{2}\, m \left[\cos \omega_0 t - \cos\left(\omega_0 t - \frac{2\pi}{3}\right)\right]\right\} V_{dc}$$

$$= \frac{1}{2}\, m\, V_{dc} \sqrt{3} \cos\left(\omega_0 t + \frac{\pi}{6}\right)$$

$$V_{AB\,fund\,peak} = \frac{\sqrt{3}}{2}\, m\, V_{dc} \longrightarrow m = \frac{V_{AB\,fund\,peak}}{\sqrt{3}\,\frac{V_{dc}}{2}}$$

$$m = \frac{V_{AB\,fund\,peak}}{\sqrt{3}\,\frac{V_{dc}}{2}} = \frac{\sqrt{2}\, V_{AB\,fund\,RMS}}{\sqrt{3}\,\frac{V_{dc}}{2}} = \frac{\sqrt{2}\, V_{AN\,fund\,RMS}}{\frac{V_{dc}}{2}}$$

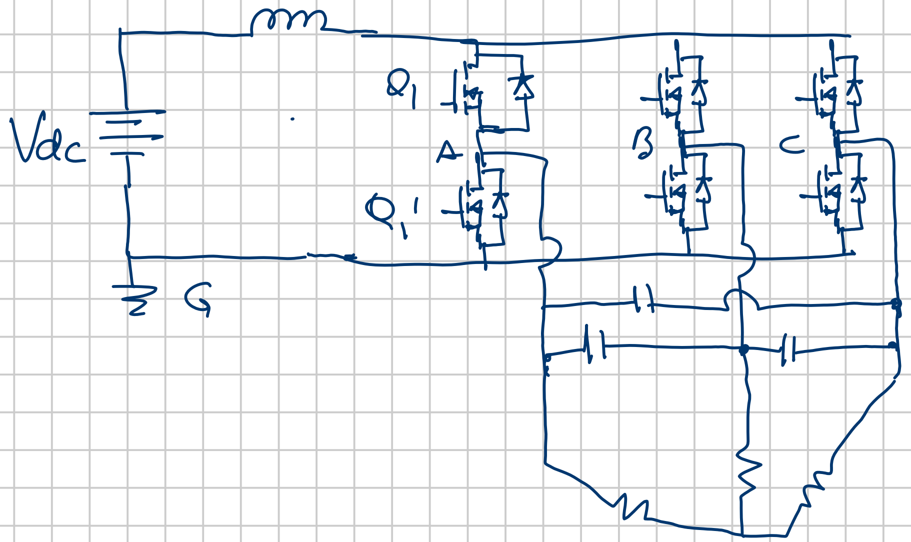
$$\boxed{m = \frac{V_{AN\,fund\,peak}}{V_{dc}/2}}$$

→ half the gain than a 1-phase inverter

Still we have a VSI ⇒ Voltage input, current output

Other topologies are:

- Current source interface (CSI)



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504 IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS, VOL. 39, NO. 2, MARCH/APRIL 2003

Z-Source Inverter

Fang Zheng Peng, *Senior Member, IEEE*

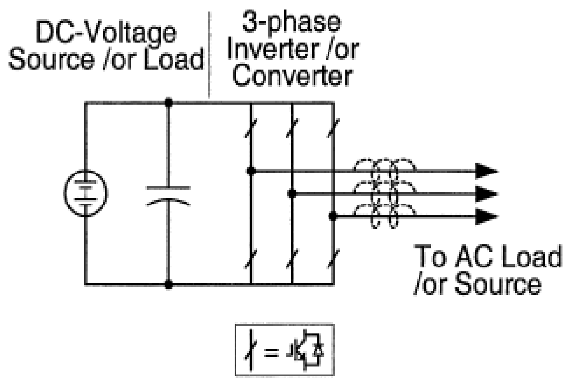

Fig. 1. Traditional V-source converter.

only reduces the input voltage

inconvenient output filter

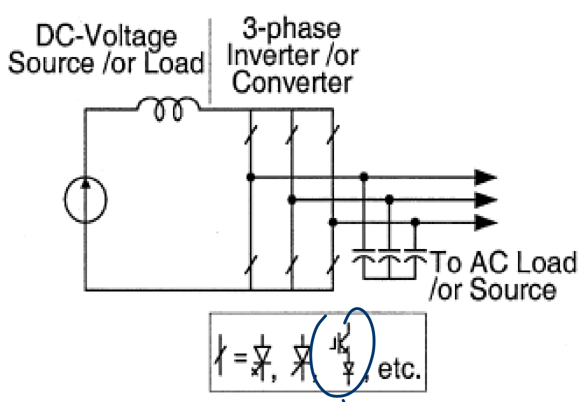

Fig. 2. Traditional I-source converter.

only boost input voltage

Requires additional diode to block

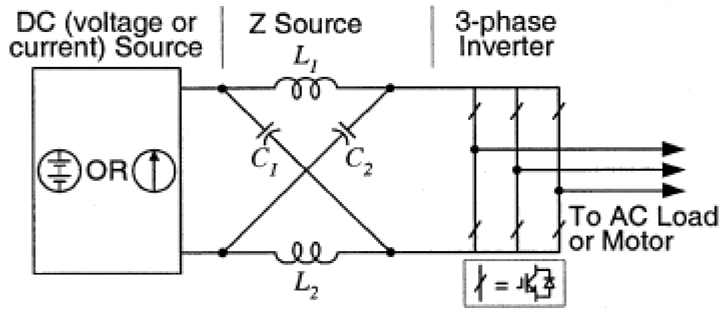

Fig. 4. Z-source converter structure using the antiparallel combination of switching device and diode.

Impedance source inverter

→ ZSI

Can both buck and boost voltage

$V_{out,\,fund\,peak\,ph} = mB\frac{V_{dc}}{2}$

boost factor → depends on the ratio between shoot-through and non-shoot-through states

mod index

$mB = B_B$

Buck-Boost factor

Can vary between 0 and 1

2:1 range in fuel cells

typical dc link voltage

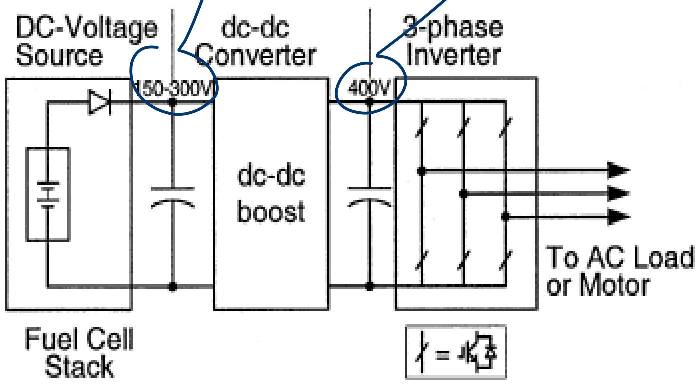

Fig. 6. Traditional two-stage power conversion for fuel-cell applications.

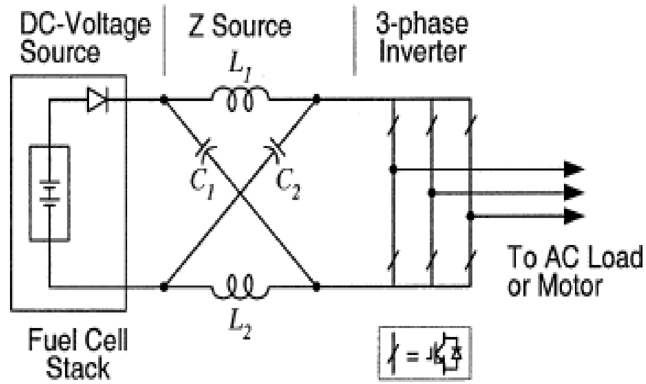

Fig. 7. Z-source inverter for fuel-cell applications.