COMMUNICATION WITH MULTIPLE SENDERS: AN EXPERIMENT

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ABSTRACT. We implement multi-sender cheap talk in the laboratory. While full-information transmission is not theoretically feasible in the standard one-sender one-dimension model, in this setting with more senders and dimensions, full revelation is generically a robust equilibrium outcome. Our experimental results indicate that fully revealing outcomes are selected in particular settings, but that partial-information transmission is the norm. We uncover a number of behavioral patterns: Senders follow exaggeration strategies, qualitatively similar to those predicted by a special case for the fully revealing equilibrium. Receivers on the other hand follow differing heuristics depending on the senders' biases, which are not always sequentially rational. When full revelation is observed it it can be explained as the intersection of the receiver heuristics with the equilibrium response.

1. INTRODUCTION

In many strategic situations decision-makers must rely on experts' advice to make informed choices. A tension arises when the decision-maker's preference over final outcomes is misaligned with the experts'. A theoretical literature on strategic information transmission, starting from Crawford and Sobel (1982), shows that with just a single informed expert strategic motives can substantially reduce information transfer. With multiple experts a new challenge for the decision-maker emerges, resolving the potentially conflicting advice. However, multiple experts also creates an opportunity, as decision makers may be able to fully extract all the available information from experts by comparing their advice. Through a series of laboratory experiments, we examine whether such fully revealing outcomes are selected, and how human subjects respond to conflicting advice. While the underlying tensions we investigate are present in the field—judicial proceedings, congressional testimony, consumers making large purchases, healthcare, *etc*—a controlled experiment allows us to isolate the strategic tensions, with data and exogenous variation not commonly available outside the laboratory.

Battaglini (2002) constructs a fully revealing equilibrium with multiple senders over a multi-issue policy, and our experiment will examine this particular equilibrium. The setup is as follows: A decision-maker (DM) needs to make a choice on a multi-issue policy, but she is uncertain over the best choice. However, she consults with a number of experts, who are perfectly informed on the ideal policy, but are known to be biased. Experts independently provide their recommendations to

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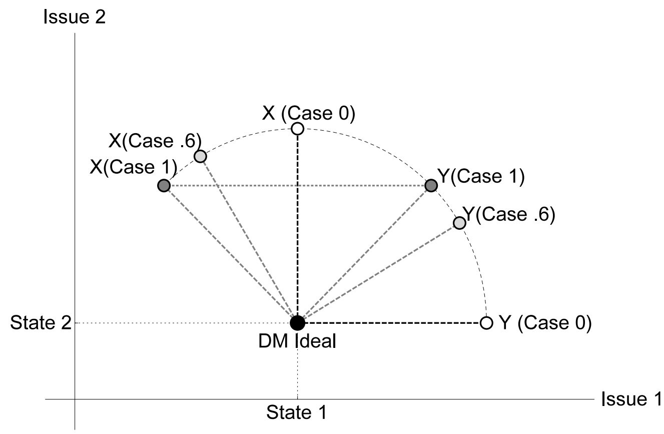


FIGURE 1. Experimental Treatments

her, and the DM then makes a policy choice. Battaglini shows by construction that an equilibrium generically exists where the DM combines the experts' recommendations and infers her best policy.

Consider the simplest case for the equilibrium construction: two experts provide guidance for a two-issue policy, with preferences as illustrated by *Case 0* in Figure 1. Here the realized state of the world, representing the DM's ideal choice for the two-issue policy, is given by the solid black point, and the ideal points for the experts are given by the two white points. In the *Case 0* environment, expert X is known to have an ideal policy coincident with the DM on issue 1, but biased upward on issue 2. Conversely, expert Y's ideal policy on issue 2 is coincident with the decision maker, but he is biased to the right on issue 1. While the DM does not know the realized state, she does know the relative position of each expert's ideal policy in relation to her own—the experts' biases. In the fully revealing equilibrium, the DM combines recommendations from the two experts, following expert X on issue 1, and expert Y on issue 2. Given the DM's reaction to their recommendations, each expert only has influence over the final choice on a single issue, and conditional on the other expert truthfully revealing, it is a best response for each to truthfully reveal themselves.

Battaglini (2002) shows that the intuitive construction in *Case 0* generalizes. For a simple illustration, consider experts' relative ideal policies in *Case 1*, labeled with dark-gray points in Figure 1. This environment is simply a rotation of the *Case 0* coordinate system, and the equilibrium construction can be similarly rotated. In fact, the equilibrium construction works whenever we can find a coordinate system that mirrors the alignment over issues in *Case 0,* reconstructing the equilibrium in the new basis. However, while the general case and *Case 0* share a common structure in different coordinate systems, the equilibrium intuition viewed within the original coordinate system is quite different. The construction in *Case 1* requires the DM to combine and compare recommendations from *both* experts in *both* canonical issues, while in contrast, *Case 0* requires no inference across issues.

Our experiment implements the two-expert, two-issue game in the laboratory using a modified state space (a pair of circles), where our treatment variable is the experts' bias directions. Achieving full revelation in our environment is certainly challenging, as the decision-maker must select the one correct point out of 129,600 possibilities, which are equally likely ex-ante. But, in the treatment that matches *Case 0*, our data is qualitatively close to a fully revealing outcome, with just under a quarter of DM choices exactly coincident with the true state, and many more close by. Moreover, the component strategies for a majority of subjects in both the expert and DM roles form a mutual best response.

However, we also document conditions under which full revelation is not achieved. Rotating the experts' biases, as presented in *Case 1* and the intermediate *Case .6*, as in Figure 1, the match to full revelation disappears. The observed failure in these cases does not seem to be driven by equilibrium selection. Subjects in experts' roles have a consistent pattern of response across all treatments. The large majority of experts can be classified as providing recommendations that add an *exaggeration* to the true state in the direction of their own self interest. Refining the equilibrium set by restricting experts to linear exaggeration strategies, we show that the only equilibria consistent with experts' exaggeration patterns is fully revealing. Given experts' behavior, a sequentially rational DM will use strategies similar to those predicted by the fully revealing equilibrium. Yet, DM choices are only consistent with the equilibrium response in *Case 0*, and the amount of information extracted across treatments is stochastically ordered by the rotation—with greater extraction in *Case 0* than *Case .6*, and *Case .6* than *Case 1*.

Further analysis of DMs' response to recommendations provides an explanation for the failure: subjects treat the canonical issues independently, as if responding in two unrelated single-issue problems. Where the equilibrium (and sequentially rational) response in *Cases .6* and *Case 1* makes substantial use of across-issue information, our estimations show subjects ignore it completely. In contrast, the receiver response within-issue is fairly sophisticated, where subjects show a significant reaction to experts' biases, with behavior clustered around the unidimensional best-response. Subjects react strategically to the incentives, demonstrating they understand the environment and tensions within issue; their failure is in not understanding the connections between the distinct choice dimensions.

A positive description of receivers' response is as follows: In issues where the two experts' ideal points lie on opposite sides of the DM's ideal, final choices are a weighted-average of the recommendations, with greater weight placed on experts with smaller biases within the issue. In issues where the expert's ideal points lie on the same side of the DM's, a common response is to identify the recommendation that seemed to be the least exaggerated, and then shade a fixed amount from it. In fact, the success for the fully revealing prediction in *Case 0* can be explained in terms of these within-issue heuristics: all the weight on one sender/shading nothing from the minimum.

The failures in the rotated environments and success in *Case 0* suggest room for policy interventions to select fully revealing outcomes through framing. Whenever it is possible for the policy-maker to control the terms/framing of the debate, inducing a *Case-0*-like environment might lead to gains in information transmission. Moreover, because the behavioral responses in *Case-0* form a mutual best response for the experts and DM, full revelation is likely to be a robust long-run outcome.

1.1. **Literature.** Our paper is part of a larger experimental literature examining cheap talk and strategic information transmission. This literature starts with a number of studies examining the one-sender, one-dimension Crawford and Sobel (1982) environment.[1] Though experimental data

[1]Prominent examples are: Dickhaut, McCabe, and Mukherji (1995); Blume, DeJong, Neumann, and Savin (2002); Cai and Wang (2006); Kawagoe and Takizawa (2009); Wang, Spezio, and Camerer (2010). Outside of the standard Crawford and Sobel framework, Gneezy (2005) documents a strong behavioral aversion to sending deceptive messages in a simple sender-receiver game, making comparisons to behavior in a dictator game with similar payoffs (see also Ottaviani and Squintani, 2006; Kartik, 2009; Kartik, Ottaviani, and Squintani, 2007, for theoretical treatments). Chung

follows the broad comparative statics over the the best-case equilibrium predictions (partial revelation of information, decreasing in the sender's bias magnitude), subjects over-communicate relative to equilibrium, with excessive truth-telling and naive response by receivers. Our paper instead studies the case with two senders within a two-dimension environment, where a fully revealing equilibrium exists. Our focus is on how receivers combine senders' recommendations, and whether fully revealing outcomes are selected.

Recent experimental papers have begun to expand from the one-sender–one-receiver setting to include additional senders and receivers.[2] Closest to our paper is the independent work of Lai, Lim, and Wang (forthcoming). Their focus is the ideas in Ambrus and Takahashi (2008), who show that Battaglini-type fully revealing equilibria may not exist when the state-space is bounded, as sender messages can lead to out-of-equilibrium inferences. In a baseline treatment similar to the *Case 0* setting above, Lai, Lim, and Wang find evidence consistent with full revelation. Their main treatments manipulate the set of available states and messages, thereby changing the possibility for senders to force out-of-equilibrium inferences (messages implying an impossible state). They find significantly lower revelation where these out-of-equilibrium inferences are possible. In contrast, our paper examines environments where out-of-equilibrium inferences cannot be forced. Our concentration is on examining whether or not the same fully revealing equilibrium is selected, and the extent to which receivers incorporate the available information as the frame shifts.

The paper is structured as follows: Section 2 outlines the underlying theory; Section 3 presents our experimental design; the analysis is carried out in Section 4; finally, Section 5 discusses the results, after which we conclude.

2. THEORY

In this section we first describe the general environment and then briefly describe the fully revealing equilibrium constructed in Battaglini (2002) when the state-space is a subset of $\mathbb{R}^n$.[3] Second, we introduce our experimental environment, which alters the state-space over which the theory is constructed. Finally, we illustrate a version of the Battaglini construction, and show how the main behavioral intuition—uninformed parties take a weighted average of the informed parties' recommendations on each issue, but then add a penalizing term based on the degree of their divergence in other issues—is common to both $\mathbb{R}^n$, and our own experimental environment.

2.1. **Setup and Battaglini (2002) Construction.**

The game has three players: two senders, X and Y, and a decision-maker/receiver Z. Nature chooses a state of the world $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ according to a commonly known distribution G, and the realization is perfectly observed by X and Y, however

and Harbaugh (2012) experimentally examine persuasive cheap talk where senders have state-independent preferences (for theory cf. Chakraborty and Harbaugh, 2010) finding broad support for the theoretic predictions.

[2]Battaglini and Makarov (2014) experimentally examine the tensions when those providing information must cater to multiple audiences. While observing excessive honesty in some senders, they do find that when facing multiple audiences senders react strategically. An additional sender in one dimension (with some additional uncertainty on the preferences) is analyzed in Minozzi and Woon (2011) and a companion paper to the present one, Vespa and Wilson (in preparation), while Evdokimov and Garfagnini (2014) examine communication/centralization between elements of the same firm. Plott and Llewellyn (2014) study an environment with two biased senders and two issues, where the eventual decision is voted on by a committee of five. The preferences of the committee members differ and each member votes after receiving the public advice from the informed experts.

[3]Readers interested in the fuller development of the theoretical information transmission literature are referred to the following papers and references therein: Crawford and Sobel (1982); Gilligan and Krehbiel (1989); Krishna and Morgan (2001); Battaglini (2002); Levy and Razin (2007); Ambrus and Takahashi (2008).

the decision maker Z is uninformed. The state-space $\mathbf{\Theta}$ is 2-dimensional, where each dimension $j = 1, 2$ represents an issue over which a decision must be made. Each sender decides on a message to send to the receiver, $\mathbf{x} \in \mathcal{M}_X, \mathbf{y} \in \mathcal{M}_Y$. The decision maker observes the message pair $(\mathbf{x}, \mathbf{y})$ and selects a final decision $\mathbf{z} \in \mathbf{\Theta}$. Preferences for each player $i \in \{X, Y, Z\}$ are defined through a state-dependent ideal point, $\boldsymbol{\theta} + \boldsymbol{\delta}^i$, with preferences over the final decision $\mathbf{z}$ represented by the utility function $u_i(\mathbf{z}, \boldsymbol{\theta}; \boldsymbol{\delta}^i) = -\left\|\boldsymbol{\theta} + \boldsymbol{\delta}^i - \mathbf{z}\right\|$, so that preferences are strictly decreasing in the Euclidean distance between the final decision $\mathbf{z}$ and the ideal point. The receiver's ideal point is normalized to be the state, so $\boldsymbol{\delta}^Z = \mathbf{0}$, while the senders' relative ideal points are defined by two common-knowledge vectors, $\boldsymbol{\delta}^X, \boldsymbol{\delta}^Y \in \mathbf{\Theta}$, the *biases* for senders X and Y. The timing of the game is as follows: i) the state of the world $\boldsymbol{\theta}$ is drawn and observed by the senders; ii) X and Y simultaneously send messages $\mathbf{x}$ and $\mathbf{y}$, respectively; iii) Z observes $(\mathbf{x}, \mathbf{y})$, and selects $\mathbf{z}$, after which payoffs are realized.

Strategies for the two senders are functions $\xi_X : \mathbf{\Theta} \to \Delta\mathcal{M}_X$ and $\xi_Y : \mathbf{\Theta} \to \Delta\mathcal{M}_Y$, from states to probability distributions over messages, while a strategy for the receiver is a function $\zeta : \mathcal{M}_X \times \mathcal{M}_Y \to \Delta\mathbf{\Theta}$, taking observed message pairs into a probability distribution over final decisions. The belief $\mu_Z(\boldsymbol{\theta} \,|\mathbf{x}, \mathbf{y})$ for the decision-maker reports a posterior distribution over $\mathbf{\Theta}$ for each pair of messages in $\mathcal{M}_X \times \mathcal{M}_Y$. A *fully revealing equilibrium* (FRE) is a Perfect Bayesian equilibrium, a strategy triple $\{\xi_X^\star, \xi_Y^\star, \zeta^\star\}$ and a conditional belief $\mu_Z^\star$ updated according to Bayes' rule, which for all $\boldsymbol{\theta} \in \mathbf{\Theta}$ satisfies $\zeta^\star(\xi_X^\star(\boldsymbol{\theta}), \xi_Y^\star(\boldsymbol{\theta})) = \boldsymbol{\theta}$ with probability one.

Battaglini examines an n-dimensional version of the above model where the state-space is $\mathbb{R}^n$ and the senders' utility functions are quasi-concave at the point $\mathbf{z} = \boldsymbol{\theta}$. He constructs a fully revealing equilibrium using two supporting hyperplanes with slopes $\boldsymbol{\gamma}^X$ and $\boldsymbol{\gamma}^Y$, which strongly separate the upper-contour sets at full revelation. So for sender i, the set $\Theta_i(\boldsymbol{\theta}) := \{\mathbf{z} \,|u_i(\mathbf{z}, \boldsymbol{\theta}) \geq u_i(\boldsymbol{\theta}, \boldsymbol{\theta})\}$ is separated by the hyperplane going through the point $\boldsymbol{\theta}$ with slope $\boldsymbol{\gamma}^i$. The condition for the existence of an FRE in this construction is that the hyperplanes $\boldsymbol{\gamma}^X$ and $\boldsymbol{\gamma}^Y$ (or lower-dimensional components, thereof) form a basis for $\mathbf{\Theta}$. The construction has each sender provide a message $\xi_i(\boldsymbol{\theta})$, revealing the hyperplane through the true state that separates the other sender's upper contour set $\Theta_j(\boldsymbol{\theta})$, $j \neq i$. The message sent is therefore synonymous with sending the set-valued message

$$\xi_i(\boldsymbol{\theta}) = \left\{\mathbf{z} \,\middle|\, \exists \kappa \in \mathbb{R} \text{ such that } \mathbf{z} = \kappa \cdot \boldsymbol{\gamma}^j + \boldsymbol{\theta}\right\}$$

with certainty.

The final choice by the receiver is simply the unique intersecting point between the two hyperplane messages, so $\zeta^\star(\mathbf{x}, \mathbf{y}) = \mathbf{x} \cap \mathbf{y}$. Given the receiver's strategy and sender j's revelation that the state lies on the hyperplane $\xi_j(\boldsymbol{\theta})$, so sender i is constrained to deviations resulting in a final choice somewhere on the hyperplane revealed by j. But, by construction, $\xi_j(\boldsymbol{\theta})$ separates the set of points that i prefers to full revelation. Revealing that the state lies on the hyperplane $\xi_i(\boldsymbol{\theta})$ is therefore a best response, leading to the final choice $\zeta^\star(\xi_X(\boldsymbol{\theta}), \xi_Y(\boldsymbol{\theta})) = \xi_X(\boldsymbol{\theta}) \cap \xi_Y(\boldsymbol{\theta}) = \boldsymbol{\theta}$, and an FRE.

2.2. **Exaggeration Equilibria.** In the Battaglini construction of the FRE messages are hyperplanes, however, in our experimental environment we will force senders' messages to be specific points in the state space, so that $\mathcal{M}_X = \mathcal{M}_Y = \mathbf{\Theta}$. Given that our message-space is the same as our state-space, we will refer to each message as a "recommendation." We will now present an alternative version of the Battaglini construction using this message space, which we will refer to as an exaggeration equilibrium. Though the mathematical underpinnings of our exaggeration construction are not substantially different from the geometric construction in Battaglini (2002), one advantage of our approach is in providing a framework to test broad patterns in sender response within our data, and a decomposition through which to understand the FRE prediction for receiver's response, and analyze the observed behavior.

Senders' recommendation strategies can always be decomposed to

$$\xi_X(\boldsymbol{\theta}) = \boldsymbol{\theta} + \tilde{\mathbf{x}}(\boldsymbol{\theta}),$$
$$\xi_Y(\boldsymbol{\theta}) = \boldsymbol{\theta} + \tilde{\mathbf{y}}(\boldsymbol{\theta}),$$

where $\tilde{\mathbf{x}}(\boldsymbol{\theta})$ and $\tilde{\mathbf{y}}(\boldsymbol{\theta})$ are exaggerations, random vectors added to the true state by the senders X and Y, respectively. We introduce two simple restrictions on the exaggerations senders use in equilibrium, which refine the equilibrium set substantially. Our restrictions allow for a linear family of recommendation strategies used by the senders, where our experimental data will allow us to test the empirical validity of these restrictions. By restricting on-path behavior of senders, we show that the sequentially rational response for receivers is also linear, which allows for direct econometric testing. Moreover, the best response for receivers to a generic linear strategy for the two senders has an intuitive decomposition: take a weighted average of the recommendation within the issue, and then modify this point based upon the divergence between the recommendations in other issues.

Sender Restriction A (State Independence)**.** *Exaggerations* $\tilde{\mathbf{x}}(\boldsymbol{\theta})$ *and* $\tilde{\mathbf{y}}(\boldsymbol{\theta})$ *are independent of the state* $\boldsymbol{\theta}$*.*

Sender Restriction B (Linear exaggerations)**.** *The equilibrium exaggerations vary only in a unidimensional subspace.*

Our first restriction is a form of symmetry, asking for similarly chosen exaggeration components across all realized states $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. The second provides a linear structure for the exaggerations, and constrains our focus to equilibria where the exaggeration component acts in a specific vector direction. Exaggerations satisfying the two restrictions can be written

$$\xi_X(\boldsymbol{\theta}) = \boldsymbol{\theta} + \kappa_X \cdot \boldsymbol{\gamma}^X \tag{1}$$

$$\xi_Y(\boldsymbol{\theta}) = \boldsymbol{\theta} + \kappa_Y \cdot \boldsymbol{\gamma}^Y, \tag{2}$$

where $\boldsymbol{\gamma}^X$ and $\boldsymbol{\gamma}^Y$ are unit length vectors indicating the direction each sender exaggerates within, and where κ_X and κ_Y are the exaggeration magnitudes, scalars drawn independently from distributions F_X and F_Y.

Let senders' exaggeration directions be given by $\boldsymbol{\Gamma} := \left[\begin{array}{cc} \boldsymbol{\gamma}^X & \boldsymbol{\gamma}^Y \end{array}\right]$. Though the receiver does not observe the point $\boldsymbol{\theta}$, the information contained in the recommendation pair $(\mathbf{x}, \mathbf{y})$ and the knowledge that senders have exaggerated along linearly independent directions is enough to infer the state $\boldsymbol{\theta}$ with probability one. The proposition below provides a characterization of the linear sequentially rational response when $\boldsymbol{\Gamma}$ has full rank.

Proposition 1. *When senders exaggerate in linearly independent directions given by* $\boldsymbol{\Gamma}$ *(for any exaggeration distributions* F_X*,* F_Y *), the sequentially rational receiver response is:*

$$\zeta_{\boldsymbol{\Gamma}}(\mathbf{x}, \mathbf{y}) = \left(\begin{array}{c} \alpha \cdot x_1 + (1-\alpha) \cdot y_1 + \beta_1 \cdot (y_2 - x_2) \\ (1-\alpha) \cdot x_2 + \alpha \cdot y_2 + \beta_2 \cdot (y_1 - x_1) \end{array} \right)$$

for scalars α*,* β_1 *and* β_2 *defined by*

$$\left[\begin{array}{cc} \alpha & -\beta_1 \\ -\beta_2 & 1-\alpha \end{array} \right] = \left[\begin{array}{cc} \mathbf{0} & \boldsymbol{\gamma}_Y \end{array} \right] \boldsymbol{\Gamma}^{-1}.$$

Proof. See Appendix B. □

The characterization—which we will use to motivate our estimation procedure for receiver's strategies—tells us that to infer the state in any issue the receiver linearly combines information from both senders in both dimensions. Within each issue j the receiver considers the identifiable difference between the two recommendations $\nabla_j(\mathbf{x},\mathbf{y}) = y_j - x_j$. This initial point is then modified by an amount proportional to the divergence in recommendations in the other issue $k \neq j$, the difference $\nabla_k(\mathbf{x},\mathbf{y}) = y_k - x_k$, where the degree of this modification is weighted by the parameter β_j.

The receiver response $\zeta_{\mathbf{\Gamma}}(\mathbf{x},\mathbf{y})$ is defined for any full-rank exaggeration basis $\mathbf{\Gamma}$, but in a fully revealing equilibrium we need to make sure that senders' can not benefit from any deviation at any realized state. This leads us to the following result.

Equilibrium Restriction C. *Given linearly independent bias vectors (so* $\mathbf{\Delta} := \left[\begin{array}{cc} \boldsymbol{\delta}^X & \boldsymbol{\delta}^Y \end{array}\right]$ *has full rank) the only equilibria possible that satisfy* Restrictions A *and* B *with linearly independent exaggeration directions has the exaggeration basis* $\mathbf{\Gamma}^\star(\mathbf{\Delta}) := \left[\begin{array}{cc} \boldsymbol{\gamma}^X(\mathbf{\Delta}) & \boldsymbol{\gamma}^Y(\mathbf{\Delta}) \end{array}\right]$*, with exaggeration directions* $\boldsymbol{\gamma}^X(\mathbf{\Delta}) \perp \boldsymbol{\delta}^Y$ *and* $\boldsymbol{\gamma}^Y(\mathbf{\Delta}) \perp \boldsymbol{\delta}^X$.[4]

This restriction comes from the receiver's sequentially rational response and sender optimality. The receiver's strategy translates each sender's recommendation into an exaggeration coordinate via the $\mathbf{\Gamma}^{-1}$ transformation, extracting the $\boldsymbol{\gamma}^Y$ coordinate from sender X and the $\boldsymbol{\gamma}^X$ coordinate from sender Y. If sender Y follows the exaggeration strategy in (2), and the receiver responds through $\zeta_{\mathbf{\Gamma}^\star(\mathbf{\Delta})}(\mathbf{x},\mathbf{y})$, every possible recommendation from X leads to a final choice on the $\boldsymbol{\gamma}^Y$-hyperplane through $\boldsymbol{\theta}$. What remains is to check which points on this hyperplane maximize X's outcome.

For any recommendation $\mathbf{x}$ within a neighborhood of $\boldsymbol{\theta}$, if X sends a recommendation with exaggeration in the $\boldsymbol{\gamma}^X$ direction only, then the final outcome is $\boldsymbol{\theta}$, so X is indifferent over every exaggeration in the $\boldsymbol{\gamma}^X$-direction. With our assumed preferences (negative of the Euclidean-distance from the ideal point), the set of final choices strictly preferred to $\boldsymbol{\theta}$ by X is given by all points within a distance of $\left\|\boldsymbol{\delta}^X\right\|$ to X's ideal point $\boldsymbol{\theta} + \boldsymbol{\delta}^X$. Given this upper-contour set, there is a *unique* locally supporting hyperplane at the point $\boldsymbol{\theta}$, which is orthogonal to the $\boldsymbol{\delta}^X$ direction. If the exaggeration direction $\boldsymbol{\gamma}^Y \not\perp \boldsymbol{\delta}^X$, X would be able to profitably deviate by sending the recommendation $\mathbf{x} = \boldsymbol{\theta} + \kappa_X \cdot \boldsymbol{\gamma}^X + \epsilon \cdot \boldsymbol{\gamma}^Y$ for some $\epsilon \in \mathbb{R}$. A similar argument for Y implies that $\boldsymbol{\gamma}^X \perp \boldsymbol{\delta}^Y$. Given the sender restrictions and that the two exaggeration directions are linearly independent, the sender optimality conditions indicate that the only FRE possible are those with the exaggeration basis $\mathbf{\Gamma}^\star(\mathbf{\Delta})$.[5]

2.3. **Toroidal State Space.** While the FRE constructed above is over $\mathbb{R}^2$ with the standard topology, our experiment will instead use what's referred to as a Clifford torus, a space with a circular topology.[6] This choice will have some theoretical advantages when testing for full revelation in a

[4]In Battaglini (2002) the preferences are quasi-concave, so the hyperplane $\boldsymbol{\gamma}^Y$ through the point $\boldsymbol{\theta}$ strongly separates the convex upper-contour set $\{\mathbf{z} \in \mathbf{\Theta} \,|u_X(\mathbf{z},\boldsymbol{\theta}) \geq u_X(\boldsymbol{\theta},\boldsymbol{\theta})\}$.

[5]If the exaggeration directions are not linearly independent, and each sender exaggerates in a common $\boldsymbol{\gamma}$ direction, the sequentially rational on-path response is $\zeta(\mathbf{x},\mathbf{y}) = \mathbf{x} - \mathbb{E}\left(\kappa_X \,|\nabla(\kappa_X \cdot \boldsymbol{\gamma}, \kappa_{\mathbf{Y}} \cdot \boldsymbol{\gamma}) = \nabla(\mathbf{x},\mathbf{y})\right) \cdot \boldsymbol{\gamma}$. Full revelation without full rank for $\mathbf{\Gamma}$ requires that the supports of κ_X, κ_Y be such that $(\kappa_X \,|\kappa_Y - \kappa_X)$ is degenerate for all possible $\kappa_Y - \kappa_X$ realizations.

[6]We are not aware of theory papers examining cheap talk in toroidal state-spaces, though Filipovich (2008) characterizes one-sender cheap talk on a single circle. However, our interest in this topology is methodological. We view

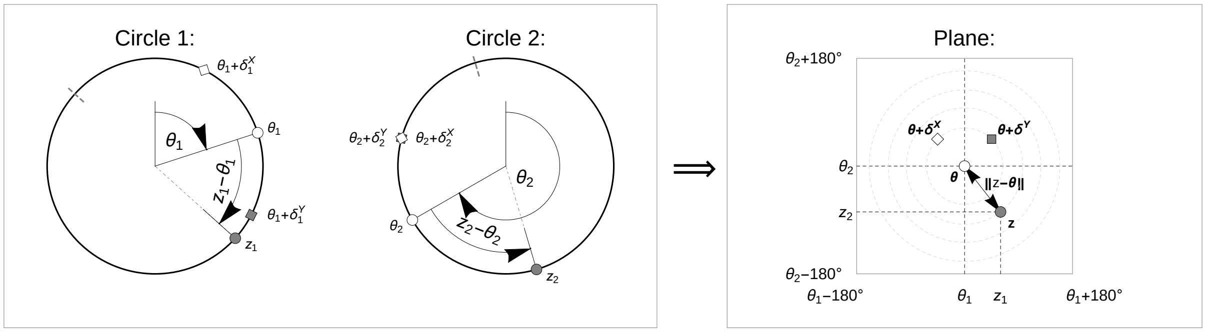


FIGURE 2. Toroidal State Space: Coordinate on Two Circles to Flat Plane

Notes: The example corresponds to Case 1 in Figure 1. The realized state is indicated with θ_i for each circle $i = 1, 2$. In Circle 1 senders are biased in opposite directions, so that θ_1 lies in between $\theta_1 + \delta_1^X$ and $\theta_1 + \delta_1^Y$. In Circle 2 senders are biased in the same direction and with equal magnitude so that $\theta_2 + \delta_2^X$ and $\theta_2 + \delta_2^Y$ overlap. z_i indicates the choice of the receiver.

laboratory setting that we will discuss next in the design section. Moreover, as we now show, the intuition for the FRE's mechanics is the same in both $\mathbb{R}^2$ and our toroidal space.

Our experiment frames the state-space $\mathbf{\Theta}$ through two separate circles, where a point on the circumference of each circle is uniformly and randomly chosen. The state-space is therefore a surface in $\mathbb{R}^2 \times \mathbb{R}^2$, where the two-dimensional realized state $\boldsymbol{\theta} = \begin{pmatrix} \theta^1, & \theta^2 \end{pmatrix}$ is indicated through a location on the two circles. The points on each circle are defined by the map $\mathcal{C} : [0°, 360°)^2 \to \mathbb{R}^2 \times \mathbb{R}^2$ given by

$$\mathcal{C}(\boldsymbol{\theta}) := \left(\begin{pmatrix} \sin \theta_1 \\ \cos \theta_1 \end{pmatrix}, \begin{pmatrix} \sin \theta_2 \\ \cos \theta_2 \end{pmatrix} \right),$$

where to mirror the experimental interface, all angular measurements will be given in degrees. Because $\mathcal{C}(\cdot)$ is a bijection, rather than work with the range (the positions on each circle's circumference) we will instead focus our analysis on the domain of $\mathcal{C}$, so $\mathbf{\Theta} = [0°, 360°)^2$, and the realized state $\boldsymbol{\theta}$ is a two-dimensional random vector, with each component uniformly distributed between $0°$ and $360°$.

The presentation on two circles is illustrated in the left panel of Figure 2. Senders perfectly observe the position $\mathcal{C}(\boldsymbol{\theta})$, and choose a position on the circumference of each of the two circles to represent their recommendations, $\mathcal{C}(\mathbf{x})$ and $\mathcal{C}(\mathbf{y})$ (for clarity, the recommendations are not shown on the figure). The receiver observes the recommendations $\mathcal{C}(\mathbf{x})$ and $\mathcal{C}(\mathbf{y})$, and then makes her own choice, the location $\mathcal{C}(\mathbf{z})$. Preferences for all agents are defined through an ideal point on each circle $\mathcal{C}(\boldsymbol{\theta} + \boldsymbol{\delta}^i)$, where each final choice $\mathbf{z}$ given the state $\boldsymbol{\theta}$ is ranked by agent i via

$$u_i(\mathbf{z}, \boldsymbol{\theta}; \boldsymbol{\delta}^i) = -\sqrt{\phi_d(z_1, \theta_1 + \delta_1^i)^2 + \phi_d(z_2, \theta_2 + \delta_2^i)^2},$$

where $\phi_d(a, b)$ is the angular length of the shortest arc (clockwise positive, counterclockwise negative) connecting two angles a and b.[7] So, in the figure, the receiver ranks the indicated decision $\mathcal{C}(\mathbf{z})$ via

$$u_Z(\mathbf{z}, \boldsymbol{\theta}; \mathbf{0}) = -\sqrt{(\theta_1 - z_1)^2 + (\theta_2 - z_2)^2}.$$

this state space as a simple test-tube for the FRE, one which replicates the main tensions and equilibrium form for the exaggeration equilibrium outlined above.

[7]So more formally, for two angles $a, b \in [0°, 360°)$ we define $\phi_d(a, b) := \mod (b - a + 180) - 180$.

The game on the two circles is locally identical (for all states/choices in $[\theta_1 - 180°, \theta_1 + 180°] \times [\theta_2 - 180°, \theta_2 + 180°]$) to a model with $\boldsymbol{\theta}$, $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ in $\mathbb{R}^2$, where agents' preferences are the negative of the Euclidean distance, $-\left\|\boldsymbol{\theta} + \boldsymbol{\delta}^i - \mathbf{z}\right\|$. The right panel of Figure 2 illustrates this equivalence, where the angles on the two circles are represented as points on the plane. The length of the shortest arcs on each circle are by construction less than $180°$ for all points in the illustrated plane, where the presentation is as if we cut the two circles at the two gray dashed lines diametrically opposite the state, and folded them out to form the two axes of the plane.[8]

Where the torus differs from $\mathbb{R}^2$ for the multi-sender game is over the measurements of differences between any two points, in particular, identifying the difference between the recommendations $\mathbf{x}$ and $\mathbf{y}$. For $\mathbb{R}^2$ the difference in dimension j is $\nabla_j(\mathbf{x}, \mathbf{y}) = y_j - x_j$. However for circles this difference will be an angular length for one of *two* possible arcs—either clockwise from x_j to y_j or counterclockwise from x_j to y_j—where the receiver must make an inference over which one is the correct measurement. For simplicity, and to fix ideas, we can think of the case where we always select the shortest arc connecting x_j and y_j (which would be the correct inference when the exaggerations are not large). The angular difference between the recommendation vectors $\mathbf{x}$ and $\mathbf{y}$ along the minor arc would therefore be given by $\nabla_j(\mathbf{x}, \mathbf{y}) = \phi_d(x_j, y_j)$.[9] So long as the inferred vector difference $\nabla(\mathbf{x}, \mathbf{y})$ is equal to the actual exaggeration difference $\xi_Y(\boldsymbol{\theta}) - \xi_X(\boldsymbol{\theta}) = \kappa_Y \cdot \boldsymbol{\gamma}^Y - \kappa_X \cdot \boldsymbol{\gamma}^X$, then the sequentially rational response of the receiver is the similar to that in Proposition 1:

$$\zeta(\mathbf{x}, \mathbf{y}; \alpha, \beta_1, \beta_2) = \begin{pmatrix} \mathbf{x} + (1-\alpha) \cdot \nabla_1(\mathbf{x}, \mathbf{y}) + \beta_1 \cdot \nabla_2(\mathbf{x}, \mathbf{y}) \\ \mathbf{x} + \alpha \cdot \nabla_2(\mathbf{x}, \mathbf{y}) + \beta_2 \cdot \nabla_1(\mathbf{x}, \mathbf{y}) \end{pmatrix},$$

for the same parameters α, β_1 and β_2 as functions of Γ.

Following the intuition for the FRE in $\mathbb{R}^2$, if the receiver uses a linear rule, it must be that senders cannot make a small deviation from their linear exaggeration strategy to improve their expected outcome. Because the toroidal environments' preferences are identical to those in $\mathbb{R}^2$ in a neighborhood of the fully revealing decision, $\mathbf{z} = \boldsymbol{\theta}$, it must be that any FRE with linearly independent exaggeration directions satisfies Equilibrium Restriction C. However, beyond this local condition, we must also check a global condition that large enough exaggerations cannot force the receiver to make an incorrect inference over the difference $\nabla(\mathbf{x}, \mathbf{y})$, and through this derive a benefit. A sufficient condition for the existence of an equilibrium satisfying this global condition is given in the following proposition:

Proposition 2. *If the biases for each sender $i \in \{X, Y\}$ satisfy $\left\|\boldsymbol{\delta}^i\right\| \leq \sqrt{5} \cdot 45°$, an FRE exists on the toroidal state-space satisfying Sender Restrictions A and B.*

Proof. See Appendix. □

[8] Our main treatments use the frame with two circles presented on the left panel of Figure 2. We also conduct robustness treatments with the frame on the right panel and results are presented in Section 4.4.

[9] If the exaggerations are opposed in sign on a particular issue j (for example, where $\kappa_X \gamma_j^X \leq 0 \leq \kappa_Y \gamma_j^Y$, for all $\kappa_i \in \text{supp} F_i$) and the exaggeration magnitudes were believed to satisfy $\left|\kappa_i \gamma_j^i\right| \in [0, 180°)$, one could instead focus on a specific arc from the two (the clockwise arc from x_j to y_j). The inferred recommendation difference would therefore be $\nabla_j(\mathbf{x}, \mathbf{y}) = \mod \in [0, 360°)$ and could therefore be any measurement from $0°$ to $360°$. For issues where the biases are in the same direction (for example where $0 \leq \kappa_X \gamma_j^X, \kappa_Y \gamma_j^Y$), and where $\left|\kappa_i \gamma_j^i\right| \in [0, 180°)$, one would still focus on the minor arc as the difference $\kappa_Y \gamma_j^Y - \kappa_x \gamma_j^X \in [-180°, 180°)$ always picks out the minor arc. More generally, the theory will work whenever the support of each exaggeration component $\kappa_i \cdot \gamma_j^i$ is assumed to be no wider than $180°$, so that the support of the observed differences is within the $360°$ resolution of the circle.

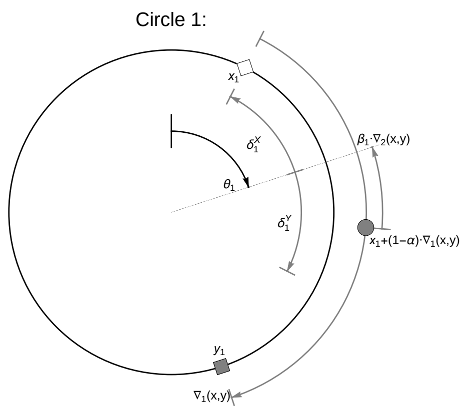


FIGURE 3. Example Inference on the Circles

Notes: The example corresponds to Case 1 in Figure 1. The realized state and messages are indicated with θ_i, x_i and y_i for each circle $i = 1, 2$ respectively. The within-dimension response is illustrated with gray circles. For each dimension the Figure also illustrates the adjustment using across-issue differences.

The sequentially rational receiver response in the FRE is exactly that characterized above for $\mathbb{R}^2$, where the parameter weights are given by

$$\begin{bmatrix} \alpha^\star & -\beta_1^\star \\ -\beta_2^\star & 1-\alpha^\star \end{bmatrix} := \begin{bmatrix} \mathbf{0} & \boldsymbol{\gamma}^Y(\boldsymbol{\Delta}) \end{bmatrix} \boldsymbol{\Gamma}^\star(\boldsymbol{\Delta})^{-1},$$

with the only added restriction being that the biases are not too large in absolute size.

The intuition for the receiver's response on the two circles is given in Figure 3. Senders X and Y have realized recommendations $\mathbf{x}$ and $\mathbf{y}$, illustrated as the white diamonds and gray squares. For Circle 1, the inferred difference is the length of the clockwise arc between x_1 and y_1, while for Circle 2 it is the clockwise arc from x_2 to y_2. The initial within-dimension response is to choose the angles illustrated by the gray circles, the weighted-average of the sender's recommendations: $x_1 + (1-\alpha) \cdot \nabla_1(\mathbf{x}, \mathbf{y})$ and $x_2 + \alpha \cdot \nabla_2(\mathbf{x}, \mathbf{y})$. These initial points are then modified using the across-issue differences. Assuming that both β_1 and β_2 are negative–so that both $\beta_j \cdot \nabla_k(\mathbf{x}, \mathbf{y})$ terms are counter-clockwise— the diagram illustrates the modification on each circle based on the across-issue difference $\nabla_k(\mathbf{x}, \mathbf{y})$.

3. EXPERIMENTAL DESIGN

3.1. **Experiment Frame.** *State Space.* Our multidimensional experiment will use the toroidal environment to examine whether or not the Battaglini FRE is selected. As noted above, the strategic tensions and FRE in both the torus and $\mathbb{R}^2$ are very similar. Why then have we chosen to use the torus? Our reasons for the choice are both methodological and theoretical. To select a random state $\boldsymbol{\theta}$ in the laboratory we need to inform subjects about the distribution it is drawn from. If we did not inform them, subjects may think that there is something meaningful to learn about the distribution, and this could add unnecessary noise to our data. The feasibility of full revelation could be tested over $\mathbb{R}^2$ if we used a distribution with full-support, such as a bivariate normal. While the equilibrium construction does not depend on the specific distribution G, it is not unreasonable to expect that non-uniform distributions might have an effect on subjects' behavior. Specifically, we worry about tensions between how likely a state was *ex ante* and what the advice they receive from senders indicates *ex post*.

Using a uniform distribution for the state introduces a new challenge. A uniformly distributed state requires a compact support, such as the unit square, $[0,1]\times[0,1]\subset\mathbb{R}^2$. However, economic theory (see Ambrus and Takahashi, 2008, for more details) shows that FRE may not exist in such state-spaces. Specific pairs of recommendations can lead to out-of-equilibrium inferences, indicating that the state implied by the two recommendations is outside of the state space—geometrically, one sender reveals a line going through the state, and the deviating sender reveals a line with an intersecting point outside the unit square. Whenever this happens, consistent receivers will conclude they are out-of-equilibrium, and take some alternative action. Depending on the off-the-path actions, senders might benefit in certain states from unilaterally forcing the out-of-equilibrium inference. We want to give the FRE its best chance, and our topological shift removes this possibility. Because of the circularity of our environment, recommendation pairs leading to an inference outside of the state space, say $(370°,-30°)'$, will within the torus indicate a point to the receiver that is still within it and on path, here $(10°,340°)'$.

Laboratory Environment. The realized angles for the state, and all recommendations/final decisions, are represented graphically to subjects as compass headings on two circles, where $360°$ is the origin (*North*) on each circles. We discretize the circles, so that the state realization $\boldsymbol{\theta}$ in our experiments is a vector of two independently distributed integers θ_1 and θ_2, drawn uniformly and independently from $\Theta_1=\Theta_2=\{1°,2°,\ldots,360°\}$ at the start of each round. In each of the first fifteen rounds of the experimental sessions, three subjects are randomly and anonymously matched together in a group. Two of the matched subjects take on the role of senders, X and Y, though within the experiment we use the neutral labels *Blue* and *Red* player (with the receiver called *Green*).[10]

Each sender perfectly observes the state of the world $\boldsymbol{\theta}$, represented as a green point on the edge of each circle. The senders then choose recommendations on each circle, angular vectors $\mathbf{x}$ and $\mathbf{y}$, which are sent to the receiver. Given the recommendations, the receiver chooses a point on each circle for the group, the angular vector $\mathbf{z}$, which determines the round payoffs. Preferences are induced by providing each of the three roles with an ideal angular location in $\boldsymbol{\Theta}$. For sender X this is parametrized by an angular bias $\boldsymbol{\delta}^X$; so that X has an ideal point δ_1^X clockwise from θ_1 on issue 1, and δ_2^X clockwise from θ_2 on issue 2. Similarly, sender Y has an angular bias $\boldsymbol{\delta}^Y$, inducing the ideal point $\boldsymbol{\theta}+\boldsymbol{\delta}^Y$. Final payments for all subjects/roles were derived using the angular distance of the chosen point $\mathbf{z}$ from the subject's ideal point. The exact monetary payment from a session is given by the sum of *two* randomly selected rounds, where the payment for the selected round is given by:

$$\max\left\{\$5,\$20-\$8\frac{\sqrt{(\text{Degrees from Ideal}_1)^2+(\text{Degrees from Ideal}_2)^2}}{45°}\right\}.$$

That is, all subjects in all roles receive \$20 if the chosen point $\mathbf{z}$ exactly matches their ideal location. Otherwise, the minimal angular distance from the ideal point (in either the clockwise or counterclockwise directions) in each circle is used to construct a cost. For every $45°$ distance from

[10]The instruction script read to subjects and slide-shows shown to them are included as an appendix to the paper, while screenshots of the interface are provided in Appendix A. Additionally, interested readers can download the *z-Tree* code (see Fischbacher, 2007) for both the session and the interactive instructions subjects followed alongside the instructions on their own screen.

their ideal point, subjects lose $\$8$, until a $\$5$ floor is reached.[11] Subjects are given both the above payoff formula, and a table indicating their payoff for every $15°$ difference from their optimal point in each issue, while the interface provides additional calculation aids.

To choose recommendations sender subjects simply click within each of the two circles (labeled Circle-A and Circle-B in the experiment) and their choice is rendered as a radial angle from the circle's center. Each sender's screen graphically illustrates their own most-preferred point (blue/red radial angles), that of the other sender (red/blue angles), and the receiver ($\boldsymbol{\theta}$, indicated by green points on the circles' circumference). While senders make their decisions, receivers view a screen graphically illustrating the relative location of the best points (the bias vectors $\boldsymbol{\delta}^X$ and $\boldsymbol{\delta}^Y$). After each sender has confirmed their chosen recommendation, the receiver final-decision stage begins. Receivers are provided with the two recommendations ($\mathbf{x}$ and $\mathbf{y}$, illustrated as blue/red radial angles, purple if coincident) alongside the senders' relative ideal locations ($\boldsymbol{\delta}^X$ and $\boldsymbol{\delta}^Y$). The receiver chooses the group action $\mathbf{z} = (z_1, z_2)'$ by clicking on the circular interface.

To review, group $g = \{i_X, i_Y, i_Z\}$ in round t is randomly formed from the session participants and the state $\boldsymbol{\theta}_{gt}$ is uniformly drawn from $\boldsymbol{\Theta}$. The state is perfectly observed by the two senders, X and Y, who chose recommendations $\mathbf{x}_{gt}$ and $\mathbf{y}_{gt}$ within $\boldsymbol{\Theta}$ in the *Message Selection* stage. The message pair $(\mathbf{x}_{gt}, \mathbf{y}_{gt})$ is observed by the receiver, who makes the group choice $\mathbf{z}_{gt} \in \boldsymbol{\Theta}$ in the *Choice Selection* stage. Finally, at the end of the round, subjects are given the feedback $\langle \boldsymbol{\theta}_{gt}, \mathbf{x}_{gt}, \mathbf{y}_{gt}, \mathbf{z}_{gt} \rangle$, along with the round payoffs: $\langle \pi_X, \pi_Y, \pi_Z \rangle$.

Each experimental session was divided into two parts. In the first part, subjects played five consecutive rounds in each role (X, Y and Z), for a total of fifteen rounds. In the second part, every subject takes on the role of the receiver Z, and were given the recommendations from groups in rounds 11–15 that they were not a member of. The second part has each subject acting as a receiver in rounds 16–20, removing potential other-regarding payment concerns, as the players that sent those recommendations are not compensated for outcomes in the second part. Final payment in the experiment was made on two randomly selected rounds from the entire session, so the maximum payment was $40 and the minimum $10.

3.2. **Treatments.** We utilize a *between-subject* design with the treatment variable $\boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\delta}^X & \boldsymbol{\delta}^Y \end{bmatrix}$ fixed in each session, matching Figure 1 in the introduction. The experimental treatments are more precisely summarized here in Table I.[12] The goal of the design is to diagnose whether fully-revealing equilibria are selected when theory indicates their existence, and examine the robustness of this selection to a simple coordinate-system transformation. This paper will focus on three treatments for $\boldsymbol{\Delta}$ where the biases are orthogonal and have symmetric magnitudes—-so $\boldsymbol{\Delta}$ always has full rank and $\|\boldsymbol{\delta}^X\| = \|\boldsymbol{\delta}^Y\|$—where an FRE with our exaggeration restrictions exists. Up to rounding error, the three treatments are simple rotations of the same coordinate system. The FRE represents the upper bound for receivers, however, in all three treatments another equilibrium exists, the uninformative babbling outcome. Babbling leads to the lower-bound outcome for

[11]The flatness in the preferences far from the true state does not affect the theoretical existence result, which requires senders have a convex upper-contour set at the fully revealing receiver decision $\mathbf{z} = \boldsymbol{\theta}$, and that the biases are not too large.

[12]However, we do experimentally vary the positive direction for biases (clockwise or counterclockwise), and dimensions (Circle-a as Issue-b), while making sure to keep the relative orientations of the senders' biases fixed.

TABLE I. Treatments

Treatment	Biases		Within	Across	Babbling/Revealing Payoff	
	$\boldsymbol{\delta}^X$	$\boldsymbol{\delta}^Y$	$\alpha^\star$	$\beta^\star$	Senders	Receiver
R(0), **P(0)**	$(0°, 60°)'$	$(60°, 0°)'$	1	0	\$5.86/\$9.33	\$5.86/\$20.00
R(.6)	$(-30°, 50°)'$	$(50°, 30°)'$	$25/34$	$-15/34$	\$5.86/\$9.63	\$5.86/\$20.00
R(1), **P(1)**, **E(1)**	$(-45°, 45°)'$	$(-45°, 45°)'$	$1/2$	$-1/2$	\$5.86/\$8.68	\$5.86/\$20.00

receivers, attaining their individually rational payoff.[13] A companion paper, Vespa and Wilson (in preparation), provides details on behavior in sessions where $\boldsymbol{\Delta}$ does not have full rank (and Battaglini-type equilibria do not exist) complementing the present work.

We refer to each of our *revelation* treatments by the label $\mathbf{R}(\tan\psi)$, where ψ is a counterclockwise rotation of the coordinate system, and $\tan\psi$ indicates the ratio of sender's bias magnitudes on each issue. In each treatment the bias vector magnitude for each sender is approximately $60°$, with small differences across the rotations to obtain round numbers for the experiment. Senders' bias directions are summarized by the rotation angle ψ: sender X has a bias in the direction $\begin{pmatrix} -\sin\psi, & \cos\psi \end{pmatrix}'$ and sender Y has a bias in the direction $\begin{pmatrix} \cos\psi, & \sin\psi \end{pmatrix}'$. Because the bias directions are orthogonal, senders' equilibrium exaggerations are predicted to be in the directions of their own biases, so $\boldsymbol{\Gamma}^\star(\boldsymbol{\Delta}) = \boldsymbol{\Delta}$, and the receiver's equilibrium response is

$$\zeta^\star(\mathbf{x}, \mathbf{y}) = \mathbf{x} + \begin{bmatrix} 1-\alpha^\star & \beta^\star \\ \beta^\star & \alpha^\star \end{bmatrix} (\mathbf{y} - \mathbf{x}), \tag{3}$$

where the across-issue modifiers are $\beta_1^\star = \beta_2^\star = -\frac{1}{2}\sin 2\psi =: \beta^\star$ and the within-issue weight is $\alpha^\star = \cos^2\psi \equiv \frac{1}{2} + \frac{1}{2}\cos 2\psi$.[14]

Table I indicates the FRE parameters—$\alpha^\star$and $\beta^\star$, the within- and across-issue components in the equilibrium response $\zeta^\star(\mathbf{x}, \mathbf{y})$—as well as the predicted payoffs to the senders and receiver under babbling/full revelation. The three R($\cdot$) treatments are:

R(0): The senders are each aligned with the receiver on a particular issue, but misaligned by $60°$ in the other. Within-issue the equilibrium weight parameter is given by $\alpha^\star = 1$, and there is no across-issue modification (so $\beta^\star = 0$). The clear asymmetry in bias magnitudes within issue fully resolves the problem, and the receiver response is the intuitive $\zeta^\star(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} x_1, & y_2 \end{pmatrix}'$. The response $\zeta^\star(\mathbf{x}, \mathbf{y})$ is to compose the two sources, following the recommendation of the unbiased sender in each circular issue. The receiver puts full weight on the unbiased sender within the issue and ignores across-issue information.

R(1): The fully rotated treatment has biases symmetrically opposed on one issue, $-45°$ and $+45°$, and symmetrically aligned on the other, both $+45°$ (so $\tan\psi = \frac{45}{45} = 1$)—and vector magnitude $63.6°$. Ex ante the two senders are symmetrically biased within each issue. In issue 1 they are diametrically opposed, and in issue 2 perfectly aligned. This symmetry has to be broken using the *realized* recommendations, $\mathbf{x}$ and $\mathbf{y}$. This treatment's equilibrium has a

[13] Within the toroidal state space babbling equilibria can satisfy *Sender Requirement A* (uniform exaggerations from 1 to 360 across both issues), however they necessarily violate *Sender Requirement* B, which guarantees revelation within at least a unidimensional subspace of $\boldsymbol{\Theta}$.

[14] Henceforth, we will for simplicity use $\mathbf{y} - \mathbf{x}$ to refer to the angular difference $\nabla(\mathbf{x}, \mathbf{y})$.

within-issue simple averaging response with $\alpha^\star = 1/2$, but the across-issue modifier reaches an extreme, with $\beta^\star = -\frac{1}{2}$. Because the two senders have an identical bias in issue 2, the sign and magnitude of the observable difference $(y_2 - x_2)$ is informative on which of the two senders provided the larger exaggeration, and by how much. Starting from the recommendation midpoint in issue 1, the equilibrium response $\zeta_1^\star(\mathbf{x}, \mathbf{y}) = \frac{1}{2}x_1 + \frac{1}{2}y_1 - \frac{1}{2} \cdot (y_2 - x_2)$ breaks symmetry by making a final choice in issue 1 closer to the sender who has exaggerated less in issue 2. A similar intuition matches the equilibrium response in issue 2, where the across-issue modification is given by $\frac{1}{2}(y_1 - x_1)$. The recommendation difference in issue 1 is informative about the total exaggeration magnitude. So the equilibrium response $\zeta_2^\star(\mathbf{x}, \mathbf{y}) = \frac{1}{2}x_2 + \frac{1}{2}y_2 - \frac{1}{2} \cdot (y_1 - x_1)$ is equivalent to removing the inferred average exaggeration from the average issue-2 recommendation—the larger the separation in issue 1 recommendations, the greater the receiver's shading from the issue 2 midpoint.

R(.6): The partially rotated treatment has senders with opposed biases of $-30°$ and $+50°$ in one issue, while in the other issue the senders have same-signed biases of $+50°$ and $+30°$, respectively (so $\tan\psi = \frac{30}{50} = 0.6$). The vector-magnitudes of the senders' biases are the same—here $58.3°$—but in each specific issue, one sender has a smaller bias. The receiver response $\zeta^\star(\mathbf{x}, \mathbf{y})$ is therefore inbetween that for R(0) and R(1). There is some initial asymmetry within each issue, which shows up as the receiver placing greater weight on the less-biased sender within-issue, with $\alpha^\star = \frac{25}{34} \approx 0.74$. Across issue there is still a lot to learn from the realized differences, and $\beta^\star = -\frac{15}{34} \approx -0.44$.

The design examines the simplest permutations of the special case R(0), rotations of the coordinate system. Given the experimental failures we will document in the simplest transformations (rotations), we see little reason to favor the FRE in more general transformations (rotations and shears).

In addition to these three treatments, we will later describe results from three further treatments conducted to examine robustness. These extension treatments are strategically identical to either the R(0) or R(1) environments. Two treatments modify the graphical interface to examine presentation effects. Instead of subjects making separate decisions on each circular issue, subjects in these treatments make choices jointly, as if locating a point on the plane depicted in Figure 2.[15] We refer to treatments in the *plane* interface as **P(0)** and **P(1)**, where the number again refers to the ratio of the senders' biases in each issue. The final robustness treatment extends the second part of the experiment to provide subjects with further experience in the receiver role. For the first fifteen rounds the treatment is identical to R(1), however, after round fifteen, all subjects face a sequence of fifteen rounds as a receiver instead of five. In these final rounds the state and recommendation data come from previous R(1) sessions. We refer to the *extended* second-part treatment as **E(1)**. Instructions and additional details for these extensions are included in online appendices.

4. RESULTS

In this section we present results using data from sessions conducted at NYU's Center for Experimental Social Science with a total of 249 subjects recruited from the student population. The section is organized as follows: After briefly summarizing the main economic findings, section 4.1 (alongside Appendix C) describes the response by senders. Section 4.2 then describes the receiver choices, and provides our main empirical findings. This is followed by Section 4.3, which provides

[15] Screenshots are provided in Appendix A.

a positive description of receiver's behavior in this environment. Section 4.4 then summarizes our robustness treatments and results.

Before we document the observed behavior in the experiment in greater detail, we first outline the main *economic* findings. We present final outcomes through an efficiency measure that captures the distance between receiver's final choice and the true state:

$$\Upsilon = \frac{\text{Babbling Distance} - \text{Observed Distance}}{\text{Babbling Distance} - \text{Fully Revealing Distance}}.$$

Values close to 100 percent reflect full revelation, with zero observed distance from the true state, while values close to 0 percent reflect no information transfer over the prior, the babbling outcome.

In the baseline, R(0), we find very high efficiency levels. Looking at the last five rounds, the average efficiency is 77 percent, and 58 of the 240 observations are at the upper boundary, with receivers making choices with zero distance from the true state—exactly choosing the true state from the 129,600 possible locations. In contrast, despite minimal *strategic* differences as we rotate the coordinate system, the efficiency drops to 56 percent in R(.6) and 39 percent in R(1), while just a single round across the 665 observations in these two treatments is coincident with the true state.

What makes the R(0) treatment special? Why does the efficiency drop as we rotate the coordinate system? We will show that across treatments, subjects' behavior in the sender roles are qualitatively close to the fully revealing exaggeration strategies presented in the theory section—so much so that the receiver empirical best response is not starkly different from the equilibrium response. There are certainly deviations from the sender restrictions, and behavior becomes noisier and more heterogeneous as the coordinate system is rotated. However, noise in the senders' response accounts for at most half of the observed efficiency losses. We will show that most of the efficiency losses in the rotated treatments can be directly ascribed to a failure in sequential rationality by receivers. Though receivers do well at incorporating within-issue information, they systematically fail to incorporate across-issue information, approaching each issue in isolation. Due to this failure, receiver response in our rotated treatments is not consistent with any perfect Bayesian equilibrium. The goal of the results section will be to document and explain these findings.

Summary statistics for each of the R($\cdot$) treatments are provided within Table II. Results are split into three parts: i) the recommendation exaggerations $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ added to the true state by the senders X and Y, respectively, as coordinates on the plane; ii) the final choice error $\tilde{\mathbf{z}} = \mathbf{z} - \boldsymbol{\theta}$ made by the receiver Z as a coordinate on the plane, again with units of angular degrees; iii) the round profits for each of the three subjects, π_X, π_Y and π_Z, given in dollars; and iv) the final efficiency Υ, as well as the fraction of subject-rounds with perfect revelation, where $\Upsilon = 1$.[16] For each variable, in each treatment, the table provides the average, as well as the within- and between-subject standard deviations obtained through a fixed-effects panel estimate, and the number of subject-round choices, N.

4.1. **Senders and exaggeration strategies.** Across our treatments the average exaggeration is close to the sender's bias, with the interpretation that recommendations are centered around the sender's ideal points. For instance, the average X-subject in treatment R(0) sends a nearly unexaggerated recommendation in the unbiased issue 1, but a recommendation that adds 47.8° to the true state in *Issue-2*, where their induced best point adds 60° to the true state. However, the table

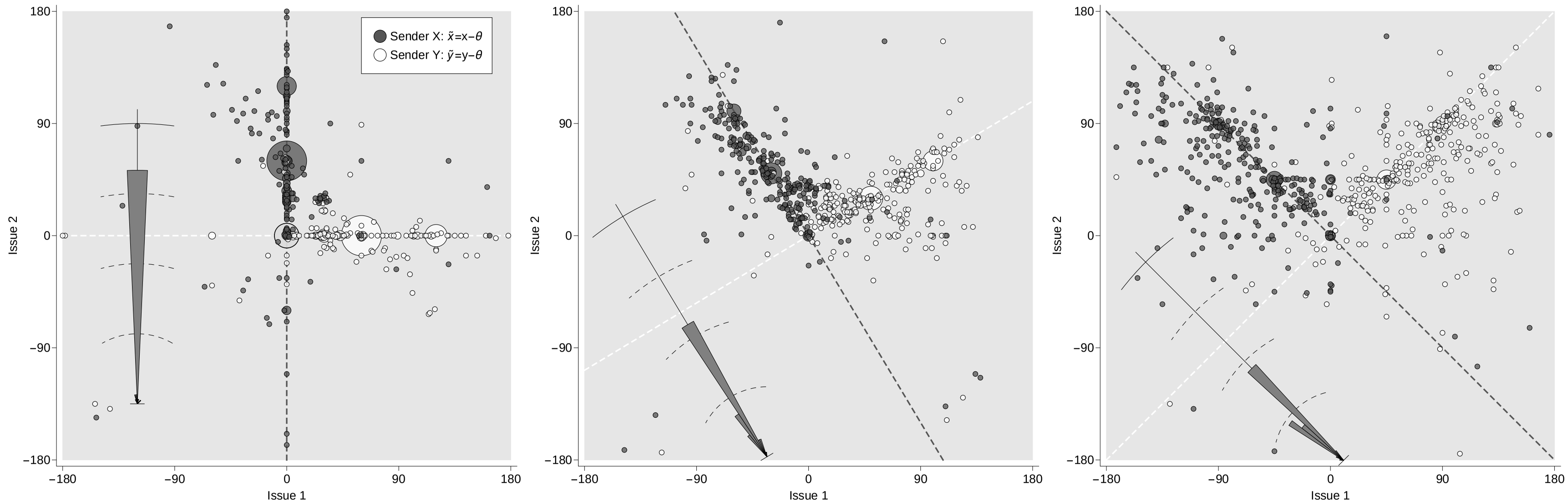
[16] All angular differences reported are the shortest arcs between the two points, so all measurements are clockwise differences in $[-180^\circ, +180^\circ)$.

TABLE II. Summary Statistics

	R(0), 4 Sessions				**R(.6)**, 4 Sessions				**R(1)**, 3 Sessions[†]			
	Avg.	σ^{B}	σ^{W}	N	Avg.	σ^{B}	σ^{W}	N	Avg.	σ^{B}	σ^{W}	N
Exaggerations:												
$\tilde{\mathbf{x}} = \mathbf{x} - \boldsymbol{\theta}$	$\begin{pmatrix} 0.2 \\ 47.8 \end{pmatrix}$	$\begin{pmatrix} 15.6 \\ 30.8 \end{pmatrix}$	$\begin{pmatrix} 24.5 \\ 44.7 \end{pmatrix}$	300	$\begin{pmatrix} -29.1 \\ 49.8 \end{pmatrix}$	$\begin{pmatrix} 23.9 \\ 31.5 \end{pmatrix}$	$\begin{pmatrix} 31.7 \\ 29.6 \end{pmatrix}$	300	$\begin{pmatrix} -61.1 \\ 53.1 \end{pmatrix}$	$\begin{pmatrix} 38.1 \\ 34.7 \end{pmatrix}$	$\begin{pmatrix} 44.8 \\ 34.1 \end{pmatrix}$	345
$\tilde{\mathbf{y}} = \mathbf{y} - \boldsymbol{\theta}$	$\begin{pmatrix} 50.1 \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 32.6 \\ 11.7 \end{pmatrix}$	$\begin{pmatrix} 40.7 \\ 19.7 \end{pmatrix}$	300	$\begin{pmatrix} 50.3 \\ 28.9 \end{pmatrix}$	$\begin{pmatrix} 30.5 \\ 16.6 \end{pmatrix}$	$\begin{pmatrix} 33.1 \\ 28.1 \end{pmatrix}$	300	$\begin{pmatrix} 62.0 \\ 52.1 \end{pmatrix}$	$\begin{pmatrix} 36.0 \\ 30.0 \end{pmatrix}$	$\begin{pmatrix} 43.1 \\ 34.9 \end{pmatrix}$	345
Choice error:												
$\tilde{\mathbf{z}} = \mathbf{z} - \boldsymbol{\theta}$	$\begin{pmatrix} 9.5 \\ 9.7 \end{pmatrix}$	$\begin{pmatrix} 17.0 \\ 13.5 \end{pmatrix}$	$\begin{pmatrix} 35.7 \\ 34.5 \end{pmatrix}$	540	$\begin{pmatrix} 2.9 \\ 25.9 \end{pmatrix}$	$\begin{pmatrix} 12.9 \\ 22.3 \end{pmatrix}$	$\begin{pmatrix} 51.0 \\ 42.4 \end{pmatrix}$	540	$\begin{pmatrix} 2.5 \\ 18.7 \end{pmatrix}$	$\begin{pmatrix} 23.7 \\ 25.1 \end{pmatrix}$	$\begin{pmatrix} 78.0 \\ 55.0 \end{pmatrix}$	450
Round Payoffs:												
π_X	\$9.31	\$1.57	\$3.07	300	\$9.56	\$2.50	\$4.00	300	\$8.08	\$2.03	\$3.72	345
π_Y	\$9.24	\$1.36	\$3.09	300	\$9.23	\$2.08	\$3.87	300	\$8.11	\$2.19	\$3.70	345
π_Z	\$15.19	\$2.83	\$4.92	540	\$11.12	\$2.30	\$4.09	540	\$9.28	\$1.99	\$4.22	450
Efficiency:												
Υ	76.7%	14.8%	28.5%	540	57.3%	15.8%	28.2%	540	40.4%	18.6%	37.0%	450
$\Upsilon = 1$	23.0%	20.1%	39.4%	540	0.2%	1.3%	4.3%	540	0.0%	–	–	450

Note: Results are from fixed-effect panel estimates across subjects and session periods, with the relevant variables regressed on a constant term. The columns headed σ^{B} provides the standard deviation *between* subjects' average levels; columns headed σ^{W} provide the standard deviation *within* each subject. The results for one R(0) session and one R(.6) session only have data for the first period from periods 16-20 due to a coding error, for this reason we conducted an additional session for each of these two treatments. The total number of unique subjects in each session is 15.

[†]Sender data for R(1) includes data from 24 subjects in the first part of two E(1) sessions.



(A) **R(0)** (B) **R(.6)** (C) **R(1)**

FIGURE 4. Sent Exaggerations

Note: In sub-figures (A–C), each circular point represents the difference between the message/recommendation sent by one of the sender subjects and the true state for periods 1 to 15. Dark gray points represent the message exaggerations $\tilde{\mathbf{x}}$ sent by sender X, while white points represent the message exaggerations $\tilde{\mathbf{y}}$ sent by sender Y. The dotted lines represents the Battaglini (2002) straightforward equilibrium strategies. Angular histograms within each scatter-plot indicate subject-level exaggeration directions $\omega + \psi$ for sender X, binned into $\frac{\pi}{36}$ radian cones, with the scale aligned to the rotation direction ψ. N.B. panel (C) contains sender exaggeration data pooled across R(1) and E(1).

also indicates substantial variation both within and between subject. We show that this variation is primarily in the directions predicted in the theory section. To illustrate this, Figure 4 displays all the exaggeration data in each treatment. The figure provides a scatter for the exaggerations $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ sent in every subject-round in each treatment. Each point represents an exaggeration added to the true state, gray points for X, white for Y, where the area of each plotted point is proportional to the mass of observations at each exaggeration vector.[17] Across the 129,600 possible realizations, there is substantial variation, but there are also clear patterns.

Exaggerations are clustered around the bias direction rays–qualitatively matching not only the linear restrictions for sender on-path behavior, but also matching the equilibrium direction. However, even though there are strong patterns conforming to these restrictions, there are also differences, noticeably so in the R(.6) and R(1) treatments. Though these differences will lead to efficiency losses, they do not substantially alter receiver's optimal response.

Exaggeration Strategies. In the theory section we specified three restrictions on behavior that we now examine. The first, *Sender Restriction A*, asked that the exaggeration components, $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$, are independent of the realized state $\boldsymbol{\theta}$. To explore this restriction we take each subject-round recommendation (with subscript it) as the unit of observation and compute the correlation between the exaggerations, $\tilde{\mathbf{x}}_{ait}$ and $\tilde{\mathbf{y}}_{ait}$, and state $\boldsymbol{\theta}_{bit}$ across issues $a, b \in \{1, 2\}$.[18] Computing the eight correlations in each treatment (four for each sender) we find that the highest absolute value for a correlation in any treatment is 0.036, and no relationships are significant. We conclude that there is no evidence for dependence between exaggerations and the state.

Sender Restriction B and *Equilibrium Restriction C* require that agents' exaggerations are linear and in the direction of the biases, respectively. Hence, one simple measure for *B* and *C* jointly is the fraction of exaggerations in the bias direction. In R(0), 69 percent of the exaggerations sent are *exactly* coincident with the equilibrium exaggeration direction, while the figures for R(.6) and R(1) are much smaller, at 13 percent and 10 percent, respectively. Allowing for some noise (a $10°$ band either side of the bias directions) these figures jump up to 82 percent adherence in R(0), 59 percent in R(.6), and 49 percent in R(1). While there is a clearly a qualitative match with the assumptions, we also observe deviations that increase as we rotate the coordinate system. In Appendix C we provide a detailed analysis of restrictions *B* and *C*, but here we provide only a brief summary of the main results:

i) *Sender Restriction A*: There is no evidence for dependence between exaggerations and the realized state.
ii) *Sender Restriction B*: Deviations from *linear exaggeration* account for approximately half of the observed efficiency losses, where the magnitude of these deviations increases as the coordinate system is rotated. However, the gains to receivers from relaxing *Restriction B* and responding with a non-linear best response are small while the additional complexity is very large. We therefore treat deviations from the restriction as white noise.
iii) *Equilibrium Restriction C*: The modal response for senders across treatments is to exaggerate in the direction of their bias. At the aggregate level exaggeration directions are not significantly different from the bias direction in R(0) and R(.6), though there is a significant difference in R(1) it is quantitatively small.

[17]For R(1), we pool data from the 24 subjects who make choices as senders within the first 15 rounds of E(1). The first 15 rounds in E(1) have identical treatment to R(1), and the treatments differ only in the length of the second part.

[18]Because the random variable we calculate correlations for is defined on a circle we use the circular correlation measure of Fisher and Lee (1983).

In the next section we analyze subjects' behavior as receivers. Given the sender results, our approach will be to examine sequential rationality under the assumption that *Restrictions A* and *B* hold, and then examine receivers' response in comparison to the Proposition 1 prediction.

4.2. **Receiver Behavior and Full Revelation.** As described, outside of the R(0) case, the full revelation equilibrium is not validated by the data from our experiments. Two strategically similar rotated treatments, R(.6) and R(1), have substantially lower information transmission. Below, we first characterize receiver outcomes in more detail across the three treatments. We then document a failure in sequential rationality in R(.6) and R(1), and outline why this failure occurs, that subjects do not use across-issue information. Given this failure, section 4.3 then goes on to outline a descriptive model of how receivers process conflicting advice.

Final Outcomes. To measure the information extracted by receivers we use the distance of the receiver's final choice from the true state, $\|\tilde{\mathbf{z}}_{it}\|$. We focus on the last five rounds of the experimental sessions, rounds 16–20, because: i) receivers have had experience with all roles in the environment, ten rounds as senders, and five rounds as a receiver; ii) the last five rounds use sender/state data from rounds 11–15, but are paid as decision problems, isolating potential other-regarding confounds; and iii) all subjects play the receiver role in the last five rounds, so we have a large number of receiver-subject observations at a consistent point in the experimental session.[19] Given the set of subjects in each treatment, $\mathcal{I}$, we construct the subject-level efficiency for each $i \in \mathcal{I}$

$$\Upsilon_i = 1 - \frac{\frac{1}{5}\sum_{t=16}^{20} \|\tilde{\mathbf{z}}_{it}\|}{\mathbb{E}\,\|\boldsymbol{\theta}\|}.$$

Empirical CDFs for Υ_i over the set of subjects $\mathcal{I}$ are presented for each R($\cdot$) treatment in Figure 5(A). Inspecting the figure, we see that not only is there a significant response in the mean efficiency (the circle labeling the CDF with the treatment label located at the average efficiency with the 95 percent confidence interval illustrated), but there is a clear first-order stochastic relationship between treatments,

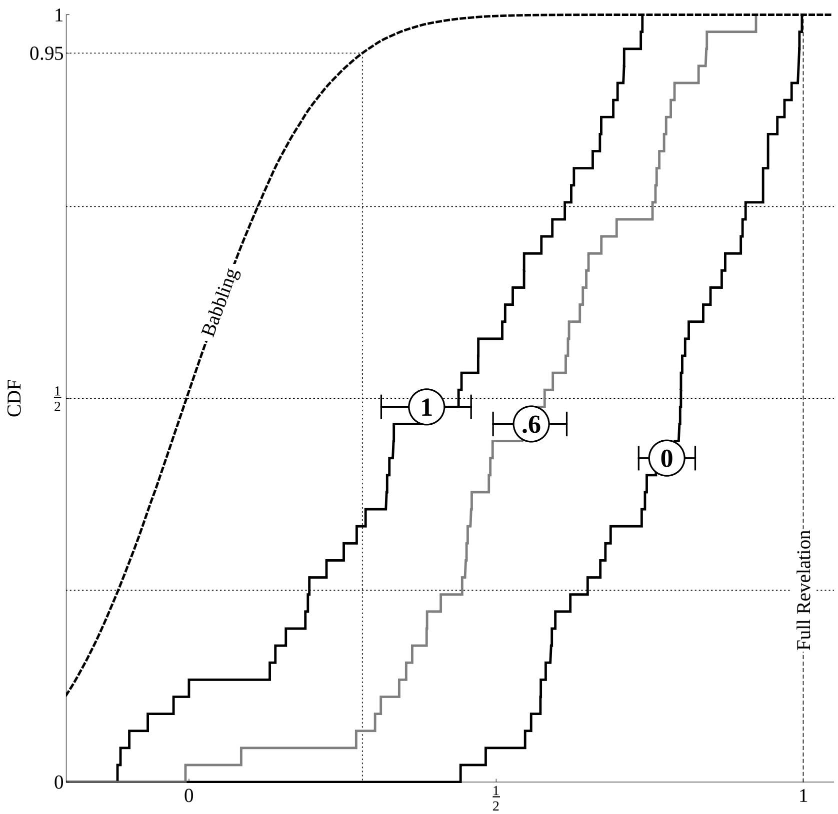
$$\text{Full Revelation} \succ_1^{\star\star\star} \Upsilon\left(\text{R}(0)\right) \succ_1^{\star\star} \Upsilon\left(\text{R}(.6)\right) \succ_1^{\star\star} \Upsilon\left(\text{R}(1)\right) \succ_1^{\star\star\star} \text{Babbling}.$$

Though none of our treatments attains the (degenerate) full-revelation upper bound, it is also clear that we have more information transmission than the lower bound, babbling.[20] In addition to stochastic dominance, a one-sided test for a babbling null (by subject instead of jointly over the distribution) can be inferred from the figure. The babbling distribution's 95th percentile is indicated, and it is clear that we can reject babbling at this level for every subject in R(0). Similarly we can reject babbling for 93 percent of subjects in R(.6), and 67 percent in R(1).

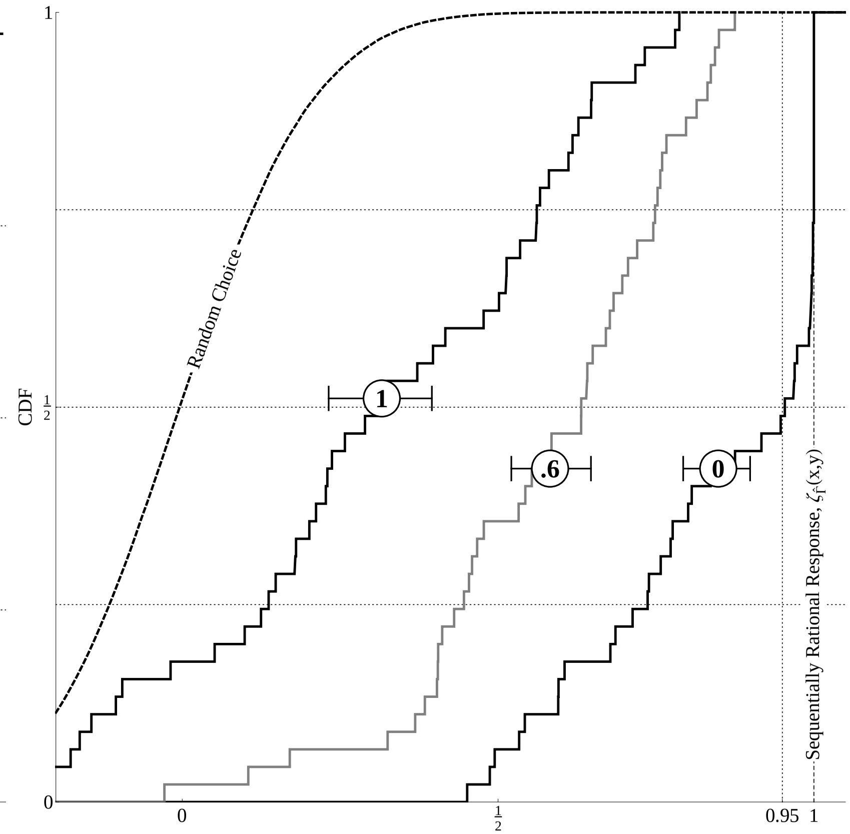
As we outlined in our discussion of sender response, deviations from *Sender Restriction B* increase as the coordinate system is rotated, and the additional noise could be the primary driver for the observed efficiency losses. To show this is not the case, we construct a second efficiency measure, which controls for losses due to noise in the observed recommendations, which we call

[19] Our findings do not change if we used the rounds in part 1 when subjects acted as receivers.

[20] All of our distributions are significantly different from one another using a Kolmogorov-Smirnoff test for non-equality of the distributions, at any conventional level. The reported significance levels for the first-order stochastic dominance relation $\succ_1$, following Barrett and Donald (2003), use a bootstrap of size 1,000 to calculate p-values. The observed stochastic ordering is significant at the following levels: ***-1 percent; **-5 percent; *-10 percent . The order $\succ_1$ is complete and transitive over the five distributions at the 5-percent level, as $\Upsilon\left(\text{R}(0)\right) \succ_1^{\star\star} \Upsilon\left(\text{R}(1)\right)$ and all treatments are stochastically dominated by full revelation, and dominate babbling at the 1 percent level.



(A) Subject Efficiency, Υ_i



(B) Receiver Efficiency, $\hat{\Upsilon}_i$

FIGURE 5. CDFs for subject's average decision efficiency

Note: Plot labels are located at the average efficiency level within the treatment; bars through the plot label represent the 95 percent confidence region for the distribution mean, extracted from a bootstrap. There is a reference CDF for Υ_i under babbling because the babbling equilibrium distance is a random variable. The fully-revealing equilibrium's efficiency is a degenerate random variable with a certain value of 1 in every subject-round.

receiver efficiency. Receiver efficiency measures the distance between the observed receiver decision $\mathbf{z}_{it}$ and the Proposition 1 sequentially rational response. Given the best-fit exaggeration basis $\hat{\mathbf{\Gamma}}$ (given in Table C5 in the appendix), we compute the counterfactual response $\hat{\zeta}_{it} = \zeta_{\hat{\mathbf{\Gamma}}}(\mathbf{x}_{it}, \mathbf{y}_{it})$. The subject-level receiver efficiency is defined as

$$\hat{\Upsilon}_i = 1 - \frac{\frac{1}{5}\sum_{t=16}^{20}\left\|\mathbf{z}_{it} - \hat{\zeta}_{it}\right\|}{\mathbb{E}\left\|\boldsymbol{\theta}\right\|}.$$

Receiver efficiency is the average distance between the subject's choice and the sequentially rational response, normalized by the expected distance under babbling (or equivalently here, the expected distance between $\hat{\zeta}_{it}$ and a uniform random choice). This is our measure for sequential rationality, where $\hat{\Upsilon}_i$ controls for noise in the sender recommendations, as both $\mathbf{z}_{it}$ and $\hat{\zeta}_{it}$ respond to the same (potentially noisy) recommendations $(\mathbf{x}_{it}, \mathbf{y}_{it})$. A subject with full receiver efficiency makes final choices exactly coincident with the sequentially rational response in all five final rounds, while a subject with zero receiver efficiency has the same average distance from $\hat{\zeta}_{it}$ as would a random uniform choice over $\mathbf{\Theta}$.

Empirical CDFs for $\hat{\Upsilon}_i$ are provided by treatment in the Figure 5(B). Again, treatments differ significantly both over the average receiver efficiency, and are completely ordered by first-order stochastic dominance at the 5 percent level. The clearest difference in comparison to Figure 5(A), is the stronger indication that many subjects in R(0) follow the sequentially rational response. Approximately a quarter of subjects in R(0) *exactly* match the response $\zeta_{\hat{\mathbf{\Gamma}}}(\mathbf{x}_{it}, \mathbf{y}_{it}) = (x_{1it}, y_{2it})'$ in *all* five rounds, while 56 percent have final choices with receiver efficiency greater than 95 percent—corresponding to an average choice within a $4.9°$ radius of $\hat{\zeta}_{it}$. Deviations by senders from *Restriction B* account for these subjects not having values for Υ_i closer to the fully efficient level.

In contrast to R(0), in R(.6) and R(1) not one subject has a receiver efficiency above 95 percent, so subjects in these treatments are consistently far from the sequentially rational response. We conclude then, that receivers in the rotated treatments are not following the $\zeta_{\hat{\mathbf{\Gamma}}}(\mathbf{x}_{it}, \mathbf{y}_{it})$-response.[21]

Having provided evidence of what they are not doing, we now turn to more constructive evidence for the strategies receivers do use. In particular, receivers do fairly well at incorporating the available information within each issue (the α terms). Crucially though, we find no evidence that subjects use *any* available information across issues (the β terms).

Receiver Strategies. To understand how subjects use the provided recommendations we estimate how receivers' final choices respond to the recommendations they received. In particular, we examine a generalization of the Proposition 1 response, where we estimate the values α_k , β_k and η_k in the following econometric identity:

$$(4) \quad \begin{pmatrix} z_1^{it} \\ z_2^{it} \end{pmatrix} = \begin{bmatrix} \alpha_1 & -\beta_1 \\ -\beta_2 & 1-\alpha_2 \end{bmatrix} \begin{pmatrix} x_1^{it} \\ x_2^{it} \end{pmatrix} + \begin{bmatrix} 1-\alpha_1 & \beta_1 \\ \beta_2 & \alpha_2 \end{bmatrix} \begin{pmatrix} y_1^{it} \\ y_2^{it} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \epsilon_1^{it} \\ \epsilon_2^{it} \end{pmatrix}.$$

Given our orthogonal design, equilibrium strategies in treatment R$(\tan\psi)$ fit in to the above estimation equation as $\alpha_1^\star = \alpha_2^\star = \cos^2\psi$ and $\beta_1^\star = \beta_2^\star = -\frac{1}{2}\sin 2\psi$, with zero offset ($\eta_1^\star = \eta_2^\star = 0$).

[21] In fact, through a similar exercise we can also conclude that receivers in the rotated treatments follow neither a more sophisticated affine best-response, nor the unrestricted, non-parametric best response $\bar{\zeta}(\mathbf{x}_{it}, \mathbf{y}_{it})$.

TABLE III. Estimated Receiver Strategy

	Estimated Aggregate Receiver Strategies					
	Within Issue		**Across Issue**		**Offset**	
	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\eta}_1$ [0.0]	$\hat{\eta}_2$ [0.0]
R(0)	0.87 [1.0]	0.86 [1.0]	-0.01 [0.0]	0.06 [0.0]	2.5	-0.4
	(0.04)	(0.05)	(0.03)	(0.03)	(1.5)	(3.0)
R(.6)	0.65 [0.74]	0.62 [0.74]	-0.06 [-0.44]	0.04 [-0.44]	2.5	-21.1
	(0.07)	(0.06)	(0.07)	(0.05)	(5.5)	(6.0)
R(1)	0.38 [0.5]	0.54 [0.5]	0.02 [-0.5]	-0.05 [-0.5]	-2.7	-29.3
	(0.08)	(0.07)	(0.08)	(0.03)	(10.4)	(5.3)

Note: Estimation results are from an iterated GMM regression enforcing the identifying constraints across the two equations (moments conditions are that all regressors are uncorrelated with the errors across both issues) using data from the rounds 16–20. For each treatment there are 225 observations, with 3 sessions, and 45 subjects. Theoretical values in square brackets next to the estimate where applicable. Standard errors are given in parentheses below estimate.

Moreover, given state-independent exaggerations in independent directions given by $\mathbf{\Gamma}$, the sequentially rational response $\zeta_{\mathbf{\Gamma}}(\mathbf{x},\mathbf{y})$ fits into the above family with $\alpha_1 = \alpha_2$ and $\eta_1 = \eta_2 = 0$.[22]

We estimate parameters in (4) using an iterated GMM approach, with data across all subjects in rounds 16–20, where the results are provided in Table III. Examining the estimated coefficients (and comparing them to the equilibrium predictions in the square brackets), the most striking features are:

i) The within-issue weights ($\hat{\alpha}_1$ and $\hat{\alpha}_2$) shift significantly by treatment in the directions of the equilibrium prediction. The only treatment where we can reject the equilibrium value at the 95 percent level is R(0), where the equilibrium coefficient is located at a boundary.
ii) The across-issue terms ($\hat{\beta}_1$ and $\hat{\beta}_2$) are not significantly different from zero (with the exception of issue 2 in R(0) where equilibrium/best response does predict zero), but are significantly different from the equilibrium prediction in R(.6) and R(1).
iii) The offset terms ($\hat{\eta}_1$ and $\hat{\eta}_2$) are only significantly different from zero when the biases of the senders have the same sign within the issue—issue 2 in R(.6) and R(1).

To briefly summarize the regression results, as these estimates constitute one of the paper's *main findings*: Receivers **do** react to senders' biases within issue, the α_j terms. However, where the full-revelation strategies require a multidimensional response, subjects in our treatments do **not** exhibit **any** reaction to *across-issue* differences (the β_j terms).[23]

[22] Parameters for the best affine response to the distributions of senders' exaggerations are provided in Table A4 in the appendix.

[23] A similar linear estimation to (4) with fewer free parameters conducted at the subject level mirrors the intuition from Figure 5(B). In R(0), 49 percent of subjects have choices with excellent fit to the model (an R^2 coefficient of 0.99 or greater), and have estimated parameters quantitatively close to the equilibrium point prediction. When we perform the same exercise in the R(.6) and R(1) treatments, the fraction of subjects with even adequate fit to the model (an R^2 coefficient of 0.8 or greater) is 13 and 20 percent respectively. Of this subset, none have estimated parameters close to the equilibrium values.

4.3. **Receiver reactions within each issue.** In section 4.4 we examine the robustness of subjects' failure to respond multidimensionally with additional treatments. However, we first provide a positive characterization of subject's within-issue response. In each specific issue, senders have two general orientations: either their biases are *opposed* (each wants the receiver's choice to move in a different direction) or they are *aligned* (both want the receiver's choice to move in the same direction) with treatment R(0) representing a boundary case in between the two. We study these two cases in turn, and provide evidence for two distinct types of unidimensional response by the receivers: an averaging response $\zeta_{\text{Avg}}(\tilde{x}_j, \tilde{y}_j)$, and a response that identifies the minimal recommendation and shades a fixed amount from it $\zeta_{\text{Min}}(\tilde{x}_j, \tilde{y}_j)$.

Opposed Issue. The senders have opposed biases in issue 1 for the R(.6) and R(1) treatments, and in a limiting sense both issues in R(0). If both senders use linear exaggeration strategies in the bias direction, the received recommendations in issue *1* would be $x_1 = \theta_1 - \kappa_X \cdot \left|\delta_1^X\right|$ and $y_1 = \theta_1 + \kappa_Y \cdot \left|\delta_1^Y\right|$ for the exaggeration magnitudes κ_X and κ_Y. The exaggeration difference $y_1 - x_1$ is directly observable, and is equal to $\kappa_Y \cdot \left|\delta_1^Y\right| + \kappa_X \cdot \left|\delta_1^X\right|$. In general—that is, outside of the R(0) case where one bias is zero—there will be a continuum of (κ_X, κ_Y) pairs consistent with the difference $y_1 - x_1$, each leading to a different inference on the precise location of θ_1.

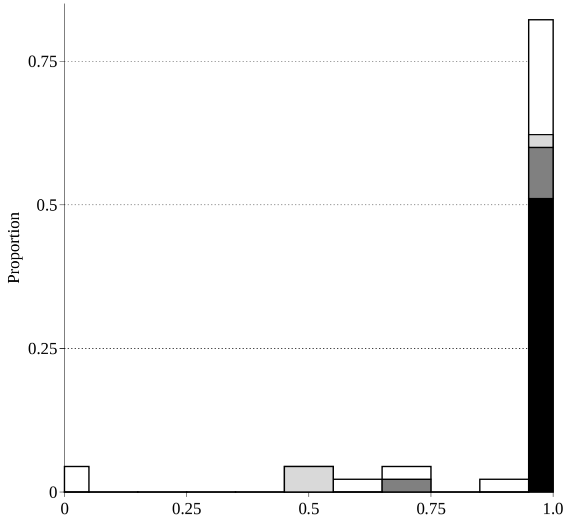
The FRE response $\zeta^\star(\mathbf{x}, \mathbf{y})$ uses the across-issue difference $y_2 - x_2$ to eliminate the degree of freedom on the location of θ_1, pinning down unique values for both κ_X and κ_Y. As we have described, subjects do not use this across-issue difference. Instead, we will show that in opposed issues the majority of subjects act *as if* responding to a symmetric conjecture on the exaggeration magnitudes that pins down the response, that $\kappa_X = \kappa_Y$. Given this conjecture, the within-issue best response is a weighted average of the recommendations, so $z_1 = \alpha \cdot x_1 + (1 - \alpha) \cdot y_1$, with the weight parameter $\alpha = \left|\delta_1^Y\right| / \left(\left|\delta_1^Y\right| + \left|\delta_1^X\right|\right)$. In R(0) the biases are 0° and 60° for the two senders in each issue, so this type of response leads to a final choice equal to the recommendation sent by unbiased sender in the issue (matching the equilibrium response). In R(.6) the biases in issue 1 are -30° and 50°, so a symmetric conjecture on the exaggeration magnitudes leads to $z_1 = {}^5\!/\!_8 \cdot x_1 + {}^3\!/\!_8 \cdot y_1$, while in R(1), the senders' biases in the first issue are -45° and $+45^\circ$, leading to the simple-average response, $z_1 = {}^1\!/\!_2 x_1 + {}^1\!/\!_2 y_1$.

For each subject $i \in \mathcal{I}$ we estimate the best-fitting parameter α_i in the following averaging-response $\zeta_{\text{Avg}}(x_1, y_1; \alpha_i) = \alpha_i \cdot x_1 + (1 - \alpha_i) \cdot y_1$ in the opposed-sign issues.[24] Figures 6(A–C) provide histograms illustrating the distribution of the estimated $\hat{\alpha}_i$ parameters by treatment. The histogram illustrates the fraction of subjects with values with model-fit greater than 99 percent (95 and 90 percent, respectively) with black (gray and light-gray) bars.[25]

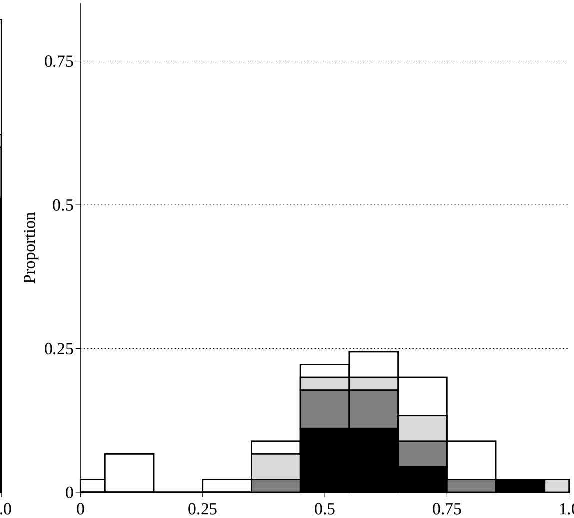
Inspecting the three opposed-issue histograms, the modal response clearly follows the bias-weighted average motivated above. The starkest pattern is within the R(0) treatment, where just over 50 percent of subjects have an estimated weight on the unbiased sender of 0.95–1.00, with very strong fit to the model. Clearly, given that this averaging process mirrors the sequentially

[24]That is, we estimate the parameter α_i that minimizes the objective $\sum_{t=16}^{20} \left|z_{1it} - \zeta_{\text{Avg}}(x_{1it}, y_{1it}; \alpha_i)\right|$ for each subject. Because data is circular, there are two potential midpoints in-between x_1 and y_1, we always choose the midpoint to minimize the error $\left|z_{1it} - \zeta_{\text{Avg}}(x_{1it}, y_{1it})\right|$. For R(0) because both issues are opposed, we pool observations across the two issues using sender symmetry, and minimize the objective $\sum_{t=16}^{20} |z_{1it} - (\alpha_i \cdot x_{1it} + (1 - \alpha_i) \cdot y_{1it})| + |z_{2it} - (\alpha_i \cdot y_{2it} + (1 - \alpha_i) \cdot x_{2it})|$.

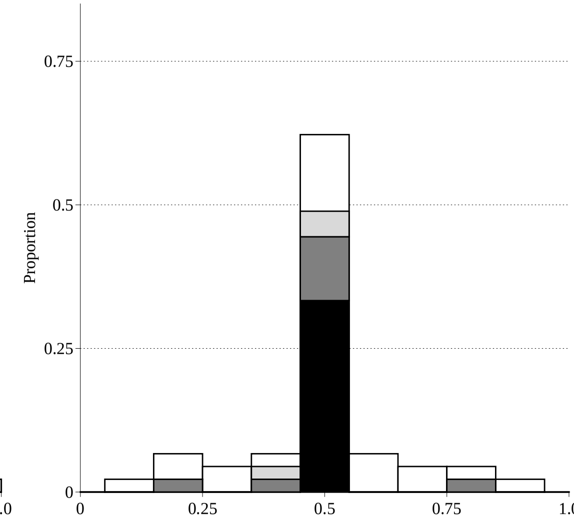
[25]Using the residuals, $\epsilon_{1it}(\hat{\alpha}_i) = z_{1it} - \zeta_{\text{Avg}}(x_{1it}, y_{1it}; \hat{\alpha}_i)$, we calculate subjects' goodness-of-fit via $R_i^2 := 1 - \hat{\text{Var}}(\epsilon_{ijt}(\hat{\alpha}_i)) / \text{Var}(\theta_j)$.



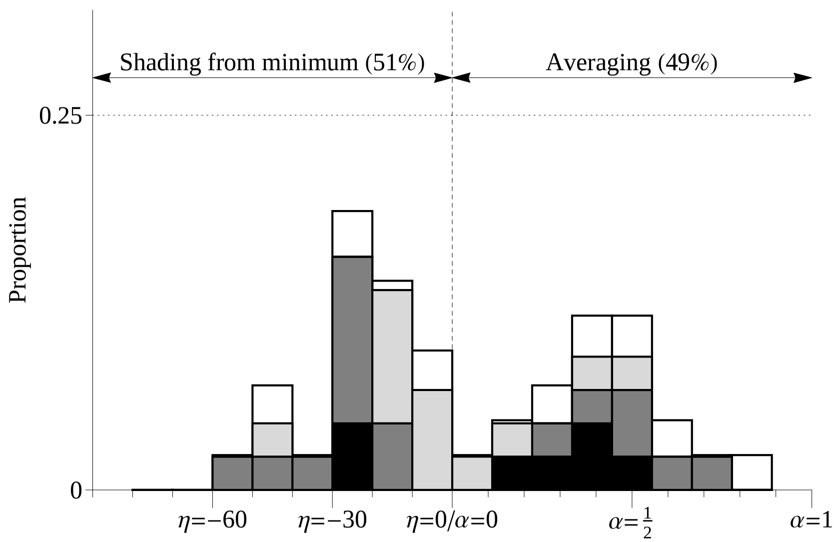
(A) **R(0)** opposed-sign averaging



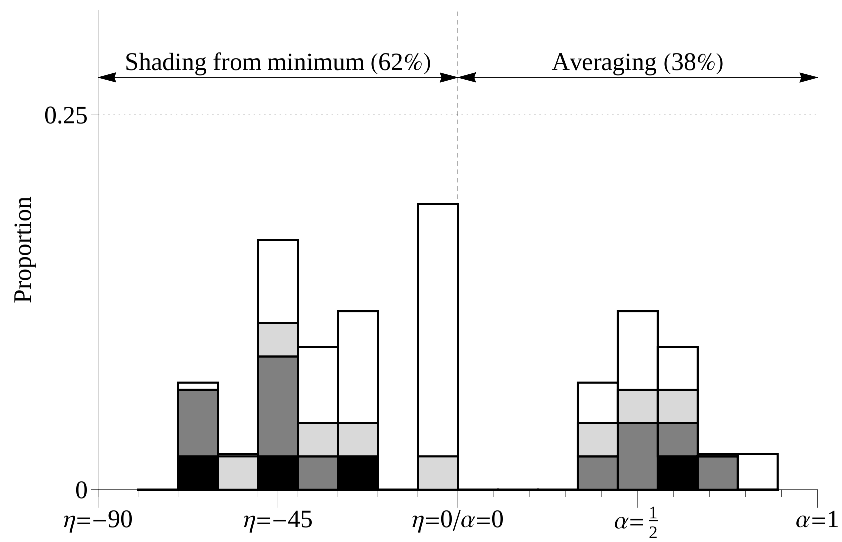
(B) **R(.6)** opposed-sign averaging



(C) **R(1)** opposed-sign averaging



(D) **R(.6)** same-sign shading/averaging



(E) **R(1)** same-sign shading/averaging

FIGURE 6. Subject-level within-issue behavior models

Note: Shading represents the subjects the assessed coefficient of determination R^2 relative to babbling within the issue. Shading represents R^2 greater than: black–0.99; dark gray–0.95; light gray–0.9 .

rational response, these are the same subjects with high receiver efficiencies in Figure 5. However, the remaining subjects' strategies seem to fall between those using a simple average (a form of *false equivalence*, given the known asymmetric sender biases in the issue) and those leaning more towards the less-biased sender.[26] In R(.6) the modal responses is for weights in 0.55–0.65, placing a greater weight on the recommendation from the sender with the bias of $-30°$, and less weight on the sender with the $+50°$ bias. Subjects are therefore again clustered around the bias-weighted average of 0.625. Finally, in R(1), we again see a large spike at the bias-weighted point, this time indicating subjects who use a simple averaging strategy, equally weighting the two recommendations. Given the responses of R(.6) and R(0), the mass of subjects in the 0.45–0.55 bin might best be thought of as the union of false-equivalence subjects, and those using a bias-weighted average.[27]

Same-sign Issue. When the senders have opposed biases, the recommendation difference *within the issue* allows receivers to observe the sum of the exaggerations, but provides no information on the relative exaggeration of each sender. In contrast, when senders have same-signed biases, the converse holds: within the issue the recommendation speaks to the relative exaggerations of each sender, but there is no information on the absolute magnitude. The clearest case for this is issue 2 in R(1). Both senders have the same bias relative to the receiver, $\delta_2^X = \delta_2^Y = +45°$. Given linear exaggerations in the bias direction, the observable recommendation difference is $y_2 - x_2 = (\kappa_Y - \kappa_X) \cdot \delta_2^X$. So, the recommendation difference is informative on which sender exaggerated by more, and by how much. But without the across-issue difference, the absolute values of κ_X and κ_Y are again not pinned down.

Examining receiver responses, subjects seem to use two main forms of response in same-signed issues: i) identifying the sender who exaggerated the least and shading the recommended point by a constant amount η, so the response is $\zeta_{\text{Min}}(x_j, y_j; \eta) := \min\{x_j, y_j\} + \eta$; or ii) averaging the two recommendations via $\zeta_{\text{Avg}}(x_j, y_j; \alpha)$. In order to distinguish between these two very-different types of response we estimate both models for each subject, and then provide the model-parameter $\hat{\eta}_i \in (-180, 0]$ for shading or $\hat{\alpha}_i \in (0, 1]$ for averaging, that attains the best fit from the two.[28]

[26]Mann-Whitney tests indicate significant differences across all treatments for the median $\hat{\alpha}_i$, with R(0) larger than R(.6) larger than R(1), at the 5 percent level.

[27]The results in R(0) and R(.6) also make clear the aggregate within-issue parameters assessed in Table III. However, the subject-level results here control for a common mistake in the first issue in the R(1) treatment, choosing the wrong arc. When both senders exaggerate to a large degree, receivers sometimes make choices on the "wrong" arc, where x_{1it} is clockwise and y_{1it} counter-clockwise, the opposite directions from the biases. This happens in 28 percent of the R(1) data. This error leads to a choice very far from the true state, and as such creates a bias in the GMM estimates. Controlling for this mistake (looking at the weight parameter α_i on the *chosen* arc) the subject-level estimates more clearly indicate the bias-averaging strategy.

[28]For $\zeta_{\text{Min}}(x_j, y_j; \eta)$ we use a circular variant of the min function: we find the least clockwise recommendation on the smallest arc connecting x_j and y_j. A parameter estimate $\hat{\eta}_i$ captures a fixed amount added to the minimal recommendation, which is found by minimizing $\sum_{t=16}^{20} \left| z_{2it} - \zeta_{\text{Min}}(x_{2t}, y_{2t}; \eta) \right|$ across $\eta \in (-180, 0]$. An estimate $\hat{\alpha}_i$, capturing a weighted averages response, is found by minimizing $\sum_{t=16}^{20} \left| z_{2it} - \zeta_{\text{Avg}}(x_j, y_j; \alpha) \right|$ across $\alpha \in (0, 1]$. We then compare each model through the fitted residual magnitudes, $\frac{1}{5}\sum_{t=16}^{20} \left| z_{2it} - \zeta_{\text{Min}}(x_{2t}, y_{2t}; \hat{\eta}_i) \right|$ and $\frac{1}{5}\sum_{t=16}^{20} \left| z_{2it} - \zeta_{\text{Avg}}(x_{2t}, y_{2t}; \hat{\alpha}_i) \right|$. The two models were chosen to match clear patterns while not overlapping, (the minimum shading model makes choices counterclockwise from the smallest arc, the averaging model makes choice on the smallest arc).

The subject-level distribution for the two same-signed issues—issue 2 in R(.6) and R(1)—are provided as histograms in Figure 6(E–F), with a non-linear scale to account for the differing model supports. We again illustrate the model's goodness-of-fit through the histogram bin shading.

Subjects whose response as a receiver matches the shading from the minimum strategy represent 51 percent of subjects in R(.6) and 62 percent of subjects in R(1). While the shading strategy represents the majority, a large minority acts as if averaging the recommendations. Interestingly, the subjects who are averaging do seem to react to the bias magnitudes, with the distribution closer to 0 in R(.6) than in R(1). For the subjects shading from the minimum recommendation, the treatments R(.6) and R(1) differ in the magnitude of the shading, with subjects in R(1) on average removing a larger amount.[29]

A common strategy seems to be removing one multiple of the bias from the minimal recommendation, matching an as-if conjecture that the more honest player recommends their ideal point, so $\min\{\kappa_X, \kappa_Y\} = 1$. In R(.6) the minimal sender is frequently the less-biased X, with a bias of $+30°$. The histogram makes clear that the larger mass of subjects (with good fit to the shading model) removes between $20°$ and $30°$ from the minimal recommendation. In contrast, the most common strategy estimated with an good fit in R(1) is to remove between $40°$ and $50°$ from the minimal recommendation.

Taken together the three treatments indicate the following general patterns to subjects' within-issue response:

i) In opposed-sign issues, the vast majority of subjects act as if computing a bias-weighted average. A small minority exhibits false equivalence, and computes simple averages of the senders' recommendations.
ii) The majority of subjects in same-signed issues act as if identifying the minimal recommendation, and removing a constant amount from it. The average amount removed responds to the treatment variable $\min\left\{\left|\delta_j^X\right|, \left|\delta_j^Y\right|\right\}$, the size of the least-biased sender. A (somewhat large) minority of subjects use averaging strategies.
iii) Subjects who do not adhere to one of our assessed within-issue models fare much worse than those who do.[30]

The modal response by subjects is in fact somewhat close to a *within-issue* restricted best response. Consider the within-issue family of strategies that takes averages with weight $\alpha \in [0, 1]$ on sender X in opposed issues and adds $\eta \in [-180, 0)$ to the minimal recommendation in same-signed issues. Within this family the best response parameters are $\alpha = 0.64$ and $\eta = -32°$ in R(.6), and $\alpha = 0.5$ and $\eta = -37°$ in R(1). Comparison with the modal response in Figure 6 makes clear that many subjects are close to this within-issue best response. The efficiency attainable with a within-issue response is relatively high, achieving 75 percent efficiency in R(.6) and 59 percent in R(1). In comparison, the efficiency attained by the multidimensional, sequentially rational response $\zeta_{\hat{\mathbf{r}}}$ (cf. Υ_S in Table C5) is 81 percent in R(.6) and 72 percent in R(1). However, though the within-issue response does well, outside of the R(0) special case, it cannot be part of an FRE, where

[29] This difference is not significant when looking at *all* shading-strategy subjects, however it is significant at the 1 percent level for subjects with good fit, an $R_i^2 > 0.9$, using a Mann-Whitney test.

[30] We break subjects into those with good fit ($R_i^2 > 0.95$) and those with poor fit ($R_i^2 < 0.95$) and examine subject efficiency Υ_i. Subjects with good fit to the averaging strategy in the *opposed-sign* issue have efficiencies 20–25 greater than those with a poor fit. When we look at the *same-signed* issue we find a similar efficiency efficiency gain for subjects with good-fit to a *shading from the minimum* response, in relation to those with poor fit to any response. Subjects with good-fit to an *averaging response* in the same-signed issue have efficiencies 5 percent lower than those with *poor fit* to either model.

the averaging strategies in particular give both senders an incentive to exaggerate further in those issues.[31]

4.4. **Robustness of Receiver Unidimensional Response.** A prominent result from comparing treatments R(0)–R(1) is subjects' failure to use across-issue information. To examine this result we conduct two robustness exercises. The first changes the graphical interface used within the experiment, and seeks to understand whether the presentation, which had subjects make separate decisions in each issue rather than jointly, might be producing the across-issue failures. The second exercise provides extra opportunities for subjects to learn the across-issue response. It also increases the statistical power to estimate the subject-level response, doubling the number of rounds each subject plays as a receiver. For both exercises the evidence indicates that the findings we reported above are not qualitatively changed.

Plane Interface. In our R(·) treatments, receivers make separate choices in each issue. The graphical environment presents recommendations on two separate circles, and the final choice is made by clicking in each in turn. It is possible that the strategic connections between issues are more readily understood when recommendations are presented jointly, and receivers make one multidimensional choice. Our first robustness exercise seeks to understand whether receivers integrate across-issue information when the geometric connection between issues is clearer. To address this concern we conducted two additional treatments, denoted P(0) and P(1), with exactly the same underlying strategic structure as R(0) and R(1), respectively. The only change is the graphical interface in the experiment. Instead of depicting states/recommendations/choices as angles on two separate circles, the interface depicts them as points on a two-dimensional square. The interface is similar to a Cartesian representation of a globe, where our graphical interface shows the toroidal state-space's fundamental polygon—opposed edges of the square are identified with one another. Choices for both senders and receivers in P(0) and P(1) are therefore inherently multidimensional, with receivers viewing senders' recommendations (bias directions) as points (vectors) on the plane.[32]

We conducted two sessions each of P(0) and P(1), recruiting 15 new subjects for each session. Procedures were nearly identical to the previous sessions, with the exception that the projected instruction presentation and subjects' instruction interface reflected the new interface.

Results from the two treatments P(0) and P(1) mirror our findings in R(0) and R(1), where we provide comparable figures to the R(·) treatments in Appendix A. Subjects use strategies as senders that are qualitatively close to *Restrictions A* and *B*, with a majority of subjects exaggerating in their bias direction, per *Restriction C*. As receivers, subjects perform very well in the P(0) treatment, with approximately half of the subjects mirroring the sequentially rational response. However, receiver subjects in P(1) again fare badly, and the two treatments are stochastically ordered, so that final efficiency distributions satisfy $\Upsilon(\mathrm{P}(0)) \succ_1^{\star} \Upsilon(\mathrm{P}(1))$.[33] Examining subjects' receiver

[31]Our companion paper, Vespa and Wilson (in preparation), compares the results from R(1) to two unidimensional environments, where senders have the same opposed- or same-signed bias in *both* issues. The results make clear that increased noise in R(1) mostly stems from increased exaggeration in any environment where receivers take averages.

[32]The interface initially centers the square on the point (180,180), and allows subjects to recenter on any coordinate, any number of times, by right-clicking with their mouse. Instructions for these additional treatments, which include extensive screenshots are included in the online appendices.

[33]Subjects actually do better in R(0) than P(0), and $\Upsilon(\mathrm{R}(0)) \succ_1^{\star\star} \Upsilon(\mathrm{P}(0))$, so the separation of issues might actually be beneficial. However, a similar exercise examining receiver's distance from the sequentially rational response (receiver efficiency) indicates that the difference in efficiency is due to greater noise in sender recommendations within

response we see the same patterns:strategic thinking within the horizontal and vertical coordinates presented (averaging/shading), but subjects continue to treat the issues independently, as if solving each coordinate in isolation.

Extended Sessions. Though many receiver subjects in our experiments do well through a non-linear, *within-issue* response, such response cannot be part of an equilibrium outcome in R(.6) and R(1). In particular, any response by receivers that chooses a midpoint strictly between the within-issue recommendations provides an incentive for greater exaggeration by senders. Matching this idea, greater noise in the R(1) exaggerations is at least partly due to some sender subjects exaggerating more in the opposed-sign issues, where receivers response is a simple average. For the FRE to succeed in the long-run, it must be that receivers learn to incorporate across-issue information. To examine whether greater experience might lead to across-issue inferences, our second robustness exercise increases the number of periods subjects act as a receiver in part two of the session, while holding constant the senders' response.

We ran two further sessions (recruiting 24 subjects), for an extended version of R(1), which we label E(1). Subjects in E(1) followed identical procedures to R(1) in the first fifteen rounds—as such we incorporate the sender data into our R(1) analyses.[34] The difference in treatment came in the second part of the E(1) sessions. Instead of five rounds where all subjects are receivers (facing states/recommendations from the same session) we ran the second part for 15 rounds, using state/recommendation data randomly chosen (without replacement) from rounds 11–15 in the three previous R(1) sessions. Because data came from previous sessions, all 24 subjects in both E(1) sessions made choices $\mathbf{z}_{it}$ in reaction to an identical sequence of underlying states and recommendations, $\{\boldsymbol{\theta}_t, \mathbf{x}_t, \mathbf{y}_t\}_{t=16}^{30}$.

We present a more detailed analysis in Appendix D, which we summarize here. Performing the within-issue estimation detailed in section 4.3, we classify 14 out of 24 subjects as taking the midpoint in issue 1 and shading from the minimum in issue 2. Because the average efficiency for this group is more than double the efficiency for the remaining subjects we focus our analysis on this subset of E(1)'s subject pool. In particular we examine whether they start to use across-issue information with more experience.

While we do observe some efficiency gains as the session evolves, we cannot attribute those gains to the median subject using across issue information. In fact, compared to an average/shade within-issue response, subjects' relative efficiency decreases slightly. To assess whether subjects use across-issue information we examine the difference in their response compared to a fixed within-issue strategy: a simple average in issue 1, shading 45° from the minimal recommendation in issue 2. We then test whether these differences are related to the recommendation difference across issue: are deviations from the midpoint in issue 1 related to $y_2 - x_2$; is the amount of shading in issue 2 related to $y_1 - x_1$. Under a null of independence, we test across-issue information use by subject in each issue.

We find that there is a small minority for whom we can reject independence in one of the two issues. However, there are no subjects for whom we reject independence in both issues. We

P(0). In comparison, we find no significant first-order stochastic relationships between $\Upsilon\left(\mathrm{R}(1)\right)$ or $\Upsilon\left(\mathrm{P}(1)\right)$, nor indeed the extended treatment $\Upsilon\left(\mathrm{E}(1)\right)$ we detail below.

[34]We do not find any statistical differences between the E(1) sessions and R(1) sessions in the first 15 rounds with respect to senders' and receivers' choices.

conclude that while it is possible that some subjects begin to incorporate across-issue information, it is not the case that the large majority of subjects stop treating issues independently.[35]

5. DISCUSSION

The data indicate clear patterns for both senders and receivers in a multidimensional, multi-sender cheap talk setting. A large majority of senders can be modeled as using a strategy that exaggerates in the direction of their own bias. In our experimental setting this behavior intersects with a component strategy in a fully revealing equilibrium. Receivers, however, do not use information in the optimal manner, and fail to adjust their choice in one dimension with information learned in another. Instead, their response treats each dimension independently. Though many receiver's within-dimension response shows strategic sophistication with a significant response to the biases, their behavior is not consistent with multidimensional sequential rationality. In our rotated environments final outcomes are far from full-revelation, and we observe increased noise in sender response. However, in one case, R(0), the *specific* framing of the multidimensional problem makes the receiver best response entirely within-issue. In this treatment, not only are the final outcomes qualitatively close to full-revelation, but the component behavior for both senders and receivers forms a mutual best response.[36]

Our findings suggest several avenues for future research on behavioral models. The data illustrates a failure of sequential rationality. Despite calculating fairly sophisticated conditional expectations within each dimension, subjects completely fail to understand connections between choice dimensions. Because of this failure of sequential rationality, behavioral concepts may better organize the data than equilibrium predictions. One alternative would be to extend models such as Jehiel (2005)'s analogy-based expectations, which attempt to modify equilibrium thinking to allow conditional expectations to be miscalculated. Our receivers are effectively failing to compute a conditional expectation: instead of assessing $\mathbb{E}\left(\theta_a \left| \mathbf{y} - \mathbf{x} \right.\right)$ in each dimension a (conditioning on the vector difference $\mathbf{y} - \mathbf{x}$), many subjects act as if calculating $\mathbb{E}\left(\theta_a \left| y_a - x_a \right.\right)$.[37]

Related cheap talk papers have modeled sender/receiver behavior (in one-sender, one-dimension settings) with level-*k* models, with agents best-responding to simple conjectures about the other

[35]Our findings suggest that learning, even if it occurs, is not fast. It is still possible that with a much longer horizon more subjects stop treating issues separately.

[36]Except for R(0), the joint behavior of senders and receivers does not constitute an equilibrium. Given that receivers' behavior is not consistent with sequential rationality, senders could benefit from further exaggerating. The finding that senders are not using an empirical best response is common to experiments with one sender and one dimension (e.g.Cai and Wang (2006)). One approach in the literature uses disequilibrium concepts (level-k, cognitive hierarchy) to better organize the data. Below we comment on the challenges that these approaches face to rationalize our data.

[37]In this sense, one can think of extensions to the Jehiel (2005) model to allow analogy classes within subspaces of the problem rather than simply subsets, where subjects take unconditional expectations over the variable $y_b - x_b$, as if responding to $\mathbb{E}\left[\mathbb{E}\left(\theta_a \left| \mathbf{y} - \mathbf{x} \right.\right) \left| y_a - x_a \right.\right]$. Alternatively, a simple dimension-specific agent-form model can accommodate the findings. In this model, receivers simply believe that they are responding to recommendations from a different pair of senders in each dimension.

player's response.[38] However, for level-*k* models to explain behavior in sequential multi-agent environments such as ours, they must address how conjectures on others respond to observed play. With multiple senders and multiple issues, receivers in our environment obtain many observables, and many conjectures over senders' behavior can be falsified by comparing recommendations. Any level-*k* model of behavior must address how agents modify their conjectures in response to conflicting information. Our data offers some examples for how subjects respond. On the one hand, when subjects are confronted with information about senders' exaggerations within dimension, they readily adapt. Where senders' biases have opposed signs, receiver response mirrors that of an updated *symmetric* conjecture over exaggerations.[39] When senders' biases have the same sign, our results point to receivers understanding the realized asymmetry, and shading from the identified minimal sender. On the other hand, our results suggest a stark failure to resolve conflicts in the conjectured response *across* dimensions. Receivers' behavior reveals an inconsistency in conjecture over each individual sender. For example, a receiver might react as if a specific sender is naively truthful in one dimension, but that same sender has exaggerated by multiples of his bias in another dimension—they are fully cursed over dimensions of choice. Any extensions of level-k to multi-sender, multi-dimension settings needs to address the extent to which conjectures are constrained and updated with respect to observables. Our evidence suggests that conjectures across dimensions are unresponsive, while our within-issue models of response suggest much greater receiver sophistication.

Our results suggest several directions for future experimental research on multidimensional cheap talk. An unsettling finding is that the behavior of senders and receivers in our rotated treatments is not a mutual best response. While we do not observe senders displaying quantitatively large deviations from bias-direction exaggerations (at the aggregate level), it is possible that greater experience may have an effect. One natural way to investigate this in the laboratory would be with computers taking the role of receivers, following some predetermined strategy.[40] Two types of computer strategy would seem natural candidates: the equilibrium strategy $\zeta^\star(\mathbf{x}, \mathbf{y})$, and the behaviorally focal within-issue strategy (averaging in opposed-sign issues, shading one multiple of the bias in same-signed issues). In particular, response to a computerized within-issue strategy would help understand the steady state behavior, whether outcomes converge to low-efficiency, babbling-like outcomes, or whether some partially revealing equilibrium emerges. Sender behavior with computers programmed to use the equilibrium response $\zeta^\star(\mathbf{x}, \mathbf{y})$ in an R(1)-like setting would help understand the degree to which the increased noise we observe from senders in R(1) is caused by receiver's within-issue response. Additionally, experiments using computerized receivers would be useful for understanding collusion. For example, a common market interest rate used in contracts (the London Interbank Offered Rate) is set via periodic survey of banks' borrowing rates, where the published rate is a trimmed mean. Ongoing investigations by various fiduciary

[38]Cai and Wang (2006) find support for a level-k behavioral model in a Crawford-Sobel setting, while Wang, Spezio, and Camerer (2010) further analyze subject behavior with eye-tracking data, providing further evidence for level-k explanations (with further evidence in Kawagoe and Takizawa 2009.). A companion paper to this, Vespa and Wilson (in preparation), provides evidence from the same toroidal interface with just one sender, uncovering very similar level-*k* behavior to the previous literature.

[39]Indeed, even the subjects who are closest in spirit to a level-0 response have a symmetric conjecture. Subjects exhibiting false equivalence (both in opposed and same-sign issues) can be thought of as responding to a belief of truthful response for each sender with symmetric white noise in recommendations.

[40]In addition to receiver automation, similar exercises with automated senders could help identify the conditions/instruction required for receivers to begin incorporating across-issue information.

authorities have alleged collusion on the part of the bankers being surveyed. Automated-receiver experiments might help understand the coordination requirements for successful collusion, and whether multidimensional reporting could inhibit it.

Another aspect of our experimental design that might be further explored is sender response. Our treatments do not address whether senders' behavior—exaggerating in the direction of their own bias—is an artifact of the orthogonal biases, or whether this behavior holds more generally. Environments with linearly independent but non-orthogonal biases have differing equilibrium predictions, and would help provide evidence on how senders behave in reaction to generic competing voices. Similarly, varying other environmental variables would help provide greater insight into receiver response. How do subjects respond to senders with differing bias magnitudes? When the prior provides a focal choice does this influence the response? Greater experimental variation here will help refine our understanding.

A related open question is the degree to which interested parties select experts with particular biases. Do receivers choose experts with large biases but in R(0)-like directions, and fully extract, or do they instead choose experts with smaller biases in R(1)-like directions? When we seek out second opinions, do we select those with diametrically opposed interest? Similarly, do interested parties (for instance, the defending/prosecuting sides in a jury trial) select experts to take advantage of receiver's heuristics? Further exploration of these questions would seem fruitful.

Finally, while our results might lead us to be skeptical toward the FRE in generic situations, from a policy perspective, our results do suggest a behavioral avenue. The FRE's theoretical existence result is generic. In contrast, our experimental findings indicate success only in a special case. However, the full-revelation existence proof works by showing that the generic environment, through a change in multidimensional frame, is the special case. Behaviorally, we can use this result, but in the opposite direction. In situations where policymakers can manipulate the frame—through mandating specific reporting metrics, instructing jurors on how to understand testimony, manipulating the message space, and so on—generic environments might be translated into an R(0)-like setting.[41] In this sense, environments with low-information transmission, such as R(.6) or R(1), might be avoided by institutional designs aware of human behavior. By changing the frame, it might be possible to affect equilibrium selection.

6. CONCLUSION

We examine three cheap-talk environments with multiple senders providing recommendations to a receiver in multiple dimensions. In each environment, full revelation can be supported as an equilibrium outcome using a similar construction to that in Battaglini (2002), where our treatment variables are reframings, rotations of the coordinate system. Though senders' strategic response is qualitatively similar across the three environments, there are large differences in the amount of information extracted by receivers, and whether the observed strategies are consistent with an equilibrium outcome. In one frame, when each sender is perfectly aligned with the receiver only in one dimension, $R(0)$, the majority of receivers succeed in extracting information, and come close to matching the fully revealing equilibrium. The majority of subjects in all roles use strategies that form a fully revealing mutual best response.

However, when we reframe the environment, and the sequentially rational response has receivers combine information across dimensions, we document substantial failures. Our results

[41] Indeed, the results of Lai et al. show that message-space restrictions can increase information transfer.

suggest that receivers treat each choice dimension independently, as if solving two isolated one-dimension problems. Therefore, though there are particular frames under which fully revealing equilibria emerge, generically, their selection seems unlikely. However, the results do suggest a potential policy intervention: if it is possible to reframe an environment by bringing it closer to $R(0)$, potentially large gains in information transmission can result.

Our paper offers a simple, positive description of how receivers process conflicting advice–averaging opposed interest senders according to their biases, directly following unbiased senders, and shading from the minimum for senders with aligned-interests. The results indicate that most subjects do not suffer from false-equivalence, that they react to the magnitudes and directions of bias, and by and large follow sensible heuristics. Where they fail is in their treatment of multidimensionality, treating strategically connected dimensions as separate.

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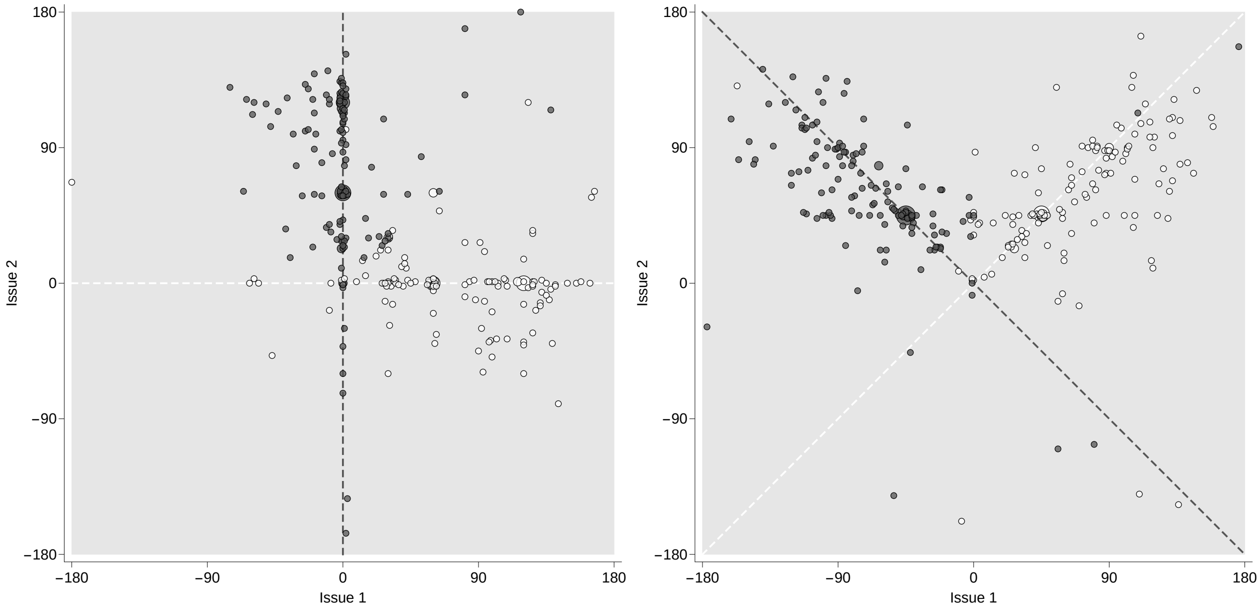


(A) **P(0)** (B) **P(1)**

FIGURE A7. Sender Exaggerations

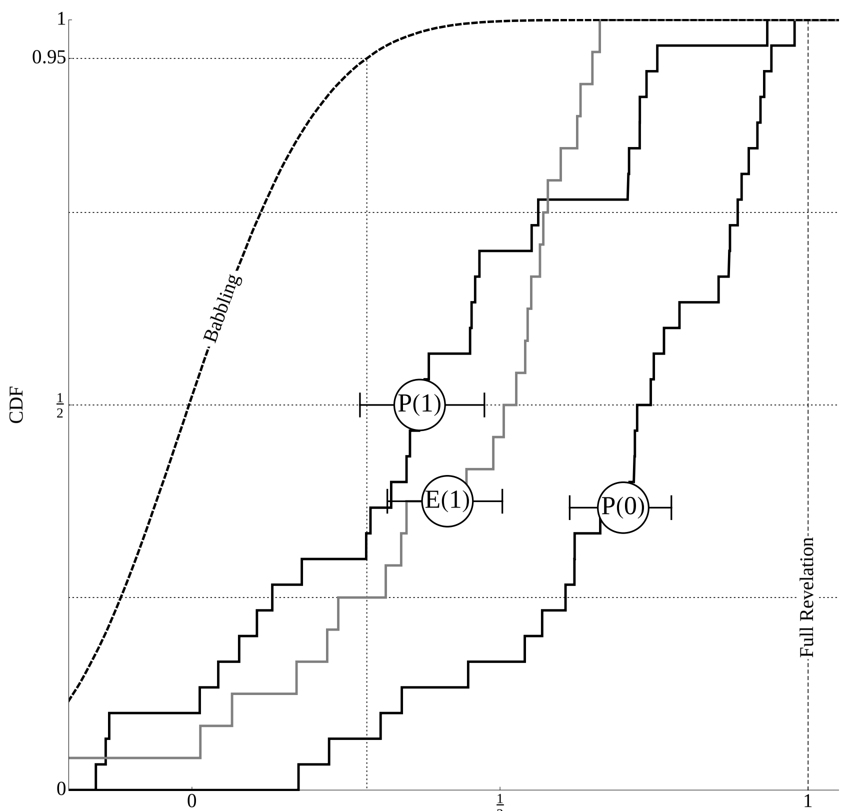
TABLE A4. Best Affine Receiver Strategy

	Receiver Strategy					
	Within Issue		**Across Issue**		**Offset**	
	$\bar{\alpha}_1$	$\bar{\alpha}_2$	$\bar{\beta}_1$	$\bar{\beta}_2$	$\bar{\eta}_1$	$\bar{\eta}_2$
R(0)	1.00	1.00	0.00	0.00	0.0	0.0
R(.6)	0.69	0.62	-0.43	-0.24	-2.9	-19.2
R(1)	0.48	0.52	-0.38	-0.24	-2.5	-25.7

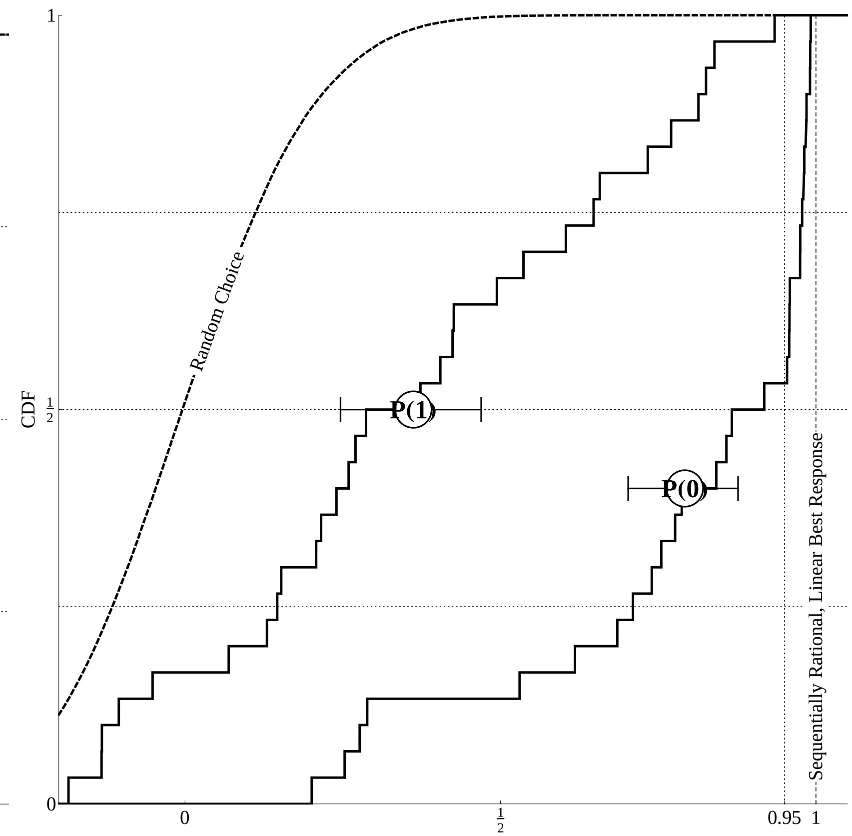
Note: The strategies above minimize the expected choice error for a receiver using the following affine response

$$\zeta(\mathbf{x},\mathbf{y}) = \left[\begin{array}{cc|cc} \alpha_1 & -\beta_1 & 1-\alpha_1 & \beta_1 \\ -\beta_2 & 1-\alpha_2 & \beta_2 & \alpha_2 \end{array}\right] \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix},$$

against the observed message pairs $(\mathbf{x},\mathbf{y})$.



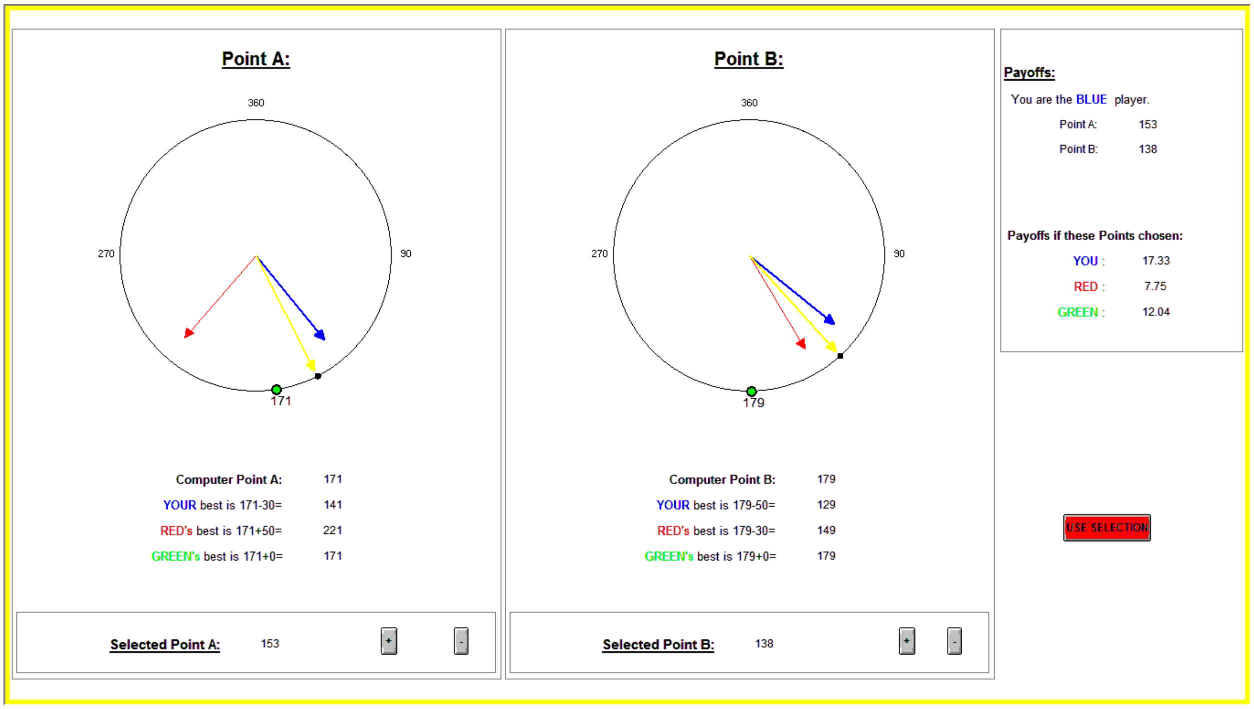
(A) Absolute Efficiency



(B) Receiver Efficiency

FIGURE A8. CDFs for subject's average decision efficiency

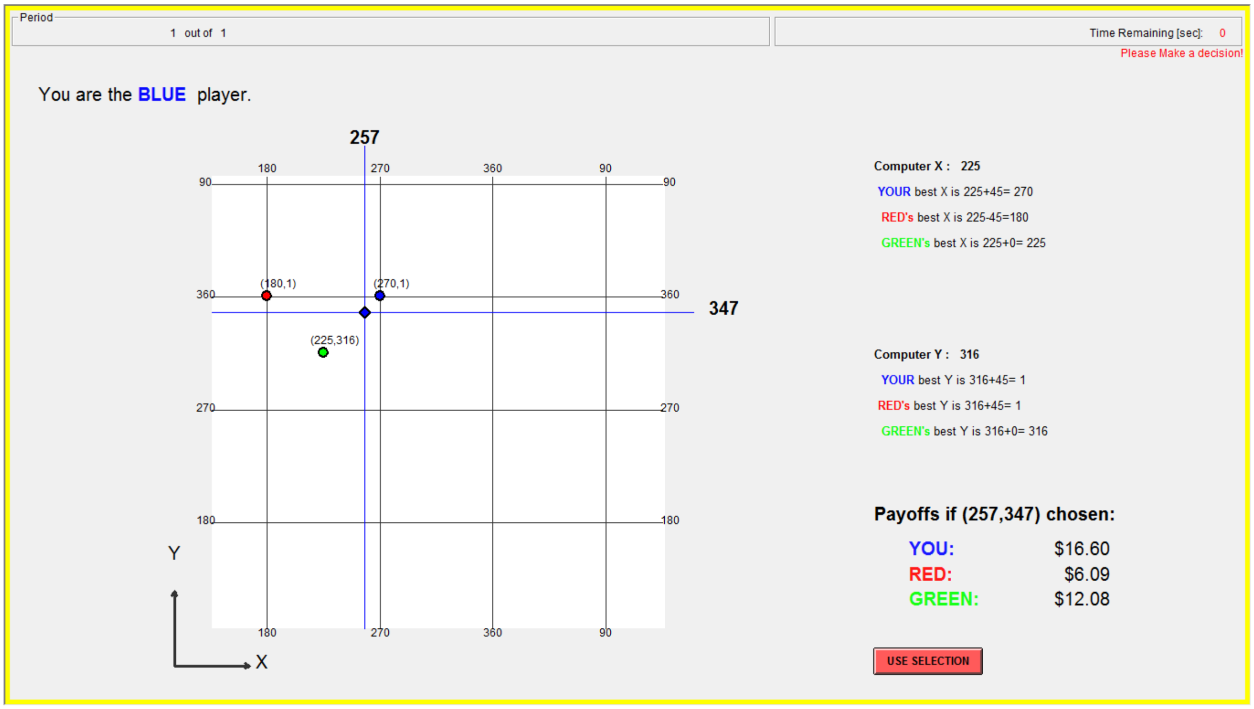
Note: Plot labels are located at the average efficiency level within the treatment; bars through the plot label represent the 95 percent confidence region for the distribution mean, extracted from a bootstrap.



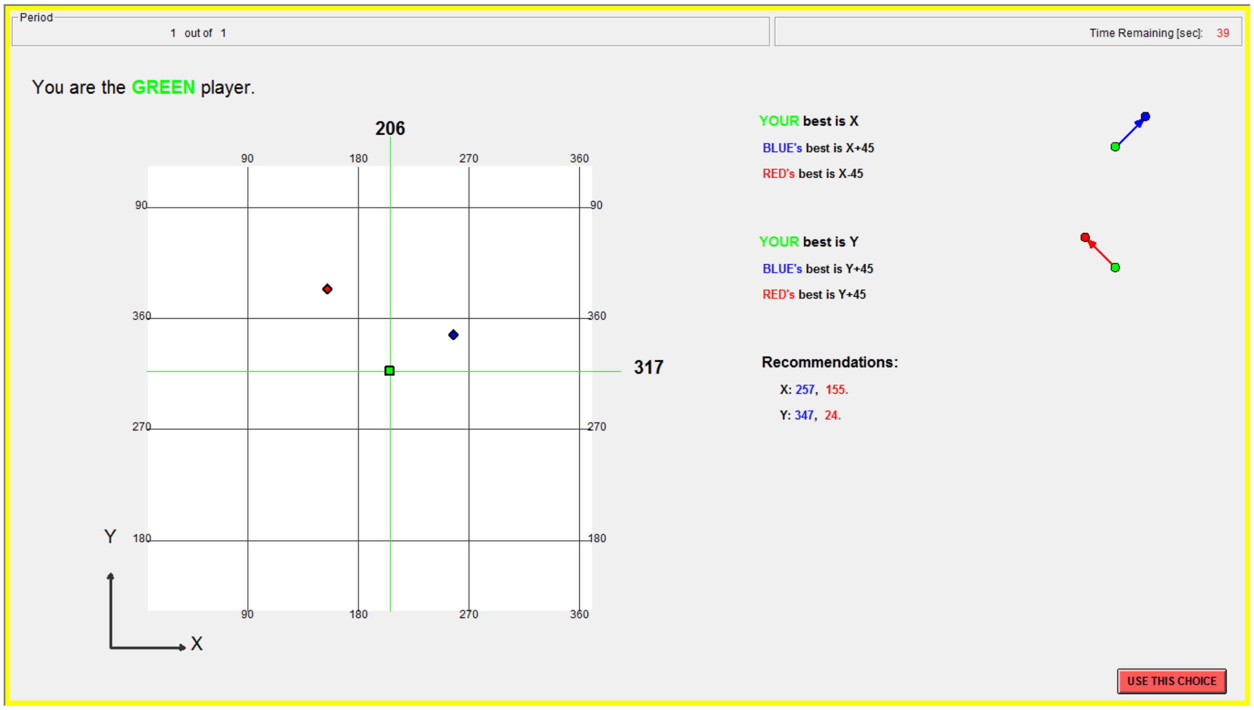
(A) Sender Interface

(B) Receiver Interface

FIGURE A9. Main Interface Screenshots



(A) Sender Interface



(B) Receiver Interface

FIGURE A10. Plane Interface Screenshots

APPENDIX B. PROOFS AND CONSTRUCTION ON TORUS

Proof of Proposition 1.

Proof. The sequentially rational response for all recommendations $(\mathbf{x}, \mathbf{y})$ solves

$$\min_{\mathbf{z}} \mathbb{E}_{\boldsymbol{\theta},\kappa_X,\kappa_Y} \{\|\mathbf{z} - \boldsymbol{\theta}\| \,|\mathbf{x}, \mathbf{y}\},$$

given beliefs that senders follow the strategy (1) and (2). The receiver has 4 equations in four unknowns, θ_1, θ_2, κ_X and κ_Y. [42]

Because $\boldsymbol{\Gamma} = \left[\begin{array}{cc} \boldsymbol{\gamma}^X & \boldsymbol{\gamma}^Y \end{array}\right]$ has full rank, the posterior belief over $\boldsymbol{\theta}\,|\mathbf{x}, \mathbf{y}$ is degenerate. The sequentially rational response is

$$\zeta_{\boldsymbol{\Gamma}}(\mathbf{x}, \mathbf{y}) = \left[\begin{array}{cc} \mathbf{0} & \boldsymbol{\gamma}_Y \end{array}\right] \boldsymbol{\Gamma}^{-1}\mathbf{x} + \left[\begin{array}{cc} \boldsymbol{\gamma}_X & \mathbf{0} \end{array}\right] \boldsymbol{\Gamma}^{-1}\mathbf{y}, \tag{5}$$

for any inferred exaggeration levels $\left(\begin{array}{cc} -\hat{\kappa}_X, & \hat{\kappa}_Y \end{array}\right)' := \boldsymbol{\Gamma}^{-1}(\mathbf{y} - \mathbf{x})$, in the support of F_X and F_Y, respectively. This receiver's response transforms each sender's recommendation into an exaggeration-direction coordinate through the transformation $\boldsymbol{\Gamma}^{-1}$. Given the sender's strategies

$$\xi_X(\boldsymbol{\theta}) = \boldsymbol{\theta} + \boldsymbol{\Gamma} \left(\begin{array}{c} \kappa^X \\ 0 \end{array}\right) \text{ and } \xi_Y(\boldsymbol{\theta}) = \boldsymbol{\theta} + \boldsymbol{\Gamma} \left(\begin{array}{c} 0 \\ \kappa^Y \end{array}\right),$$

the receiver's response selects the $\boldsymbol{\gamma}_Y$-coordinate from X, and the $\boldsymbol{\gamma}_X$-coordinate from Y, which results in the final choice $\zeta_{\boldsymbol{\Gamma}}(\xi_X(\boldsymbol{\theta}), \xi_Y(\boldsymbol{\theta})) = \boldsymbol{\theta}$, the maximal outcome for the receiver. The proposition characterization follows from $\left[\begin{array}{cc} \mathbf{0} & \boldsymbol{\gamma}_Y \end{array}\right] \boldsymbol{\Gamma}^{-1} + \left[\begin{array}{cc} \boldsymbol{\gamma}_X & \mathbf{0} \end{array}\right] \boldsymbol{\Gamma}^{-1} \equiv \mathbf{I}$, and the fact that $\text{tr}\left\{\left[\begin{array}{cc} \mathbf{0} & \boldsymbol{\gamma}_Y \end{array}\right] \boldsymbol{\Gamma}^{-1}\right\} = \text{tr}\left\{\boldsymbol{\Gamma} \left[\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right] \boldsymbol{\Gamma}^{-1}\right\} = 1$, by circularity of the trace operator. □

Proposition 3. *A fully revealing perfect Bayesian equilibrium in the two-senders on two-circles cheap-talk game exists when: i) senders' biases satisfy* $\left\|\boldsymbol{\delta}^X\right\|, \left\|\boldsymbol{\delta}^Y\right\| < \sqrt{5} \cdot 45°$*; ii)* $\boldsymbol{\Delta}$ *has full rank and* $\boldsymbol{\delta}^X \perp \boldsymbol{\delta}^Y$.

Proof. Given the bias vector $\boldsymbol{\Delta} = \left[\begin{array}{cc} \boldsymbol{\delta}^X & \boldsymbol{\delta}^Y \end{array}\right]$ define the normalized matrix $\boldsymbol{\Gamma} = \left[\begin{array}{cc} \boldsymbol{\gamma}^X & \boldsymbol{\gamma}^Y \end{array}\right] := \left[\begin{array}{cc} \frac{\boldsymbol{\delta}^X}{\|\boldsymbol{\delta}^X\|} & \frac{\boldsymbol{\delta}^Y}{\|\boldsymbol{\delta}^Y\|} \end{array}\right]$. We will examine the case where $\gamma_1^Y, \gamma_2^Y, \gamma_2^X \geq 0$ and $\gamma_1^X \leq 0$, (where similar argument holds for other orthogonal vector directions, where we can simply change the positive direction, and/or relabel the coordinates). Define the clockwise angular measure

$$\phi(x) := \text{mod}\,(x, 360°)\,.$$

and the minor arc length (clockwise as positive) as

$$\phi_d(x) := \phi(x + 180°) - 180°,$$

where we let both functions act coordinate-by-coordinate on vectors. Let senders' message strategies be

$$\begin{aligned} \xi_X^\star(\boldsymbol{\theta}) &= \phi\left(\boldsymbol{\theta} + \kappa_X \cdot \gamma^X\right), \\ \xi_Y^\star(\boldsymbol{\theta}) &= \phi\left(\boldsymbol{\theta} + \kappa_Y \cdot \gamma^Y\right), \end{aligned}$$

[42] Whenever the support for the exaggerations means that the approximate position of $\boldsymbol{\theta}$ relative to $\mathbf{x}$ and $\mathbf{y}$ can always be worked out—one sufficient condition being that $\kappa^i/\max\{\delta_1^i, \delta_2^i\} \in [\sigma°, 180° - \sigma°)$ for both senders—then the analysis on the plane illustrated in the paper's Figure 2 is going to be equivalent to working on the two circles.

where in general we could consider strategies to be mixtures over κ_X and κ_Y, however, for simplicity and to prove the best case for existence, let $\kappa_X = \kappa_Y = 0$ on the path. Define the recommendation difference as $\nabla(\mathbf{x},\mathbf{y}) = \phi_d(\mathbf{y}-\mathbf{x})$. The receiver's response to all message pairs $\mathbf{x},\mathbf{y} \in \Theta := [0,360)^2$ is given by:

$$\zeta^\star(\mathbf{x},\mathbf{y}) = \phi\left(\mathbf{x} + \tfrac{1}{2}\nabla(\mathbf{x},\mathbf{y}) - \mathbf{\Gamma}\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\mathbf{\Gamma}^{-1}\nabla(\mathbf{x},\mathbf{y})\right),$$

with corresponding beliefs that $\boldsymbol{\theta} = \zeta^\star(\mathbf{x},\mathbf{y})$ with certain probability for all $\mathbf{x},\mathbf{y}$ (per the paper's Proposition 1, except that the final choices are constrained to be on the circle here).

Along the path the arc length is always $\nabla(\mathbf{x},\mathbf{y}) = \mathbf{0}$ and the arc midpoint is the state, $\mathbf{x} + \frac{1}{2}\nabla(\mathbf{x},\mathbf{y}) = \boldsymbol{\theta}$, so the chosen point is $\zeta^\star(\xi^\star_X(\boldsymbol{\theta}), \xi^\star_Y(\boldsymbol{\theta})) = \boldsymbol{\theta}$.[43] The receiver's reaction is therefore sequentially rational, and the on path beliefs are consistent, while all possible recommendations lead to an inferred point $\zeta^\star(\mathbf{x},\mathbf{y}) \in \Theta$.

What remains to be shown is that senders can not unilaterally deviate. Let sender X choose an arbitrary recommendation $\mathbf{x} = \phi(\boldsymbol{\theta} + \tilde{\mathbf{x}}) \in \Theta$ where we take $\tilde{x}_1, \tilde{x}_2 \in [-180°, 180°)$, and Y follows the strategy. The final outcome is a point in the $\boldsymbol{\gamma}^Y$ direction from $\boldsymbol{\theta}$ given by

$$\zeta^\star(\boldsymbol{\theta} + \tilde{\mathbf{x}}, \boldsymbol{\theta}) = \phi\left(\boldsymbol{\theta} - \left(\gamma^X_2 \tilde{x}_1 - \gamma^X_1 \tilde{x}_2\right)\cdot \boldsymbol{\gamma}^Y\right).$$

By orthogonality and unit length, $\left|\gamma^X_i \gamma^Y_i\right| \leq \frac{1}{2}$. Given the domains for $\tilde{x}_1$ and $\tilde{x}_2$, the final choice can be located anywhere in $\left\{\phi\left(\boldsymbol{\theta} + \lambda\cdot\boldsymbol{\gamma}^Y\right) \middle| \lambda \in \left(-180°(\gamma^X_2 - \gamma^X_1), 180°(\gamma^X_2 - \gamma^X_1)\right)\right\}$.The direction $\boldsymbol{\gamma}^Y$ locally separates X's upper contour set, however, because of the circular topology, we need to make sure that it does so globally. We need to check that exaggerations cannot lead to an inference so far along the vector $\boldsymbol{\gamma}^Y$ that the response $\zeta^\star(\boldsymbol{\theta} + \tilde{\mathbf{x}}, \boldsymbol{\theta})$ gets close to $\boldsymbol{\theta} + \boldsymbol{\delta}^X$ from the opposite direction, given cyclicality every $360°$ units. The condition then is that for all feasible λ

$$\left\|\phi_d\left(\boldsymbol{\theta} + \lambda\cdot\boldsymbol{\gamma}^Y - \boldsymbol{\delta}^X - \boldsymbol{\theta}\right)\right\| \geq \left\|\phi_d\left(\boldsymbol{\theta} - \boldsymbol{\delta}^X - \boldsymbol{\theta}\right)\right\|.$$

A sufficient condition for this is that bias has a magnitude $\left\|\boldsymbol{\delta}^X\right\|$ smaller than $\sqrt{5}\cdot 45°$.[44] The X sender is therefore indifferent over any exaggeration that satisfies $\left(\gamma^X_2 \tilde{x}_1 - \gamma^X_1 \tilde{x}_2\right) = 0$, but strictly prefers these points to any other. □

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[43] For mixed exaggeration strategies, this is also true so long as the distributions are chosen so that $\kappa_Y \gamma^Y - \kappa_X \gamma^X \in (-180, 180)^2$. A sufficient condition for this is that $\kappa_X, \kappa_Y \in (-90°, 90°)$.

[44] This is the bias level at which a circle of radius $\left\|\boldsymbol{\delta}^X\right\|$ centered at the point $\begin{pmatrix} \theta_1 + \delta^X_1 + 360° \\ \theta_2 + \delta^X_2 \end{pmatrix}$ touches the circle of radius $180°(\gamma^X_2 - \gamma^X_1)$ centered at $\boldsymbol{\theta}$, which occurs when the coordinate system is rotated by $\Psi = \arctan\left\{\sqrt{5} - 2\right\}$. For different coordinate-system rotations this constraint on the biases will be slack.

Appendix C. Sender Behavior Analysis

To examine *Sender Restriction B* and the subsequent *Equilibrium Restriction C,* we estimate the best-fitting exaggeration direction for sender X in each treatment, given by the unit length vector $\boldsymbol{\gamma}^X(\omega) := \begin{pmatrix} -\sin(\omega+\psi) & \cos(\omega+\psi) \end{pmatrix}'$. The parameter ω will be estimated, while ψ is the treatment rotation, so that estimates of $\omega = 0$ indicate exaggerations in the bias-direction for X. For any specific exaggeration direction $\boldsymbol{\gamma}^X(\omega)$, we can decompose an observed exaggeration $\tilde{\mathbf{x}}_{it}$ into the exaggeration in the $\boldsymbol{\gamma}^X(\omega)$-direction ($\kappa_{it}(\omega) = -\tilde{x}_{1it}\sin(\omega+\psi)+\tilde{x}_{2it}\cos(\omega+\psi)$), and a residual in the orthogonal direction ($\epsilon_{it}(\omega) = \tilde{x}_{1it}\cos(\omega+\psi) + \tilde{x}_{2it}\sin(\omega+\psi)$). To estimate the best-fitting exaggeration direction , we find the ω that minimizes.

$$Q(\omega;\psi) = \sum_{i,t} |\epsilon_{it}(\omega)| = \sum_{i,t} |\tilde{x}_{1it}\cos(\omega+\psi) + \tilde{x}_{2it}\sin(\omega+\psi)|.$$

So the estimated parameter $\hat{\omega}$ minimizes the expected deviation from the chosen exaggeration direction. Together, *Sender Restriction B* and *Equilibrium Restriction C* imply no variation outside of a specific vector direction (so $\min_\omega Q(\omega;\psi) = 0$) and that $\omega = 0$, respectively.

We pool exaggerations from subjects in both sender roles in each treatment and provide estimates for the exaggeration angle $\hat{\omega}$, and sample averages for the on-ray exaggeration $\kappa_{it}(\hat{\omega})$ and the off-ray error size $|\epsilon_{it}(\hat{\omega})|$ in Table C5.[45] The intuition suggested from Figure 4 clearly matches the estimates: the best-fit exaggeration directions are qualitatively close to the bias/equilibrium directions across all three treatments. In the case of R(1), the best-fit direction is significantly different from zero (at the 5 percent level), where the positive sign reflects exaggerations with a larger magnitude in issue 1 (the direction where the two senders have opposed biases) than issue 2 (where the senders have the aligned biases). However, from a quantitative point of view the difference with the equilibrium direction is small: the angle of the estimated exaggeration direction differs from the equilibrium by approximately $\frac{\pi}{100}$ radians.

In addition to estimates for the exaggeration angle ω at the aggregate level (the important variable for receivers given random matching), we also examined exaggerations at the subject-level, computing angle estimates $\hat{\omega}_i$ using the same objective $Q(\cdot)$ over the ten rounds each subject was a sender. Results from subject-level estimates are illustrated in Figure 4 as angular histograms in each plot, where we illustrate the estimated sender X direction $\hat{\omega}_i+\psi$. The clearest pattern in the estimates is that the mode in each treatment is to exaggerate in the direction of the bias: 83 percent of subjects in R(0) have an estimated exaggeration direction within a $\frac{\pi}{36}$ radian cone of the bias direction, where this figure is 50 percent in R(.6) and 46 percent in R(1). However, a secondary pattern is the proportion with estimates counterclockwise from the bias in rotated treatments—a third of subjects in R(.6) and just under a half in R(1).

Despite the good fit in terms of the direction (*Restriction C*), the magnitudes of the off-ray exaggerations components are not negligible, and so we cannot accept *Restriction B* at face value. Sample-average magnitudes for residuals, $\hat{\mathbb{E}}\,|\epsilon_{it}(\hat{\omega})|$, are listed in Table C5, where *Restriction B* implies this number is at the boundary, with zero variation.[46] The exaggeration strategy makes a

[45] Exaggerations from sender Y, $\tilde{\mathbf{y}}_{it}$, are reflected through the line passing through the origin and the average bias $\frac{1}{2}(\boldsymbol{\delta}^X + \boldsymbol{\delta}^Y)$ to produce an effective exaggeration $\tilde{\mathbf{x}}_{it}$. The symmetric Y exaggeration direction is therefore given by $\boldsymbol{\gamma}^Y(\omega) = \begin{pmatrix} \cos(\psi-\omega) & \sin(\psi-\omega) \end{pmatrix}'$, the reflection of $\boldsymbol{\gamma}^X(\omega)$.

[46] Allowing for subject-level variation in the exaggeration direction does not lead to large drops in the expected off-ray residuals. Within the rotated treatments R(.6) and R(1), subject-level heterogeneity in the exaggeration direction accounts for 3 percent and 9 percent, respectively, of the observed magnitudes $\hat{\mathbb{E}}\,|\epsilon_{it}(\hat{\omega})|$.

TABLE C5. Sender Estimates

Variable	**R(0)**	**R(.6)**	**R(1)**†
Exaggeration angle, $\hat{\omega}$	0.000	0.009	0.034
	(0.000,0.000)	(0.000,0.021)	(0.018,0.058)
Ray exaggeration, $\hat{\mathbb{E}}\hat{\kappa}_{it}$	49.0°	57.8°	79.2°
	$(-60.0, 134.0)$	$(-24.6, 142.3)$	$(-32.7, 179.8)$
Error magnitude, $\hat{\mathbb{E}}\left\|\epsilon_{it}(\hat{\omega})\right\|$	8.3°	14.8°	23.0°
	$(0.0, 52.0)$	$(0, 56.3)$	$(0.0, 86.0)$
$\zeta_{\hat{\mathbf{\Gamma}}}(\mathbf{x},\mathbf{y})$ parameters, (α,β_1,β_2)	(1.00,0.00,0.00)	(0.74,0.45,0.43)	(0.50,0.54,0.47)
Attainable efficiency, Υ_S	88.4%	81.4%	71.5%
Loss from *Restriction B*, $\bar{\Upsilon}_S - \Upsilon_S$	8.4%	10.0%	9.2%

Note: 95 percent confidence intervals below the $\hat{\omega}$ estimate are calculated using a bootstrap of size 1,000; confidence intervals below $\hat{\kappa}$ and $|\hat{\epsilon}|$ reflect 95 percent coverage for residuals under $\hat{\omega}$ (two-sided for κ, one-sided for $|\hat{\epsilon}|$). †R(1) estimates include data from 24 subjects in the first part of treatment E(1).

very specific restriction, and a direct test of it is binary, *all* data either satisfies the restriction or the restriction fails. This is similar to results in choice, where *any* observed cycles in revealed preference lead to a falsification of rationality. In large datasets, such violations are almost always present, and the analysis switches to quantification of the degree of the violations. Our approach will be similar, where we will add the possibility of noise, and measure the effect of this noise against a benchmark.

Allowing for noise requires some pre-specification of the scale of this noise, so that a test might assess the probabilities the data is produced by this data generation process. Because of this we focus on criteria rather than statistical tests (much as principal component analysis does). Given our paper's aim, a useful way to understand the magnitudes of the departure from linear exaggeration, is through the effect on a receiver who believes that *Restriction B* holds, and by Proposition 1 follows a linear response. For example, in R(0), a receiver following the linear best-response to senders' exaggerations (identical to the equilibrium response here) would expect to have a final choice $8.3°$ distant from the true state in each issue, or, given the payoffs, a monetary loss of $2.50.

Moreover, to facilitate comparisons with the receiver analysis in section 4.2, we will show how deviations from *Restriction B* affect the attainable efficiency with the *Proposition 1* response by receivers. Given the best-fit exaggeration directions, the exaggeration basis $\hat{\mathbf{\Gamma}} = \left[\begin{array}{cc}\boldsymbol{\gamma}^X(\hat{\omega}) & \boldsymbol{\gamma}^Y(\hat{\omega})\end{array}\right]$, the sequentially rational response $\zeta_{\hat{\mathbf{\Gamma}}}(\mathbf{x},\mathbf{y})$ is summarized by three parameters, (α,β_1,β_2), which we list in Table C5. The efficiency level attainable given this response is

$$\Upsilon_S = 1 - \frac{\hat{\mathbb{E}}_{(\tilde{\mathbf{x}},\tilde{\mathbf{y}})}\left\|\zeta_{\hat{\mathbf{\Gamma}}}(\tilde{\mathbf{x}},\tilde{\mathbf{y}})\right\|}{\mathbb{E}\left\|\boldsymbol{\theta}\right\|},$$

where $\hat{\mathbb{E}}_{(\tilde{\mathbf{x}},\tilde{\mathbf{y}})}\left\|\zeta_{\hat{\mathbf{\Gamma}}}(\tilde{\mathbf{x}},\tilde{\mathbf{y}})\right\|$ is the sample-average distance between $\zeta_{\hat{\mathbf{\Gamma}}}$ and the true state, assessed across all treatment exaggeration pairs $(\tilde{\mathbf{x}},\tilde{\mathbf{y}})$, and $\mathbb{E}\left\|\boldsymbol{\theta}\right\|$ is the analytical receiver choice distance in a babbling equilibrium. This measure incorporates the deviations from linear exaggerations, and illustrates the efficiency attainable by a receiver following the Proposition 1 sequentially rational response.

The efficiency measure Υ_S is listed by treatment in Table C5. The \$2.50 expected loss from noise in R(0) is mirrored as an 11.6 percentage point efficiency loss. In R(.6) and R(1) the increased magnitude for residuals translate to increased efficiency losses, 18.6 and 28.5 percent, respectively (with expected monetary losses to the receivers of \$4.19 and \$6.13). The effects from sender deviations from the linear restriction are substantial, and by comparing the Υ_S measure to the observed efficiency measure Υ listed in Table II, we can infer that this noise in sender behavior accounts for a slightly less than half of the final efficiency losses in each treatment.

However, we argue that though there are efficiency losses from sender deviations from *Restriction B*, the extent to which receivers can do better with more sophisticated beliefs is small. To make this case we compare the efficiency attainable under the linear exaggeration restriction (Υ_S) to an unrestricted *non-parametric* best response ($\bar{\Upsilon}_S$). The unrestricted best response is

$$\bar{\zeta}(\mathbf{x},\mathbf{y}) = \arg\min_{\mathbf{z}} \hat{\mathbb{E}}_{\tilde{\mathbf{x}},\tilde{\mathbf{y}}} \left(\|\mathbf{z} - (\mathbf{x} - \tilde{\mathbf{x}})\| \,\middle|\, \tilde{\mathbf{y}} - \tilde{\mathbf{x}} = \mathbf{y} - \mathbf{x} \right),$$

where we calculate expectations by simulation from the empirical distributions for $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$. Given $\bar{\zeta}(\mathbf{x},\mathbf{y})$ we calculate the upper bound on efficiency given senders' response

$$\bar{\Upsilon}_S = 1 - \frac{\hat{\mathbb{E}}_{(\tilde{\mathbf{x}},\tilde{\mathbf{y}})} \left\| \bar{\zeta}(\tilde{\mathbf{x}},\tilde{\mathbf{y}}) \right\|}{\mathbb{E}\, \|\boldsymbol{\theta}\|}.$$

The last row in Table C5 provides the efficiency loss from maintaining *Restriction B*, the difference $\bar{\Upsilon}_S - \Upsilon_S$. In each of our treatments, receivers best responding to the assumption of linear exaggerations lose approximately ten percentage points from the attainable efficiency.

Appendix D. Extended Treatment Analysis

In the E(1) treatment the set of subjects $\mathcal{I}$ has 24 members. As a first approach we perform the same within-issue exercise as section 4.3, and classify 14 of the 24 subjects as taking a midpoint in issue 1 (the opposed issue) and shading from the minimal recommendation in issue 2 (the aligned issue). We partition the set of subjects into two subgroups, where $\mathcal{W}$ captures the classified subjects and those unclassified are in $\mathcal{I} \setminus \mathcal{W}$.[47]

For each round t, let the median subject in subgroup $\mathcal{G}$'s efficiency level be$\Upsilon^{t}_{.5}(\mathcal{G})$. Table D6 provides averages for the median subject's efficiency in each group across all rounds, and across blocks of five rounds in the first four columns. Before we discuss the figures in the table we make a few observations that will be useful to assess the evolution of efficiency levels. Because all 24 subjects in E(1) faced the common state/recommendation sequence $\{\boldsymbol{\theta}_t, \mathbf{x}_t, \mathbf{y}_t\}_{t=16}^{30}$, the selected order for the sequence may itself have an effect in the way that median efficiency evolves. Contrarily, in the last five rounds of R(1) subjects faced information from earlier rounds of the experiment in a random order. Consequently, as a reference, the last two columns of the table provide the median efficiency and interquartile range for the 45 subjects in the R(1) treatment, averaged over rounds 16–20.

Inspecting the three columns that show the five round blocks, there seems to be an increasing efficiency level. Moreover, breaking the analysis up to look at the subgroups $\mathcal{W}$ and $\mathcal{I} \setminus \mathcal{W}$, it is clear that the increasing efficiency is primarily driven by subjects in the former subgroup, as the median subject in $\mathcal{I} \setminus \mathcal{W}$ has a decreasing efficiency level across the blocks. However, the data suggest that the improvement in efficiency is not related to subjects making a connection across dimensions, but is rather an outcome of the particular sequence that subjects faced in E(1). First, if the random variation in R(1) and E(1) are comparable, the expected efficiencies will be centered at 48.3 percent. However, the final column illustrates the interquartile range for the 5-rounds efficiency averages in R(1), showing substantial variation across subjects, even were that subject to follow the equilibrium response. Observed increases could simply be the product of the specific sequence randomly chosen for E(1). To control for this variation more carefully, we will examine two counterfactuals, and compare subject's results to these baselines.

The first counterfactual is our proxy for a within-issue strategy:

$$\bar{\zeta}\left(\mathbf{x}, \mathbf{y}\right) = \begin{pmatrix} \zeta_{\text{Avg}}\left(x_1, y_1; \alpha = \tfrac{1}{2}\right) & \zeta_{\text{Min}}\left(x_2, y_2; \eta = -45\right) \end{pmatrix}'.$$

That is, choose midpoint in issue 1, and subtracts 45° from the minimal recommendation in issue 2. The second counterfactual computes the equilibrium response $\zeta^\star(\mathbf{x}_t, \mathbf{y}_t)$, which incorporates across-issue information according to the the equilibrium strategy. Table D6 provides the efficiency levels these two counterfactual strategies would produce given the precise sequence of states/recommendations in E(1). Both strategies yield large increases across the three blocks of five rounds, with the equilibrium strategy doing somewhat better. Controlling for the information available, it seems that subjects actually do worse as the session evolves: in the first block subjects in $\mathcal{W}$ do slightly better than the within-issue strategy $\bar{\zeta}\left(\mathbf{x}, \mathbf{y}\right)$, but are doing worse than this

[47]The classification is somewhat forgiving, allowing for an averaging parameter $\hat{\alpha}_i \in (0.4, 0.6)$ in issue 1, and a shading parameter $\hat{\eta}_i \leq 0$ in issue 2. This means that we exclude those following averages in issue 2. We require both issues to have a model-fit of $R_i^2 > 0.6$. We motivate the model-fit requirement by examining the parameters α_i and η_i (and the corresponding R_i^2) that the subjects followed the equilibrium response, $\zeta^\star(\mathbf{x}_{it}, \mathbf{y}_{it})$. The equilibrium response, when estimated through a (mis-specified) within-issue response attains an R^2 of 0.84 in issue 1 with an estimate $\alpha = 0.48$, in issue 2 the estimated parameter is $\eta = -57.5$, with a model R^2 of 0.88.

TABLE D6. Efficiency Across each Block of Five Rounds

Efficiency, Υ	**Median**	**Median**			**Median,**	**Interquartile Range**
	All Rounds	**16–20**	**21–25**	**26–30**	**R(1)**	**R(1)**
All Subjects, $\mathcal{I}$	51.8%	45.2%	48.6%	61.6%	48.3%	8.7%–71.7%
$\mathcal{W}$-Subjects	59.8%	51.1%	57.5%	70.9%	64.3%	40.0%–77.2%
$\mathcal{I} \setminus \mathcal{W}$-Subjects	29.0%	36.1%	28.2%	22.8%	30.0%	0.9%–67.5%
$\bar{\zeta}(\mathbf{x}, \mathbf{y})$	62.8%	48.5%	63.2%	76.7%	63.1%	43.2%–78.8%
$\zeta^{\star}(\mathbf{x}, \mathbf{y})$	71.6%	60.6%	72.5%	81.7%	72.1%	49.2%–90.3%

Note: For the subjects in R(1), 18 are selected into the group $\mathcal{W}$ out of the 45 in this treatment; due to variation of the messages/states selected for each subject in R(1) the figures for $\bar{\zeta}(\mathbf{x}, \mathbf{y})$ and $\zeta^{\star}(\mathbf{x}, \mathbf{y})$ represent the median/interquartile range for the counterfactual strategy averaged over the five rounds.

baseline in the final two blocks of five. A better comparison can be obtained by dropping the two most extreme efficiency outliers from the first and last blocks. Examining rounds 18, 19 and 20 in the first block, and comparing this to rounds 28, 29 and 30 in the last block, the within-issue strategy yields an efficiency of 64 percent across 18–20, and 66 percent over 28–30 (the equilibrium strategy yields 80 percent and 75 percent, respectively). The median subject in $\mathcal{W}$ (and in $\mathcal{I} \setminus \mathcal{W}$) achieves 71 percent (31 percent) efficiency across 18–20 , but just 61 percent (4 percent) in rounds 28–30. We therefore conclude that there is not compelling evidence for substantial improvement in subjects' final *outcomes* as E(1) proceeds, that if anything, the evidence suggests decreases.

Despite subjects not improving their final outcomes, it might be that the observed drops stem from subjects beginning to incorporate across-issue information, albeit too coarsely. Our focus now will be on tests for whether subjects use across-issue information, which we construct using the differences between their choices and the two counterfactual responses outlined above. To motivate this comparison, Figure D11 illustrates three rounds in E(1) where the differences are pronounced. In each of the three plots, the exaggerations of the two senders $\tilde{\mathbf{x}}_t$ and $\tilde{\mathbf{y}}_t$ are illustrated as gray and white points, respectively. Each subject's chosen point as the receiver ($\tilde{\mathbf{z}}_{it} = \mathbf{z}_{it} - \boldsymbol{\theta}_t$) is labeled with the numbers 1–24, where black numbers indicate subjects in $\mathcal{W}$, white numbers in $\mathcal{I} \setminus \mathcal{W}$. Finally, each plot illustrates the two counterfactual decisions: i) the fully-revealing equilibrium choice $\zeta^{\star}(\mathbf{x}_t, \mathbf{y}_t) - \boldsymbol{\theta}_t$, depicted as the black point; and ii) the within-issue strategy $\bar{\zeta}\,(\mathbf{x}_t, \mathbf{y}_t) - \boldsymbol{\theta}_t$ depicted as the midpoint of a line interval, representing issue-2 shading strategies with $\eta \in [-90, 0]$.

The third graph in the figure illustrates round 27, where the senders' exaggerations have approximately the same magnitude. In this round both the within-issue and equilibrium strategies do well, attaining 92 and 97 percent efficiency, respectively (indeed, we dropped round 27 as an outlier in the above discussion). Unsurprisingly, subjects in $\mathcal{W}$ do very well here, with average efficiency levels of 86 percent, while subjects in $\mathcal{I} \setminus \mathcal{W}$ attain an average efficiency of 42 percent.

In comparison to the third plot, the first two illustrate rounds where the two senders have asymmetric exaggeration magnitudes (most easily seen in the issue-2 difference, $\tilde{y}_2 - \tilde{x}_2$). In both cases sender *Y* has exaggerated by less, and the equilibrium strategy therefore skews towards Y's recommendation in the issue 1. However, the first two plots also vary in the sum of the exaggerations (seen in the separation between senders in issue 1, $\tilde{y}_1 - \tilde{x}_1$), with the first plot having a smaller separation. Because of the this variation in the across-issue difference, the equilibrium strategy in the first plot (round 24) shades less than the second plot (round 25).[48]

Examining the response in Figure D11, it is hard to distinguish a clear pattern of subject response favoring the equilibrium point over the within-issue strategy. In each of the three rounds the Y sender can be identified as exaggerating less by inspecting $y_{2t} - x_{2t}$, and the simplest pattern to look for is subjects clustered to the right of $\bar{\zeta}(\mathbf{x}_t, \mathbf{y}_t)$. From just these three rounds, there is little evidence for the majority of subjects locating in this direction.

Our formal tests for across-issue inference examine all fifteen rounds in the final part of E(1), and look for associations between subjects' deviations from the within strategy, given by $\bar{\boldsymbol{\epsilon}}_{it} := \mathbf{z}_{it} - \bar{\zeta}\,(\mathbf{x}, \mathbf{y})$, and the differences predicted by incorporating across-issue information as the equilibrium does, given by $\epsilon^{\star}_t := \zeta^{\star}(\mathbf{x}_t, \mathbf{y}_t) - \bar{\zeta}\,(\mathbf{x}, \mathbf{y})$. For each subject $i \in \mathcal{W}$ and issue

[48]The round 25 figure also indicates that larger separation between the two recommendations lead some subjects to choose the wrong issue 1 arc. That is, they choose the arc where the *X* sender is clockwise, and the *Y* sender counterclockwise. In addition, the figure illustrates the general finding that this type of mistakes is more commonly made by $\mathcal{I} \setminus \mathcal{W}$ subjects.

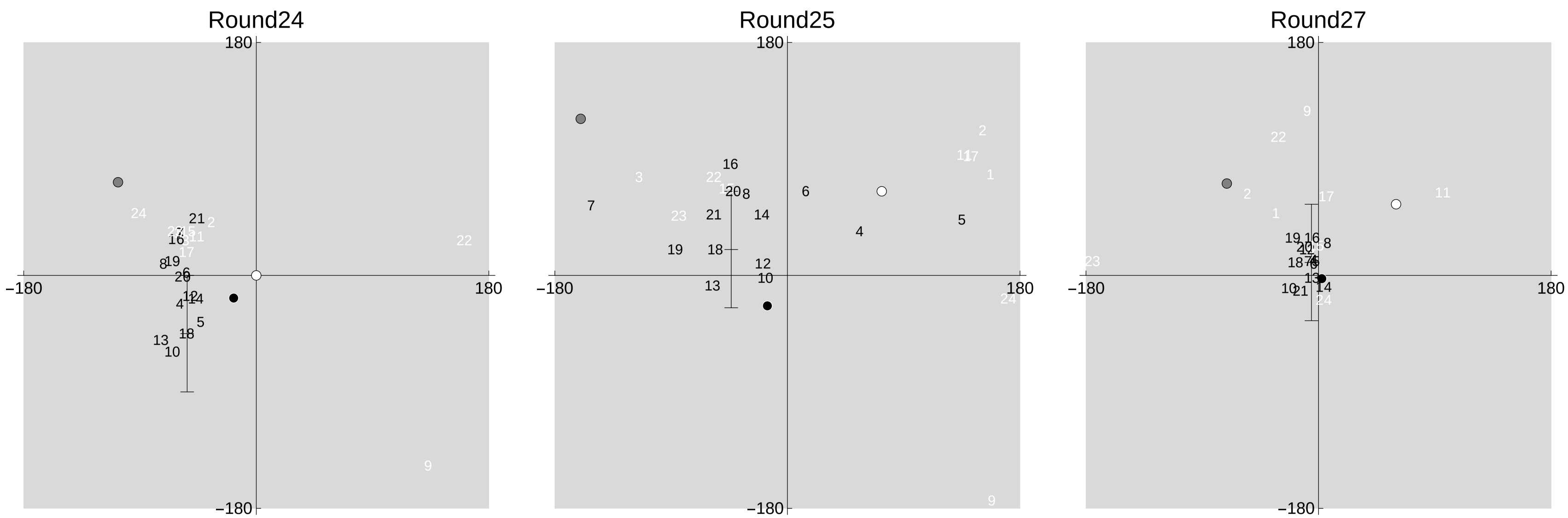

FIGURE D11. Subject Receiver Reactions, E(1)

Note: The true state is normalized to the origin. Gray point indicates $\tilde{\mathbf{x}}$, white point $\tilde{\mathbf{y}}$, black point indicates the equilibrium choice $\zeta_{\mathbf{\Delta}}(\tilde{\mathbf{x}}_t, \tilde{\mathbf{y}}_t)$, numbers represent subject choices $\tilde{\mathbf{z}}_{it}$, where numbers in black represent subjects in $\mathcal{W}$, and numbers in white those in $\mathcal{I} \setminus \mathcal{W}$. The black line indicates the $\zeta_{\text{Within}}(\mathbf{x}, \mathbf{y})$ choice, for the family of shading parameters $\eta \in [-90, 0]$.

$j \in \{1, 2\}$, we test a null hypothesis of statistical independence between $\bar{\epsilon}_{jit}$ and $\epsilon^{\star}_{jt}$, across rounds $t = 16, \ldots 30$.[49] Using a one-sided Goodman-Kruskal test—so that the alternative hypothesis specifies positive dependence—we reject independence for just two subjects in issue 1 (subjects 4 at the 10 percent level, and subject 14 at the 2 percent level) and four subjects in issue 2 (subject 16 at the 10 percent level, subjects 8, 12 and 18 at the 5 percent level).[50] There is some evidence then, that a minority of subjects begin to incorporate components of the available across-issue information. However, no subject shows a statistical relationship in both issues, with the closest being subject 14, for whom we can reject independence at the 15 percent level in issue 2. Examining just the final five rounds, 26–30, we can reject independence for four subjects in issue 1 (4, 8, 14 and 18) but we reject for no subject in issue 2.[51] Finally, we note that although a small minority does begin to adjust for across-issue inferences within their final choice, the precise level at which they do so is not close to the sequentially rational response.

[49]For subjects in $\mathcal{I} \setminus \mathcal{W}$ the test produces spurious correlations, as the common differenced component $\bar{\zeta}(\mathbf{x}_t, \mathbf{y}_t)$ does not provide a useful baseline against which to consider deviations.

[50]A Kendall's tau test rejects independence for the same subjects. Hoeffing's independence test (two-sided) rejects independence for two subjects in issue 1 (subject 4 and 12 at the 10 percent level, subject 14 at the 1 percent level), and fails to reject for all subjects in issue 2.

[51]This failure to reject in the second issue seems to be related to reduced variance in the second component of $\epsilon^{\star}_t$ in the last five rounds. All five final rounds have an *equilibrium response* adding between $-70°$ and $-49°$ to the minimal recommendation. In contrast, the previous five rounds, 21–25, have effective η ranging between $-116°$ and $-17.5°$.