ECE2795, Power electronic interfaces

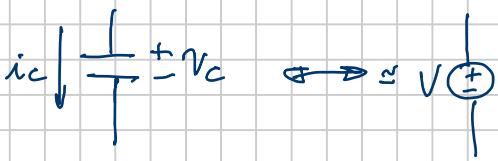
Note Title (v. 2/2015)

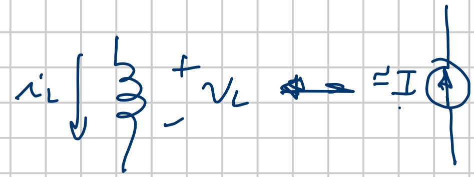
References:

P. T. Krein. *Elements of Power Electronics*. New York, NY, U.S.A.: Oxford University Press, 1998.

Fundamental concepts:

1) Energy storage $\Rightarrow$ state variables




$i_C = C\frac{dv_C}{dt}$ $\downarrow$ "Voltage source"

$i_L = L\frac{dv_L}{dt}$ $\downarrow$ "current source"

2) Operation regime

2 Regimes: - Transient
- Steady state

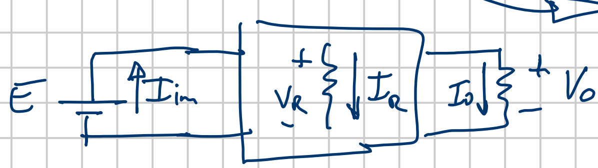
- In steady state

The sum of energy exchanges over one period is zero

$$\sum W\big|_T = 0$$

$\Downarrow$ Riemman's sum

$$\frac{1}{T}\int_0^T v(t)\, i(t)\, dt = 0$$




$$-E I_{in} T_{in} + V_R I_R T_R + V_o I_o T_o = 0$$

Implications: Inductance

$$i_L \simeq I_L \Rightarrow \bar{v}_L = \frac{1}{T}\int_0^T v_L(t)\,dt = 0$$

over one period the average voltage in the inductor is zero. If not, then $\sum W\big|_T \neq 0$ so the inductor has been charged or discharged

Capacitor

$v_C \simeq V_C \Rightarrow \bar{i}_C = \frac{1}{T}\int_0^T i_C(t)\,dt = 0$

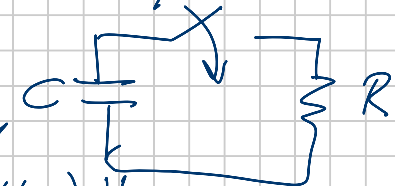
over one period the average current in the capacitor is zero. If not, then $\sum W|_T \neq 0$ and the capacitor has been charged or discharged

3) Time constants $\longrightarrow$ Fast and slow dynamics

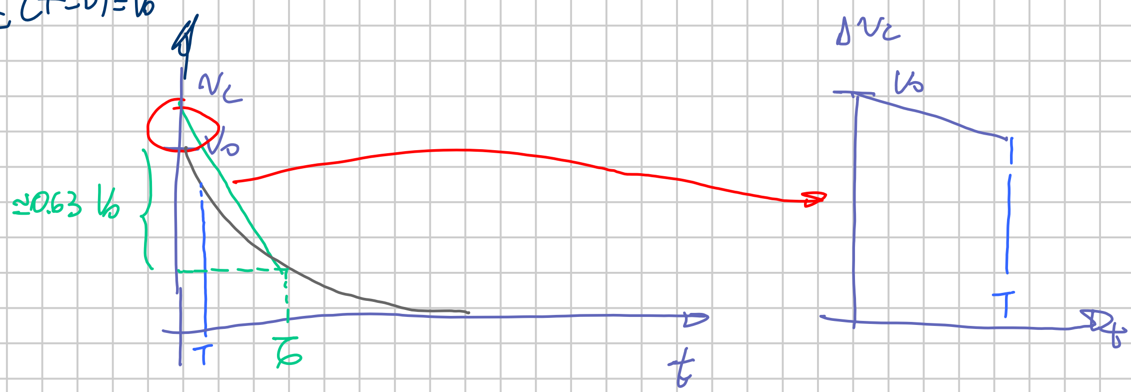
 $\longrightarrow RL \Rightarrow \tau = \frac{L}{R}$

 $\rightarrow RC \Rightarrow \tau = RC$

Fast dynamics:




$v_C(t=0) = V_0$




With fast dynamics: $T \ll \tau$

$\hookrightarrow$ we focus on the voltage change during a short interval

So the exponential decay can be approximated to a linear decay

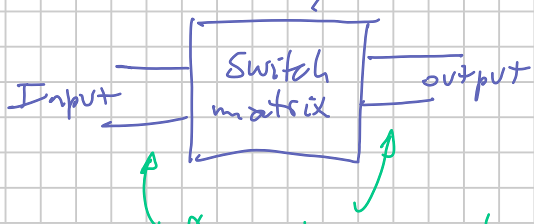
4) "Thou shall not try to violate KVL and KCL"

$\Downarrow$

Hence $\rightarrow$ it is usually not a good idea to connect capacitors in parallel or inductors in series

5) Switch matrix

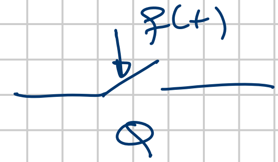
$\hookrightarrow$ It is made of a connection of at least one or more switches with at least an input and an output port,

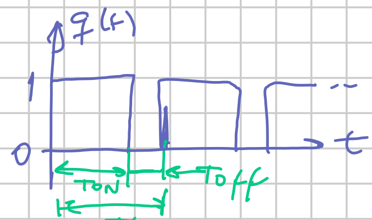



$\rightarrow$ combination of all switch states defines the matrix state: ON $\Rightarrow$ 1, OFF $\Rightarrow$ 0

A matrix converter has storage elements only at the input & output.

Control of switches is characterized by switching functions



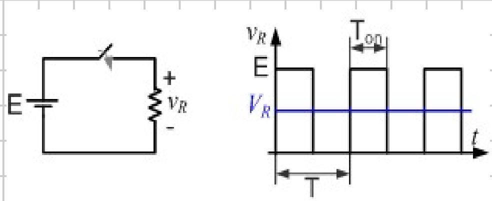

$\hookrightarrow f_{sw} = 1/T \rightarrow$ switching frequency

$\hookrightarrow T \rightarrow$ switching period

Duty cycle D $\longrightarrow D = \frac{1}{T}\int_0^T q(t)\,dt = \frac{1}{T}\int_0^{T_{on}} 1\,dt$

($\hookrightarrow$ average of $q(t)$)

$$\boxed{D = \frac{T_{on}}{T}}$$




$\rightarrow$ The dc voltage on the resistor is $V_R = ED$

## 6) Switch selection

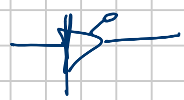
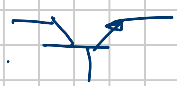
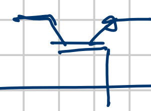
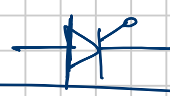
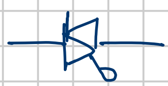
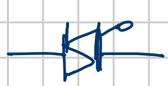
Switch caracterization is based on 2 criteria:

2 criteria → Conduction characteristics ?
Current direction and intensity
⇓
Forward vs. bidirectional conduction

→ Blocking characteristics ?
Voltage polarity and magnitude
⇓
Forward, reverse & bidirectional blocking

Forward direction ↓ + / − : If it can stay open with this polarity ↓ Forward Blocking (FB)

− / + : If it can stay open with this polarity ↓ Reverse Blocking

| Action | Simbol | Device |
|---|---|---|
| FCRB |  | Diode  |
| FCFB |  | BJT <br>IGBT  |
| FCBB |  | GTO  |
| BCFB |  | FET  |
| BCBB |  |  Ideal switch |

Also FCBB.

Also BCFB FCBB

## 7) Filter: 2 Approaches → Fourier (harmonics are ripple)

→ Linear approximation (Using concept #3)

$$V_L = L\frac{\Delta I_L}{\Delta t}, \quad I_C = C\frac{\Delta V_C}{\Delta t}$$

Let's see some definitions

$$\overline{x}(t) = \frac{1}{T}\int_0^T x(t)\,dt \longrightarrow \text{Average value}$$

$$x(t)_{RMS} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} \longrightarrow \text{rms value}$$

Instantaneous Active power $\longrightarrow p(t) = v(t)\,i(t)$

Average power $\Rightarrow P = \frac{1}{T}\int_0^T p(t)\,dt$

$$THD = \sqrt{\frac{\sum_{n=2}^{\infty} C_n^2}{C_1^2}} \longrightarrow x(t) = x(t+T) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$\eta = \frac{P_{OUT}}{P_{in}}$$

$$\%\,\text{Line regulation} = 100\,\frac{V_{OUT}\big|_{HIGHEST\ INPUT} - V_{OUT}\big|_{LOWEST\ INPUT}}{V_{OUT\,NOMINAL}}$$

$$\%\,\text{Load regulation} = \begin{cases} 100\,\dfrac{V_{OUT}\big|_{NO\ LOAD} - V_{OUT}\big|_{FULL\ LOAD}}{V_{OUT}\big|_{NO\ LOAD}} \\[2ex] 100\,\dfrac{V_{OUT}\big|_{MIN\ LOAD} - V_{OUT}\big|_{FULL\ LOAD}}{V_{OUT}\big|_{NOMINAL}} \end{cases}$$

Power factor $\rightarrow p.f. = \dfrac{\text{Average power}}{\text{Apparent power}} = \dfrac{\frac{1}{T}\int_0^T p(t)\,dt}{V_{RMS}\, I_{RMS}}$

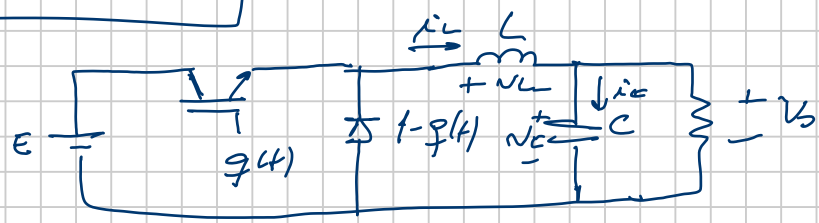
If we only have one harmonic then

$$\frac{1}{T}\int_0^T p(t)\,dt = P = V_{RMS}\, I_{RMS} \cos\varphi$$

and $p.f. = \cos\varphi$

# Modeling and analysis of dc-dc converters

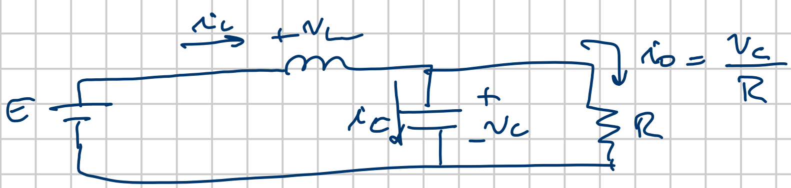
## Buck converter:




$i_L = x_1$

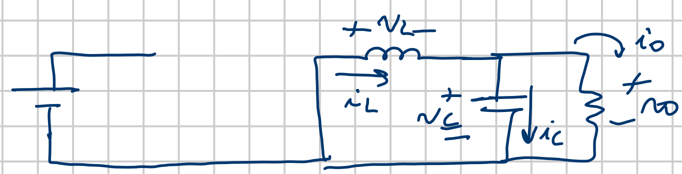
$v_C = x_2$

$g(t)=1$




$$\begin{cases} L\dot{x}_1 = E - x_2 \\ C\dot{x}_2 = x_1 - x_2/R \end{cases}$$

$g(t)=0$

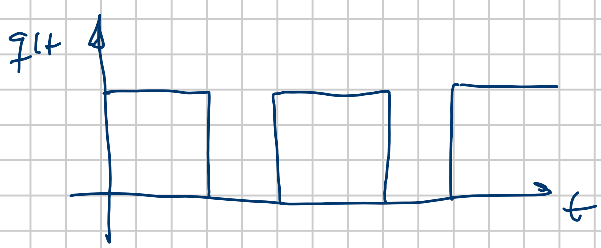



$$\begin{cases} L\dot{x}_1 = -x_2 \\ C\dot{x}_2 = x_1 - x_2/R \end{cases}$$

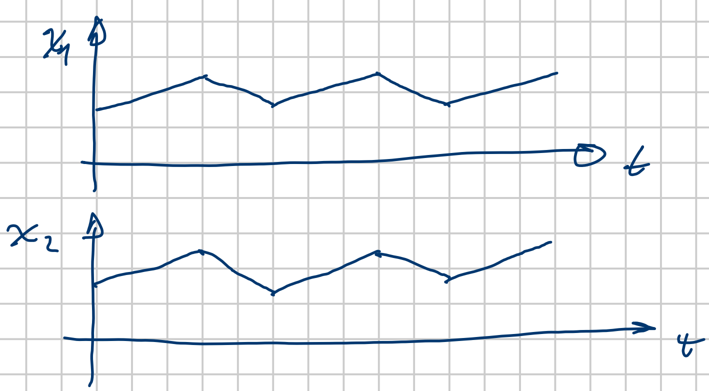
Hence

$$\begin{cases} L\dot{x}_1 = g(t)E - x_2 \\ C\dot{x}_2 = x_1 - \frac{x_2}{R} \end{cases} \quad (1)$$

(1) $\longrightarrow$ Switched system dynamic eqs.




Steady state



Apply the fast average operator to (1)

Fast average operator $\rightarrow \bar{f}(t) = \dfrac{1}{T_{sw}} \displaystyle\int_t^{t+T_{sw}} f(t)\,dt$

Laplace

Linear operator

$$\bar{F}(s) \propto \frac{F(s)}{s}$$

Low pass filter

Original representation:

$$\begin{cases} L\dot{x}_1 = g(t)E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$$

Fast average operator

$$\begin{cases} L\dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} \dot{x}_1\,dt = E\dfrac{1}{T_{sw}}\displaystyle\int_t g(t) - \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} x_2\,dt \\ C\dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} \dot{x}_2\,dt = \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} x_1\,dt - \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} \dfrac{x_2}{R}\,dt \end{cases}$$

$$\begin{cases} L\dot{\bar{x}}_1 = \bar{d}(t)E - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases} \longrightarrow \text{Fast average model}$$

If the duty cycle is fixed and constant then
$\bar{d}(t) = D$

and

$$\begin{cases} L\dot{\bar{x}}_1 = DE - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

Notice that: $\overline{v_L} = L\,\dot{\overline{x}}_1$ and $\overline{i_c} = C\,\dot{\overline{x}}_2$

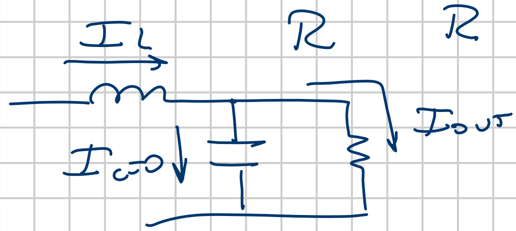
So, in steady state and since $\overline{v_L} = 0$ and $\overline{i_c} = 0$ we have that

$$\overline{v_L} = 0 = DE - \overline{x_2} \Longrightarrow \overline{x_2} = V_o = DE$$

$$\boxed{D = \frac{V_o}{E}}$$

And, from $i_c = 0$,

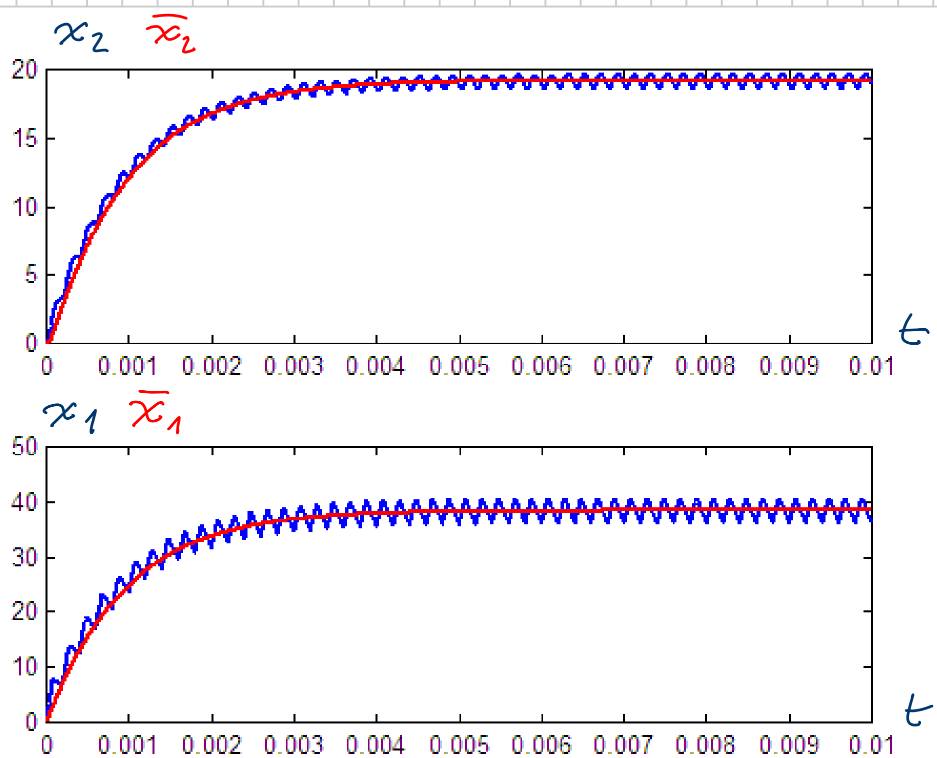
$$\overline{x_1} = I_L = \frac{\overline{x_2}}{R} = \frac{V_o}{R} = I_{out}$$




$I_L = I_{out}$

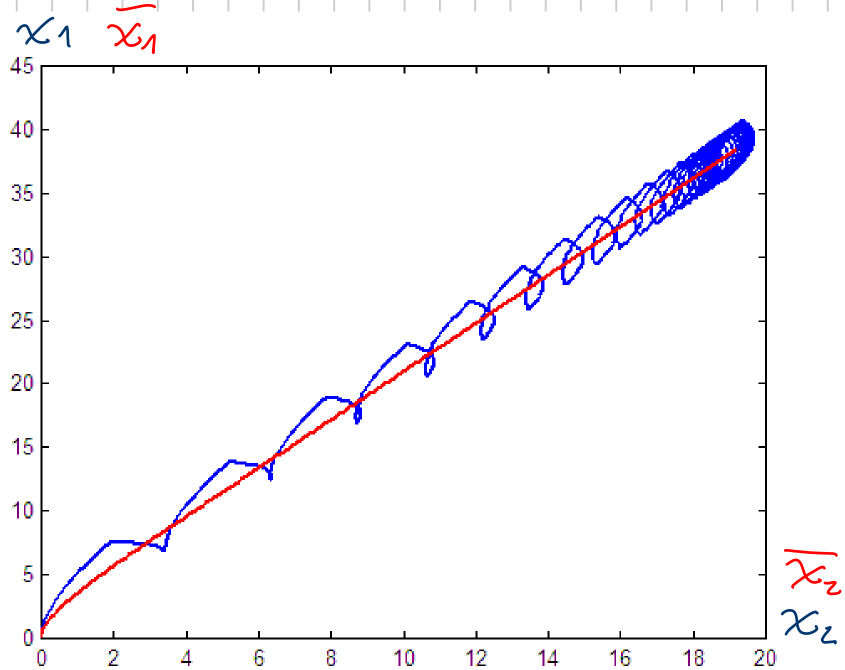
and $I_c = 0$

average current in the capacitor is zero

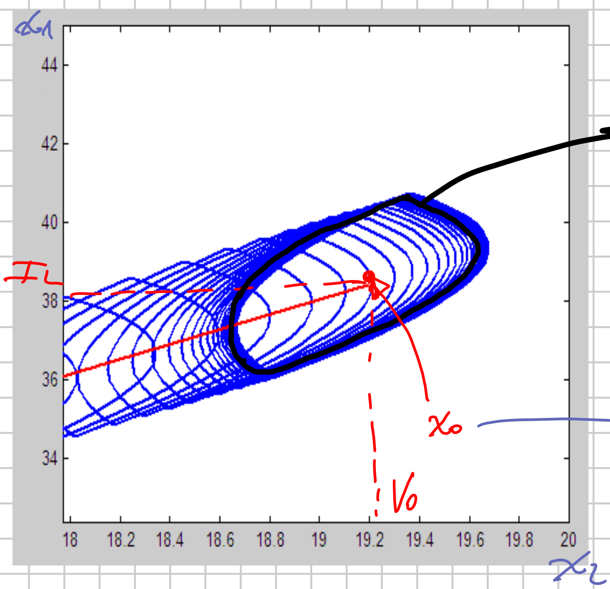



↳ Time domain representation from simulating a buck converter with

$D = 0.4$, $E = 48V$, $R = 0.5\,\Omega$, $L = 500\mu F$, $C = 100\mu F$




state space representation
(phase portrait)




Limitcycle

(1) does not lead to an equilibrium point

Equilibrium point only achieved in average.

↳ The operation of a buck converter (and for that matter, any dc-dc converter) consists on a continuous operation between two transient trajectories

The key assumption is to consider $T_{sw}$ small enough (or $f_{sw}$ large enough) so the exact behavior approximates the fast average behavior

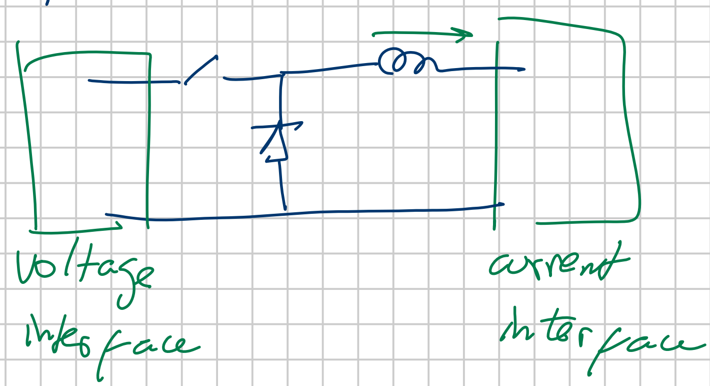
As $f_{sw}$ increases, the limit cycle gets smaller, towards the equilibrium point.

Notice that with $L$ large enough, $i_L$ will be almost constant so without a capacitor $v_R = R i_R = R i_L \simeq R I_L$

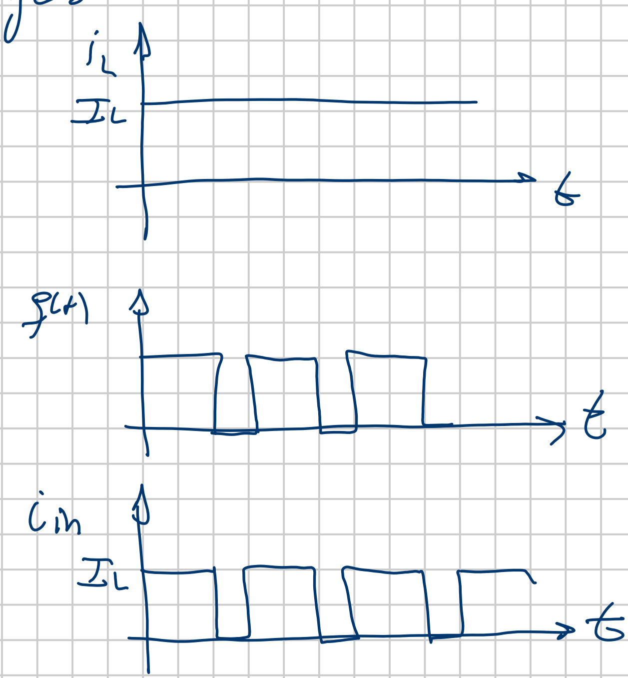
$$\downarrow$$

$$v_R \simeq V_o \rightarrow \text{constant}$$

So the output capacitance may not be needed. The reason is that the buck converter has a voltage-source input and a current-type output:

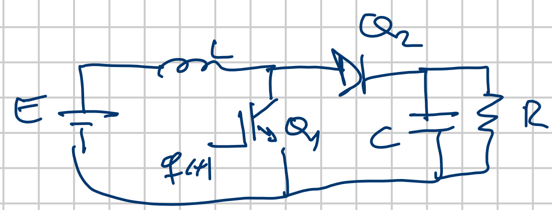



Although this configuration may work for some sources, such as PV modules, for others, such as fuel cells, is not suitable, because the current input has abrupt and deep changes:



For sources that require relatively constant current output a boost converter may be a better option:

## Boost converter




where $q'(t) = 1 - q(t)$

$q(t) = 1$
$$\begin{cases} L\dot{x}_1 = E \\ C\dot{x}_2 = -\dfrac{x_2}{R} \end{cases}$$

$q(t) = 0$
$$\begin{cases} L\dot{x}_1 = E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$$

$$\begin{cases} L\dot{x}_1 = E - q'(t)\, x_2 \\ C\dot{x}_2 = q'(t)\, x_1 - \dfrac{x_2}{R} \end{cases}$$

Fast average issue $\longrightarrow$ $\dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} q'(t)\, x_i\, dt \neq \underbrace{\dfrac{1}{T_{sw}}\int_t^{t+T_{sw}} q'(t)dt}_{D} \underbrace{\int_t^{t+T_{sw}} x_i\, dt}_{\bar{x}}$

e.g. $x_i = At$

$$\overline{(x_i q(t))} = \frac{1}{T_{sw}}\int_t^{t+T_{sw}} At\, q(t)dt = \frac{A}{2} D \left(2t + D T_{sw}\right)$$

$$\bar{x}_i\, d(t) = \left(\frac{1}{T_{sw}}\int_t^{t+T_{sw}} At\right)\left(\frac{1}{T_{sw}}\int_t^{t+T_{sw}} q(t)dt\right) = \frac{DA}{2}\left(2t + T_{sw}\right)$$

So for $T_{sw}$ very small there is no problem (again, the assumption is that $f_{sw}$ is large enough)

So, for high switching frequency $\left(f_{sw} = \frac{1}{T_{sw}}\right)$.

$$\begin{cases} L\dot{\bar{x}}_1 = E - d'\bar{x}_2 \\ C\dot{\bar{x}}_2 = d'\bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

Again, for a fixed and constant duty cycle D:

$$\begin{cases} L\dot{\bar{x}}_1 = E - D'\bar{x}_2 \\ C\dot{\bar{x}}_2 = D'\bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases} \qquad \text{where } D' = 1-D$$

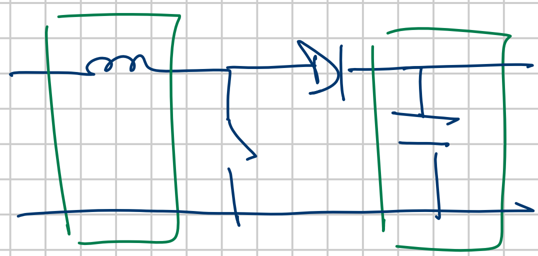
And in steady state $\bar{v}_L = L\dot{\bar{x}}_1 = 0$. So

$$0 = E - D'\bar{x}_2 \Rightarrow \bar{x}_2 = V_o = \frac{E}{D'} = \frac{E}{1-D}$$

And since $\bar{i}_C = 0$, then

$$\bar{x}_1 = I_L = \frac{\bar{x}_2}{D'R} = \frac{V_o}{D'R} = \frac{I_o}{D'}$$

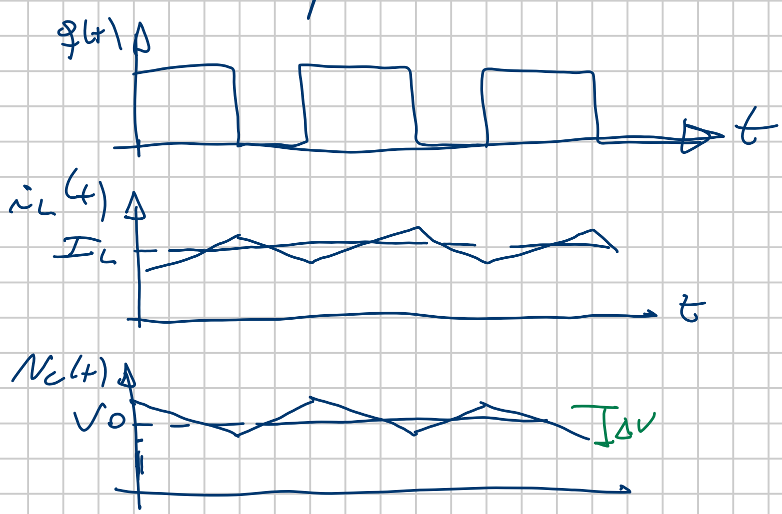
Now, the configuration of a boost converter is current-source input / voltage-source output



current source (with L large enough)

voltage source (with C large enough)

Boost converter main waveforms:



Notice that when $q(t) = 1$, $i_c(t) = i_o \simeq I_O = \frac{V_o}{R}$

So, $i_c(t) = C\frac{dV_c}{dt}$

$$\downarrow$$

$$i_c(t) \simeq C\frac{\Delta V_c}{\Delta t}$$

$$\frac{V_o}{R} \simeq C\frac{\Delta V_c}{\Delta t} = C\frac{\Delta V_c}{D\,T_{sw}} = C\frac{\Delta V_c}{D} f_{sw}$$

$$\Delta V_c = \frac{1}{RC} D\,T_{sw}\,V_o = \frac{V_o D}{RC f_{sw}}$$

Since $RC = \tau$ then $\Delta V_c = \frac{T_{sw}}{\tau} D\,V_o = \frac{V_o D}{\tau f_{sw}}$

So $\underset{\sim}{\Delta V_c}$ is lower as $T_{sw} << \tau$

voltage ripple

(see concept (3) on page 2 above)

or, what is the same

For the same $C$, $\Delta V_c$ is smaller as $f_{sw}$ is higher.

So size of components (specially energy storage devices) can be reduced as $f_{sw}$ is increased

Issues with the boost converter:

When $D = 1 \rightarrow i_L \rightarrow \infty$ and $V_c \rightarrow 0$

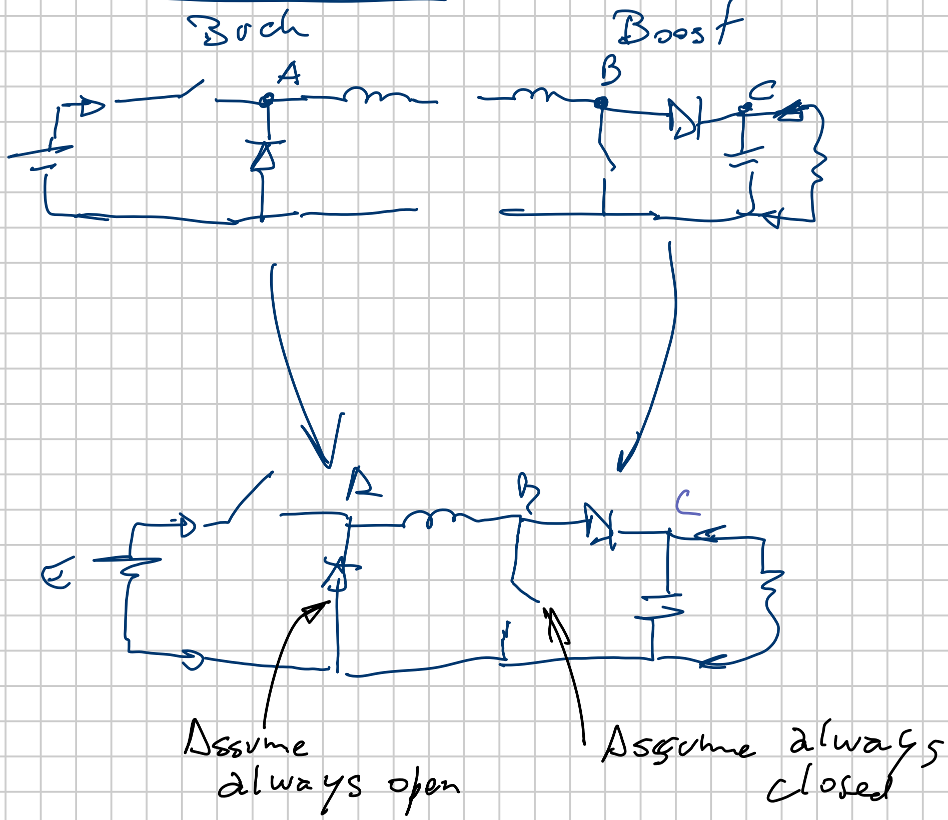
I can blow something

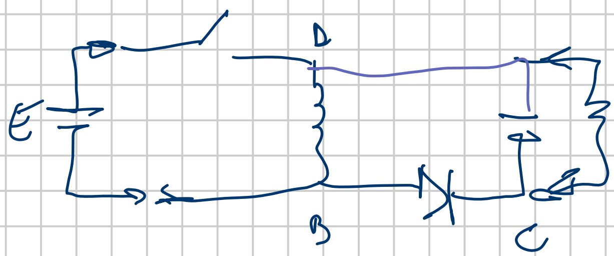
When $R \rightarrow \infty$ then $V_c \rightarrow \infty$

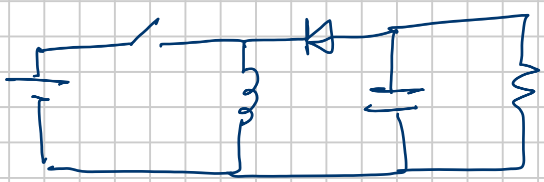
I can also blow something

No load operation

# Buck-Boost converter

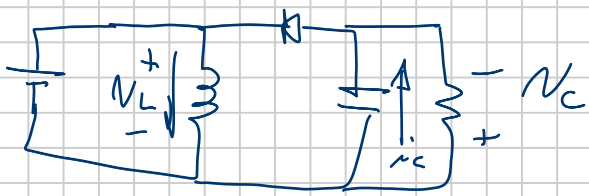


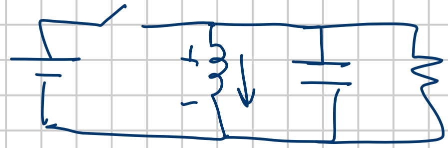
Buck-boost converter

When $q=1$




$$\begin{cases} L\dot{x}_1 = E \\ C\dot{x}_2 = -x_2/R \end{cases}$$

When $\varsigma = 0$



$$\begin{cases} L\dot{x}_1 = -x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$$

Thus

$$\begin{cases} L\dot{x}_1 = \varsigma(t)\,E - (1-\varsigma)\,x_2 \\ C\dot{x}_2 = (1-\varsigma)\,x_1 - \dfrac{x_2}{R} \end{cases}$$

$$\Big\downarrow \text{ Fast average operator}$$

$$\begin{cases} L\dot{\bar{x}}_1 = d(t)\,E - d'(t)\,\bar{x}_2 \\ C\dot{\bar{x}}_2 = d'(t)\,\bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

Again, for a fixed and constant duty cycle $D$:

$$\begin{cases} L\dot{\bar{x}}_1 = DE - D'\bar{x}_2 \\ C\dot{\bar{x}}_2 = D'\bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

In steady state:

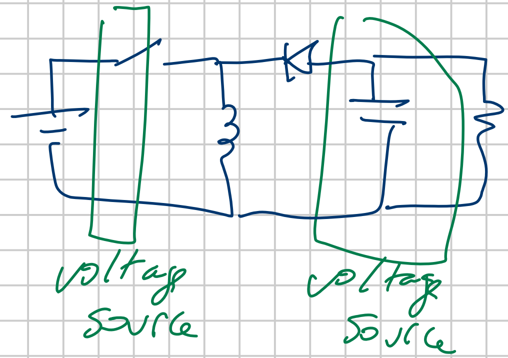
$$0 = DE - D'\bar{x}_2 \qquad \bar{x}_2 \longrightarrow V_o \longrightarrow \frac{V_o}{E} = \frac{D}{D'} = \frac{D}{1-D}$$

$$0 = I_L D' - I_o \longrightarrow I_L = \frac{I_o}{D'} = \frac{I_{in}}{D}$$

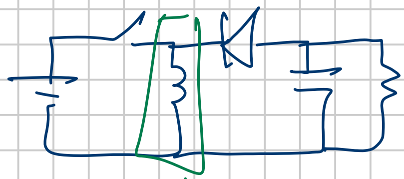
The buck-boost converter has the same issues than the boost converter with respect to $D=1$

and no load operation

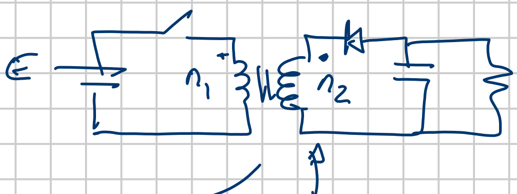
Notice that the buck-boost has a voltage source interface on both the input and the output:




The flyback converter can be derived from the buck-boost.



Let's use a coupled inductor instead




They have galvanic isolation

$$DE\frac{n_2}{n_1}$$

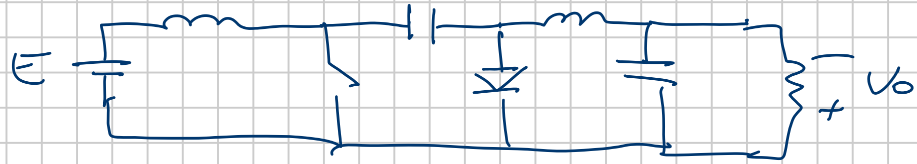
$$DE\frac{n_2}{n_1} = -(1-D)V_o$$

$$\frac{V_o}{E} = \frac{D}{1-D}\underbrace{\frac{n_2}{n_1}}$$

→ coupled inductors turns ratio

## Other useful topologies

- Cuk converter (Boost-Buck converter)




current source interface

current source interface (the capacitor is redundant)

$$\frac{V_o}{E} = \frac{D}{1-D}$$

Sepic converter


current source interface

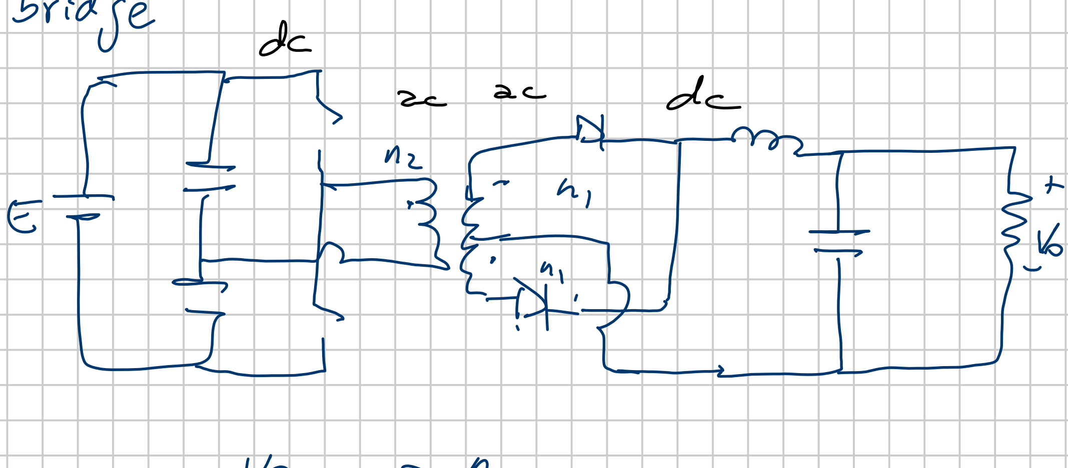
voltage source interface

$$\frac{V_o}{E} = \frac{D}{1-D}$$

- Forward converters

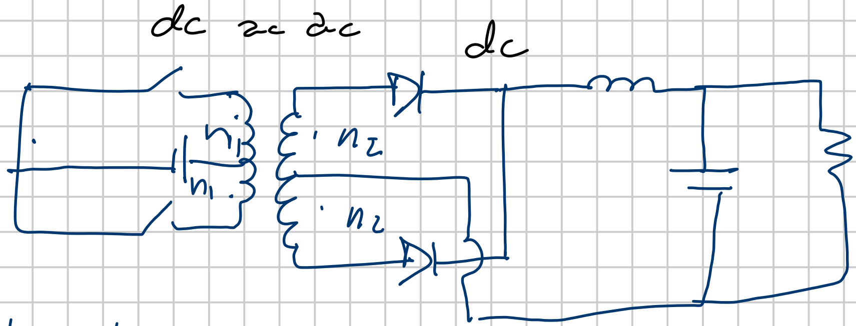
↳ they naturally have galvanic isolation

Half bridge




$$\frac{V_o}{E} = D\,\frac{n_2}{n_1}$$

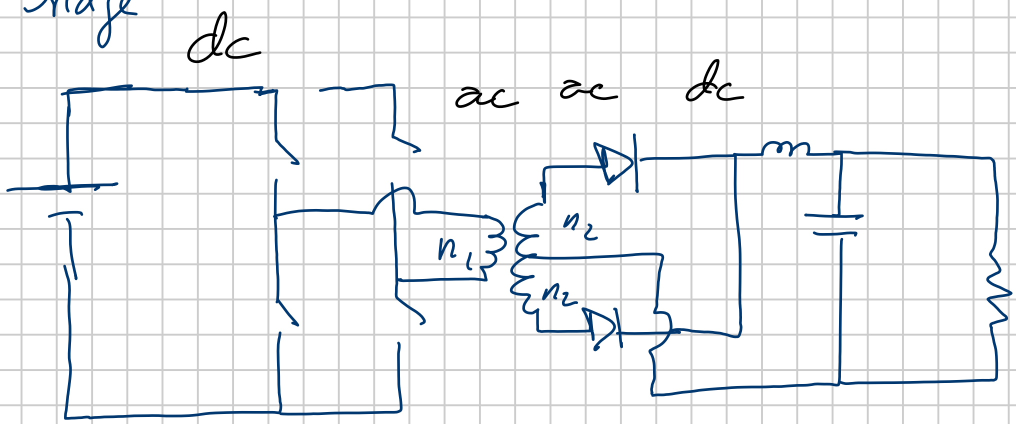
- Push pull




Traditional voltage-source configuration

$$\frac{V_o}{E} = 2\frac{n_2}{n_1} D$$

- Full bridge




Transformer

traditional voltage source configuration

$$\frac{V_o}{E} = 2\frac{n_2}{n_1} D$$