ECE 2795 Notes on reliability

Note Title

It is often claimed that microgrids can provide a more reliable power supply than the grid..

To study whether or not this statement is true we need to understand some basic concepts of reliability theory

Let's consider first that we are studying a particular device or component of a system

Reliability is the probability that an item will operate without failure for a stated period of time under specified conditions

Since reliability is a probability it can only take values between 0 and 1.

We identify the reliability of an item with R.

The complement of the reliability is the unreliability F.

$$F = 1 - R$$

Unreliability → It is the probability that a component/system fails to work continuously over a stated time interval

The use of the words "without failure" in the definition of reliability or the term "continuously" in the definition of unreliability is not arbitrary. They imply that the concept of reliability can only be applied directly to systems or repairable items.

The terms that consider a system's or a repairable item's behaviour in normal operation and after a failure are "availability" and "unavailability"

The term "availability" can be used in different senses depending on the type of system or item

1) Availability (A) is the probability that a system/item works on demand → Definition appropriate for standby systems

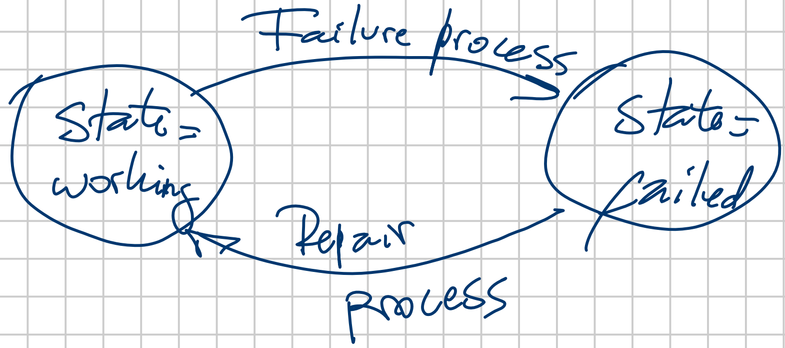
2) Availability (A(t)) is the probability that a system/item is working at a specific time t → Definition appropriate for continuously operating systems

3) Availability (A) is the expected portion of the time that a system or item performs its required function → Definition appropriate for repairable systems

Unavailability → It is the probability that a system or item does not operate at a time t
↳ U_a

$$A = 1 - U_a$$

Simple model for system behavior



- Reliability calculation:

$$F(t) = P[\text{a given item fails in } [0, t)) \qquad (1)$$

↳ continuous operation is implicit

It is a probability distribution with random variable t

The probability density function is

$$f(t) = \frac{dF(t)}{dt}$$

$$f(t)\,dt = P[\text{a given item fails in } [t, t+dt]) \qquad (2)$$

then $f(t)\,dt = F(t+dt) - F(t)$

or $F(t) = \int_0^{\tau} f(\tau)\,d\tau$

A hazard function $h(t)$ is created to characterize the transition to the failed state. $h(t)$ is the expected rate at which failures occur "given that"

$h(t)dt = P[$ an item fails between t and $t+dt$ / it has not failed until $t]$

Since $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

"A"

But any item that fails between t and $t+dt$ has not failed before so $P(A \cap B) = P(A)$. Hence,

"B"

$$h(t)dt = \frac{P[\text{component fails between } t \text{ and } t+dt]}{P[\text{no failure in } [0,t)]}$$

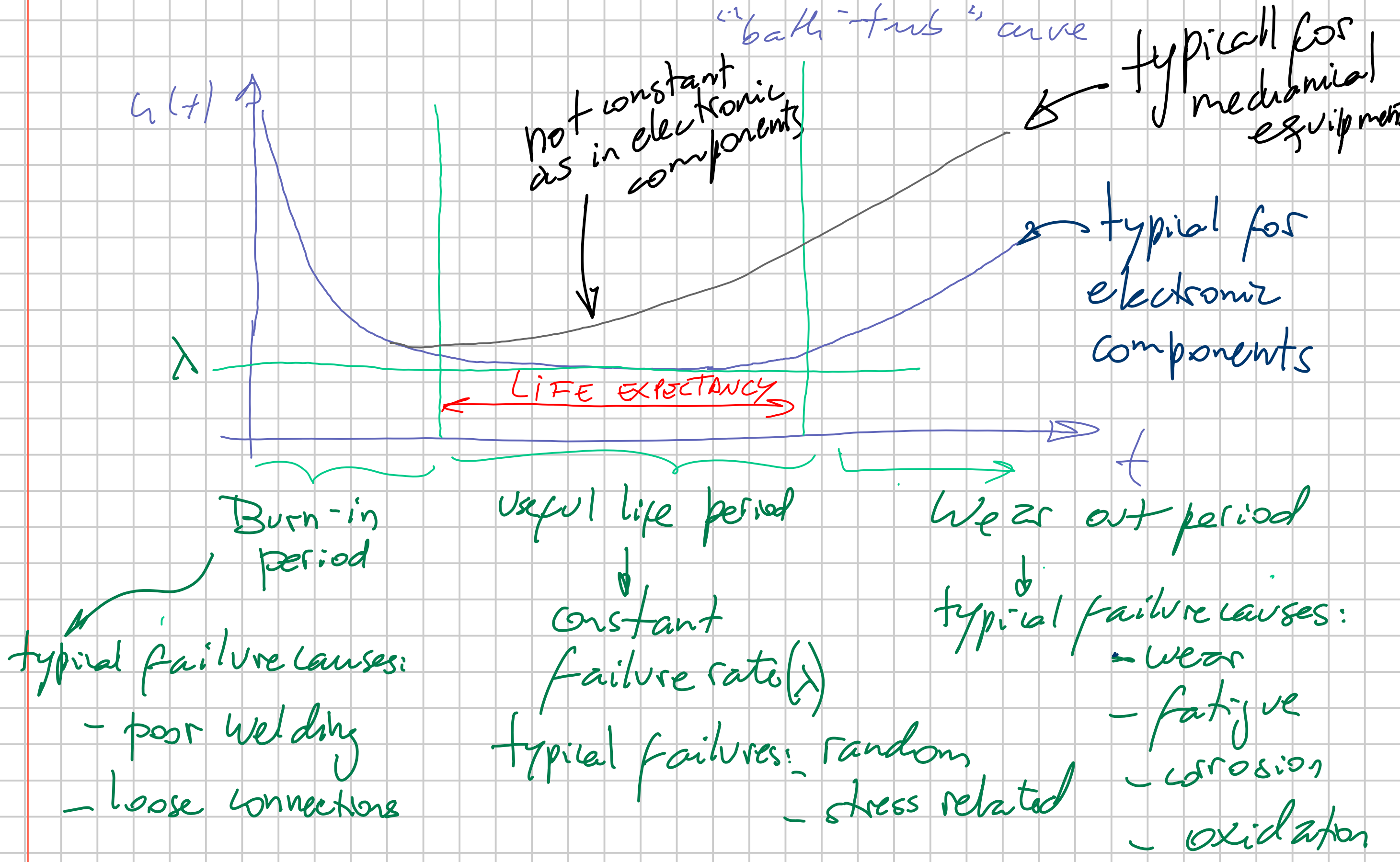
And, from (1) and (2)

$$h(t)\,dt = \frac{f(t)\,dt}{1-F(t)} \longrightarrow h(t) = \frac{f(t)}{1-F(t)}$$

$$\int_0^t h(t)\,dt = \int_0^t \frac{f(t)}{1-F(t)}\,dt = \int_0^t \frac{f(t)}{1-\int_0^t f(\tau)\,d\tau}\,dt$$

$$\boxed{F(t) = 1 - e^{-\int_0^t h(\tau)\,d\tau}}$$

Typical form for h(t):



With $h(t) = \lambda = \text{Constant}$ → units: $\frac{1}{sec}$ or $\frac{1}{hr}$ or $\frac{1}{year}$.

$$F(t) = 1 - e^{-\lambda t} \longrightarrow f(t) = \lambda e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}$$

the mean is $MTTF = \mu_T = \int_0^\infty t f(t)\,dt = \frac{1}{\lambda}$

Mean time to failure

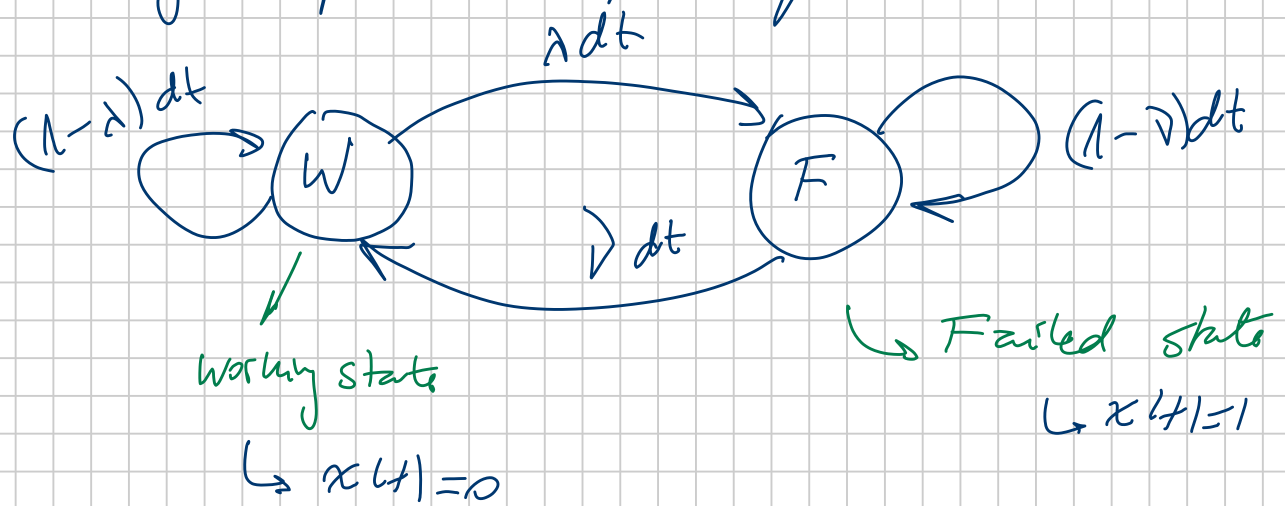
Now consider that an item or system can be repaired after it failed and be brought back into service

There are several ways to study the reliability behavior of a system or repairable item. A good one is by using Markov analysis because it provides a graphical depiction of the process and a flexible way of addressing different situations.

The equivalent of the hazard rate in repairable processes is the failure rate $\lambda(t)$

$$\lambda(t)\,dt = \frac{P[\text{component fails in } [t, t+dt])}{P[\text{component was working at } t=t]}$$

For the repair process, the equivalence to $\lambda(t)$ is the repair rate $\nu(t)$. A simple Markov's representation of a single repairable component process is:



$x(t) \rightarrow$ state.

The probability that the above item is in the failed state (unavailability) after dt is given by:

$P(x(t+dt)=1) = P[$item was working at $t=t$ AND undergoes failure during dt OR the component was failed at $t=t$ AND it wasn't repaired during $dt)$

$$\underbrace{P(x(t+dt)=1)}_{P_f(t+dt)} = \underbrace{P(x(t)=0)}_{P_w(t)} \lambda(t)dt + \underbrace{P(x(t)=1)}_{P_f(t)}(1-\nu(t)dt)$$

"AND" "OR" "AND"

$$P_f(t+dt) = P_w(t)\,\lambda(t)\,dt + P_f(t)(1-\nu(t)\,dt)$$

$$\underbrace{\frac{P_f(t+dt)-P_f(t)}{dt}} = P_w(t)\lambda - P_f(t)$$

$$\frac{dP_f(t)}{dt} = \dot{P}_f(t)$$

$$\frac{dP_f(t)}{dt} = \lambda P_w(t) - \nu P_f(t)$$

→ since $P_w(t) + P_f(t) = 1$ then $P_w(t) = 1 - P_f(t)$

$$\boxed{\frac{dP_f}{dt} = \lambda - (\lambda+\nu)P_f(t)}$$ → 1st order dif. eq.

I assume that it was initially working → initial cond: $P_f(0) = 0$

$$\boxed{P_f(t) = \frac{\lambda}{\lambda+\nu}\left(1 - e^{-(\lambda+\nu)t}\right)}$$

Assuming constant failure and repair rates

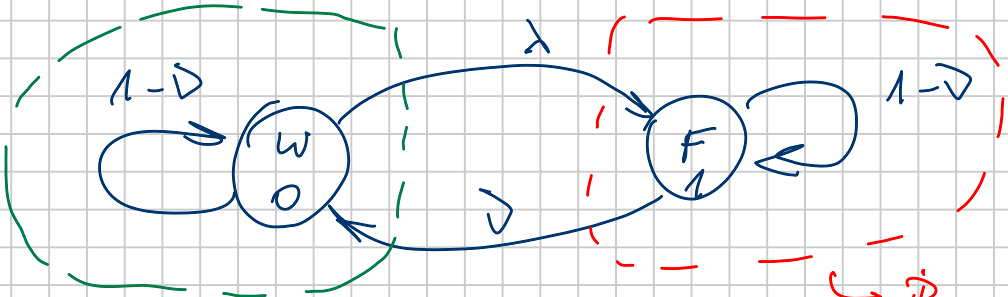
And since $P_w(t) = 1 - P_f(t)$ ⇝

$$\boxed{P_w(t) = \frac{1}{\lambda+\nu}\left(\nu + \lambda e^{-(\lambda+\nu)t}\right)}$$

We could have reached the dif. equations is from the following property of the diagram: ↘

$$\frac{dP_{state}}{dt} = (\text{Rate of entering the state}) - (\text{Rate of leaving the state})$$

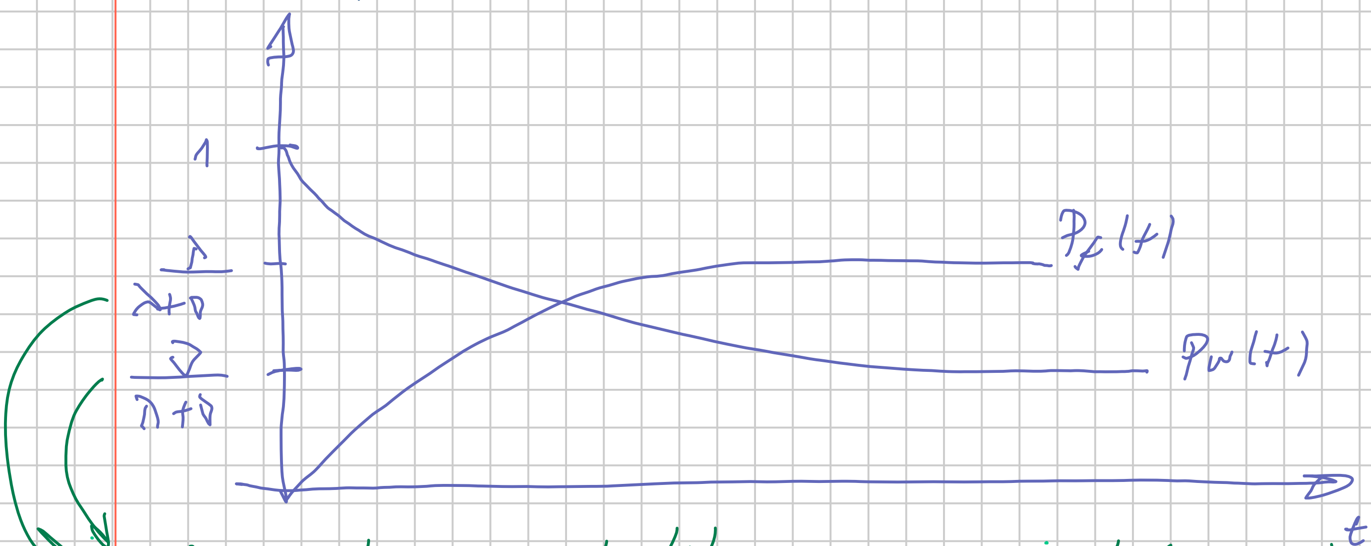
In terms of rates, Markov's diagram becomes:



$\dot{P}_F = \lambda P_W - \nu P_F$

$\dot{P}_W = \nu P_F(t) - \lambda P_W(t)$

If we plot $P_F(t)$ and $P_W(t)$ we obtain:



Steady state probabilities of being failed or working. I.e. how likely it is that after having been "there" for a long time the item is working or not.

↓

This fits the definitions of availability and unavailability.

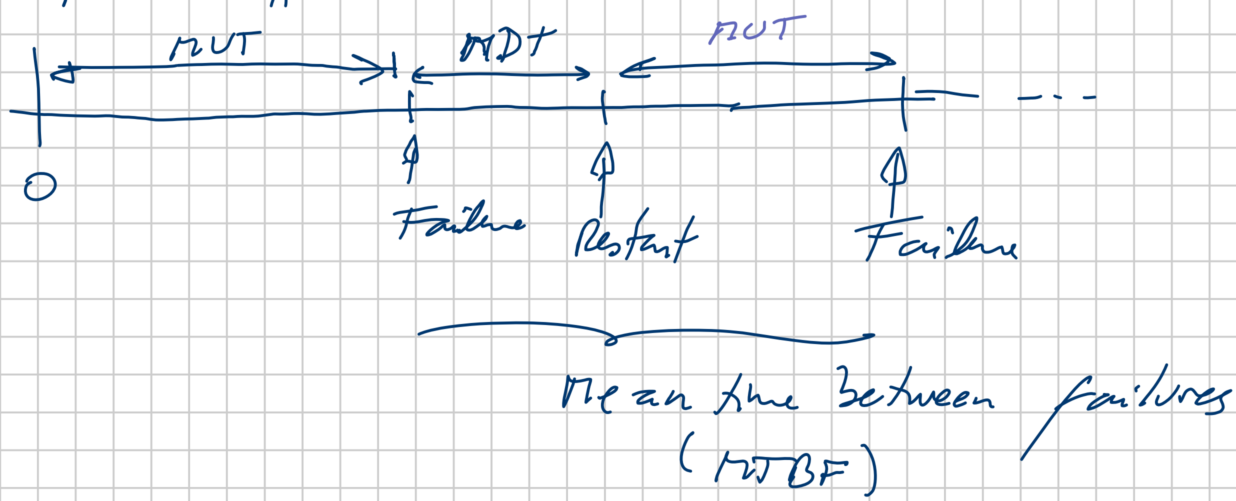
Hence, $A = \frac{\nu}{\lambda+\nu}$, $U_a = \frac{\lambda}{\lambda+\nu}$

In the same way that for the case when $h(t) = \lambda = \text{constant}$ we found out that $MTTF = \mu_F = \frac{1}{\lambda}$, now

For $\lambda(t) = \lambda = \text{constant}$ $\longrightarrow$ Mean up time $= MUT = \frac{1}{\lambda}$

For $\nu(t) = \nu = \text{constant}$ $\longrightarrow$ Mean down time $= MDT = \frac{1}{\nu}$

So the process goes like this



MDT includes : detection
- fault repair
- put the system back into service

The concepts of MUT, MDT and MTBF apply to repairable systems only

Notes: 1) $MTBF = MUT + MDT$

2) $MUT \neq MTTF$. When a system is restarted after it has been repaired, all the failed components may not necessarily have been repaired. The MUT characterizes the mean operating time until the next failure. The MTTF characterizes the mean operating time of a system which is entirely repaired (to new) before been restarted

$$A = \frac{\nu}{\nu + \lambda} = \frac{\frac{1}{MDT}}{\frac{1}{MDT} + \frac{1}{MUT}} = \frac{MUT}{MTBF}$$

$$U_a = \frac{MDT}{MTBF}$$

Reliability networks is another technique to calculate availability of systems with multiple components.

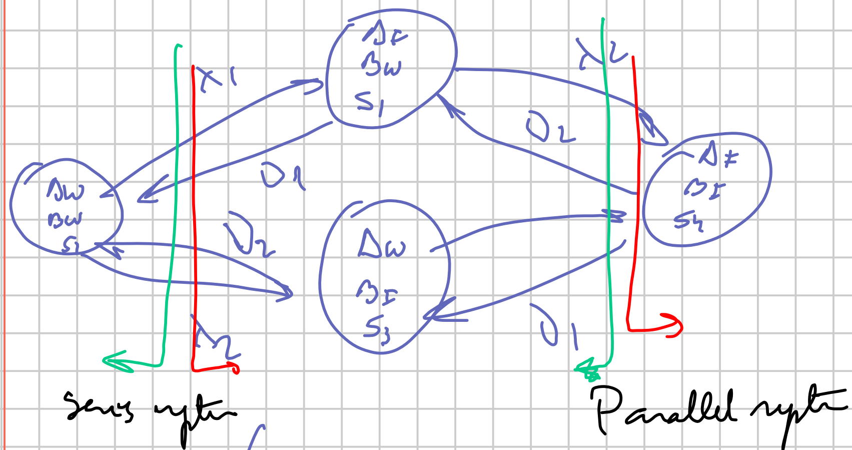
A reliability network is a representation of the reliability dependences between components of a system

The network has always the following features:

a) A starting node
b) An ending node
c) A set of nodes
d) A set of edges
e) An incidence function that associates each edge with an ordered pair of nodes

- The edges represent the components
- The nodes represent system architecture
- The expected operating condition of the system is represented by paths though the network.
  - ↳ If there is at least one path from the starting node to the ending node then the system is working
  - → If not the system has failed

Simple architectures:



– green → system working

– red → system failed

$$\dot{P}_S(t) = A\,(P_S\,t)$$

$$A = \begin{pmatrix} -(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 & 0 \\ \nu_1 & -(\nu_1+\lambda_2) & 0 & \lambda_2 \\ \nu_2 & 0 & -(\nu_2+\lambda_1) & \lambda_1 \\ 0 & \nu_2 & \nu_1 & -(\nu_1+\nu_2) \end{pmatrix}$$

$$P_{S_1}(t\to\infty) = \frac{\nu_1\nu_2}{(\nu_1+\lambda_1)(\nu_2+\lambda_2)} \qquad T_1 = \frac{1}{\lambda_1+\lambda_2}$$

$$P_{S_2}(t\to\infty) = \frac{\lambda_1\nu_2}{(\nu_1+\lambda_1)(\nu_2+\lambda_2)} \qquad T_2 = \frac{1}{\nu_1+\lambda_2}$$

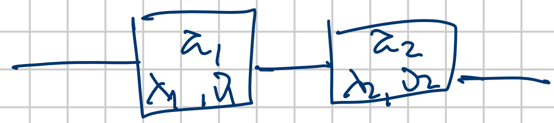
$$P_{S_3}(t\to\infty) = \frac{\lambda_2\nu_1}{(\nu_1+\lambda_1)(\nu_2+\lambda_2)} \qquad T_3 = \frac{1}{\nu_2+\lambda_1}$$

$$P_{S_4}(t\to\infty) = \frac{\lambda_1\lambda_2}{(\nu_1+\lambda_1)(\nu_2+\lambda_2)} \qquad T_4 = \frac{1}{\nu_1+\nu_2}$$

↓ Probability of being in each state

↓ Mean time in each state

a) Series system



For non-reparable n components

$$R_{sys}(t) = e^{-\lambda_{sys} t} = \prod_{i=1}^{n} R_i(t)$$

where → $\lambda_{sys} = \sum_{i=1}^{n} \lambda_i$ and $MTTF = \frac{1}{\lambda_{sys}}$

For 2 reparable components

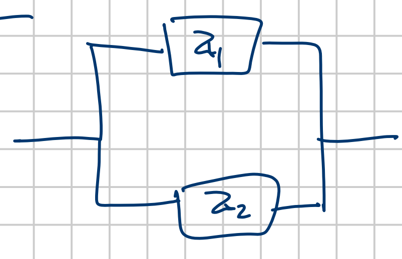
$$A = P_{S_1}(t \to \infty) = \frac{\nu_1 \nu_2}{(\nu_1 + \lambda_1)(\lambda_2 + \nu_2)} \quad \text{(see above)}$$

$$A = \frac{\nu_1 \nu_2}{(\nu_1 + \lambda_1)(\lambda_2 + \nu_2)} = \frac{\nu_1}{\nu_1 + \lambda_1} \cdot \frac{\nu_2}{\lambda_2 + \nu_2}$$

$$A = a_1 a_2$$

For n components → $A_{sys} = \prod_{i=1}^{n} a_i$

Parallel reparable systems



From above $U_a = P_{S_4}(t \to \infty) = \frac{\lambda_1 \lambda_2}{(\lambda_1 + \nu_1)(\lambda_2 + \nu_2)}$

Hence → $U_a = \frac{\lambda_1}{\lambda_1 + \nu_1} \cdot \frac{\lambda_2}{\lambda_2 + \nu_2} = (1 - a_1)(1 - a_2)$

For n components in //

$$U_{a\,sys} = \prod_{i=1}^{n} (1 - a_i)$$

<u>n+1 redundancy</u>

Suppose now that we have a modular system with a total power P_o and each module has are rated for P_m. Then, without redundancy we need

$$n = \left\lceil \frac{P_o}{P_m} \right\rceil$$

↳ The upper integer value of $\frac{P_o}{P_m}$

e.g. $P_o = 7kW$, $P_m = 2kW$ } $n = 4$

The problem with lack of redundancy is that if one module fails then there is not enough capacity to power the load.

With n+1 redundancy we provide 1 extra module from those needed. Then

$$n = \left\lceil \frac{P_o}{P_m} \right\rceil + 1$$

So with n+1 redundancy it is required that n of the n+1 modules work for full system operation

Then,

$A_{sys} = P(\text{system working}) =$

$= P[\text{n modules working}] + P(\text{n+1 modules working}] =$

OR

$= {}^{n+1}C_n \, a^n \, u_a + {}^{n+1}C_{n+1} \, a^{n+1}$

Binomial distribution

All possible arrangement of n+1 elements taken in groups of n where the order doesn't matter to distinguish among arrangements

a → availability of each module

u_a → unavailability of each module

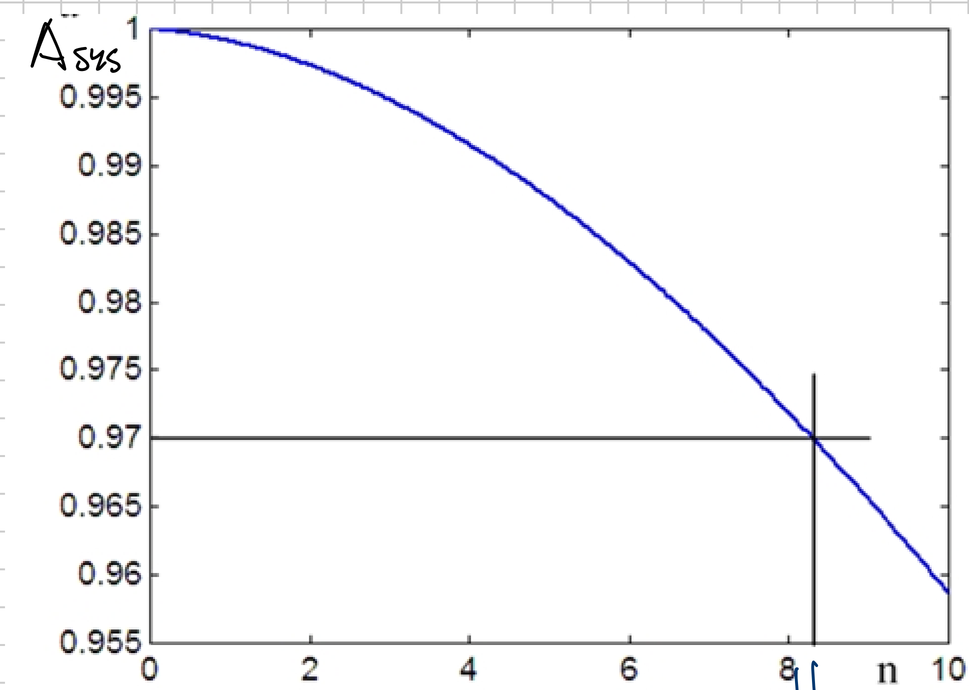
I can think of the process as having n trials and requiring k or more successes for the system to work

Recall that $^{n+1}C_n = \frac{(n+1)!}{(n+1-n)!\,n!} = \frac{(n+1)!}{n!} = n+1$

So $A_{sys} = \left((n+1)u_a + a\right)a^n = \left((n+1)(1-a) + a\right)a^n$

Is (n+1) redundancy always better than other options?

Consider a fuel cell with a = 0.97 and variable number of modules:



$A_{sys} > a$ ← | → $A_{sys} < a$

So as the number of modules increase the system availability decreases.

So the extra redundant capacity represented by the "+1" is less

The best reliability option is "1+1"

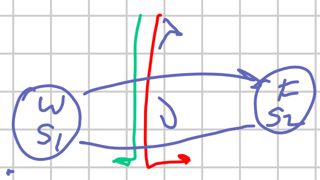
this extra capacity has it cost (\$/kW module)

parallel → But modules are larger and the extra capacity is very large (equals the load)

One option to improve economics is to use the extra capacity to power something else other than the load. For example the extra power can be injected back into the grid and in this way economics are improved.

One useful summary table.

Non-Repairable component		Repairable component
$R(t) = e^{-\lambda t}$	⟷	$A(t) = \frac{\lambda}{\lambda+\nu}\left(1 - e^{-(\lambda+\nu)t}\right)$
$F(t)$	⟷	$U_a(t) = \frac{1}{\lambda+\nu}\left(\nu + \lambda e^{-(\lambda+\nu)t}\right)$
$\mu = MTTF$	⟷	MUT → $MUT = MTTF$ only if all components are repaired
Considers all components repaired → $\tau = MTTR$	⟷	MDT
NOTHING	⟷	$MTBF$
$\lambda = \frac{1}{MTTF}$	⟷	$\lambda = \frac{1}{MUT}$ → Failure rate
$\nu = 0$	⟷	$\nu = \frac{1}{MDT}$ → repair rate
$R(t=\infty) = 0$	⟷	$A(t=\infty) = \frac{\nu}{\lambda+\nu}$
$F(t=\infty) = 1$	⟷	$U_a(t=\infty) = \frac{\lambda}{\lambda+\nu}$; $U_a \overset{\text{def not}}{=} Q$
NOTHING	⟷	see diagram below



$P_{S_1}(t) = A(t)$

$P_{S_2}(t) = U_a(t)$

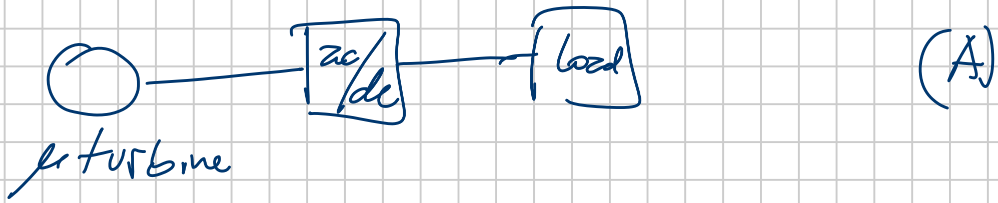
$\vec{P}^{\,t} = \vec{P}^{\,t}\mathbb{A} \rightarrow \mathbb{A} = \begin{pmatrix} -\lambda & \lambda \\ \nu & -\nu \end{pmatrix}$

$s_1 = 0$, $s_2 = -(\lambda+\nu)$

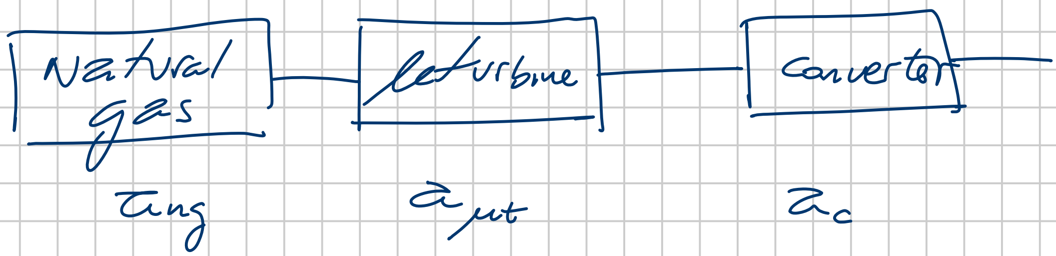
Next classes) material

(draft)

Based on these techniques we can calculate a microgrid availability. For example, let's consider the following microgrid:



The availability can be calculated with the following diagram



$$A_{sys} = a_{ng}\, a_{\mu t}\, a_c$$

What if we have a more complicated structures we can use Markov analysis or we can use the concept of paths in a reliability network:

a) *Path set*: A list of edges such that if they all work, then the system is also in the working state, i.e., any path between the start node and the end node.

b) *Minimal path set*: A path set such that if any one item is removed, the system will no longer work, i.e., any given path between the start node and the end node assuming that all other paths are interrupted due to at least one failed component.

c) *Cut set*: A list of components such that if all fail then the system is also in the failed state.

d) *Minimal cut set (K)*: A cut set such that if any one item is removed from the list, the system will no longer fail. The probability that a minimal cut set will occur is given by

$$\mathrm{P}(K) = \prod_{i=1}^{N_k} u_i$$

where u_i is the unavailability of the i-th edge of the N_k components in the minimal cut set K.

For a system with repairable components, the unavailability can be calculated from [277]

$$U = \mathrm{P}\left(\bigcup_{j=1}^{M_c} K_j\right)$$

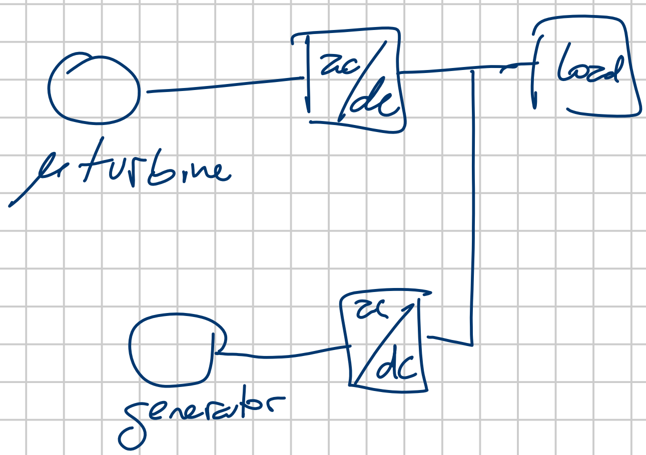
where Mc is the number of minimal cut sets in the system. Calculation of (B.6) is usually extremely tedious. However, the calculation can be simplified by recognizing that U is bounded by

$$\sum_{i=1}^{M_c} \mathrm{P}(K_i) - \sum_{i=2}^{M_c} \sum_{j=1}^{i-1} \mathrm{P}(K_i \bigcup K_j) \le U \le 1 - \prod_{i=1}^{M_c} [1 - \mathrm{P}(K_i)] \le \sum_{i=1}^{M_c} \mathrm{P}(K_i)$$

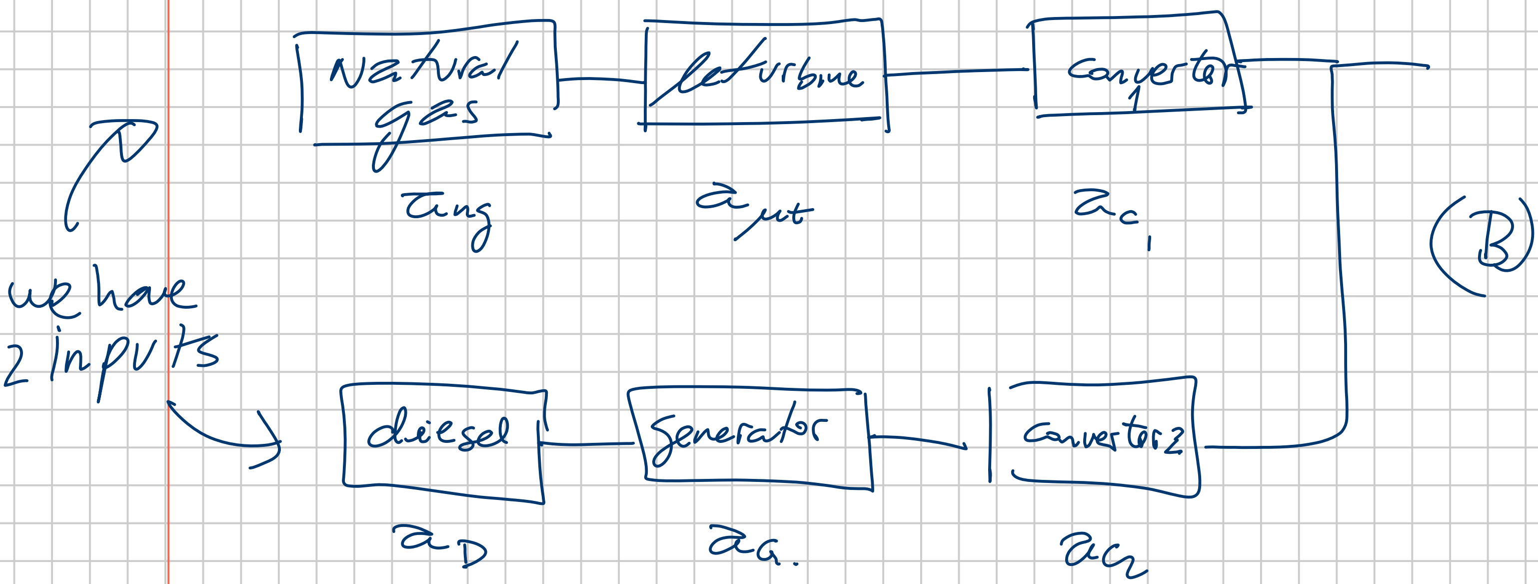
Thus, if the components are highly available, i.e., q_i << 1, then U can be approximated to

$$U \cong \sum_{j=1}^{M_c} \mathrm{P}(K_j)$$

So let's consider the following microgrid:



So the reliability network is:



Minimal cut sets: $k_1: \{ng, D\}$, $k_2: \{\mu t, D\}$, $k_3: \{C_1, D\}$

$k_4: \{ng, G\}$, $k_5: \{\mu t, G\}$, $k_6: \{C_1, G\}$

$k_7: \{ng, C_2\}$, $k_8: \{\mu t, C_2\}$, $k_9: \{C_1, C_2\}$

So $U \simeq \sum_{i=1}^{9} P(k_i)$

where $P(k_1) = u_{ng} u_D$

So, how do we know the values of the different unavailabilities?

From different sources.

Item and origin of the value	MTTF/MUT* (Hours)	MDT** (hours)	Availability a
Reciprocating Engine	823	5	0.9939
PV arrays ****	3636	14	0.996
Fuel Cell (performance degradation)	5000	166.6	0.967742
Microturbine	8000	50	0.993789
Wind turbine ****	1900	80	0.9595
ac mains	2440	2.08	0.999150
Diesel / Gas	2 M	50	0.999975

*MUT: Mean up-time (used for repairable system components)

**MDT: Mean down-time (only applicable to repairable components)

***NR: Not repairable

****Operational MUT and MDT depend on the actual energy availability

For the converters we can calculate the availability by estimating the MDT and by calculating the MUT=MTTF from

$$MTTF = \frac{1}{\lambda_{conv}}$$

where λ_{conv} can be calculated by considering that from a reliability perspective all components are in series. Hence,

$$\lambda_{conv} = \sum_{i=1}^{n} \lambda_i$$

→ each component i

The values of λ_i can be obtained from the nominal values:

Part Description	λ_{hG} (FIT)
Resistor	0.5
Capacitor Ceramic	1.0
Capacitor Tantalum	5.0
Diode	6.0
Transistor	6.0
Coil	19.0
MOSFET	20.0
IC (20 Transistors)	19.0

→ From reliability prediction handbooks such as Telcordia SR-232 and MIL-HDBK-217

Information from:

J. Kippen. "Evaluating the Reliability of DC/DC Converters." Sept. 2003,

(Unfortunately no longer available in Internet)

affected by temperature and electrical stress (e.g. voltage levels).

$$\lambda_{comp} = \lambda_n \pi_Q \pi_T \pi_E$$

π_Q → Production quality → usually = 1

λ_n → nominal value

π_T → temperature factor

π_E → electrical stress

π_T → Temperature factor → Arrhenius rate model

$$\pi_T = e^{\frac{E_a}{k}\left(\frac{1}{T_R} - \frac{1}{T_S}\right)}$$

- T_S → stress temp
- T_R → Reference temperature
- k → Boltzman constant $8.167 \cdot 10^{-5}$ eV/K
- E_a → failure activation energy → depends on failure mechanism
  - ↳ e.g. 0.6 eV

Calculation of π_E:

Part Description	Stress Level 25%	70%	80%
Resistor	0.72	1.30	1.48
Capacitor Ceramic	0.36	2.27	3.42
Capacitor Tantalum	0.23	3.25	5.87
Diode	0.48	2.01	2.85
Transistor	0.30	2.61	4.22
Coil	1.00	1.00	1.00
MOSFET	0.55	1.62	2.05
IC (25 Transistors)	1.00	1.00	1.00
IC (70 Transistors)	1.00	1.00	1.00
IC (150 Transistors)	1.00	1.00	1.00
Optocoupler	1.00	1.00	1.00

Final note: Fault tolerant strategies (to avoid single point of failures):

- Redundancy: Having more of the minimum number of the same system components
- Diversity: Having multiple paths
- Distributed systems: Spread a critical function