

Exercise Sheet 3 on Feb 8, 2018

Calculus of Variations

Exercise 7 Show that

$$\log \log \frac{2}{|x|}$$

belongs to $W^{1,n}(B)$, where B is the n -dimensional unit ball. Also show that $f \notin L^\infty(B)$.

Exercise 8 Denote the variables in \mathbb{R}^n by (x_1, \dots, x_n) , and let

$$f_j(x) := \frac{1}{\left(|x_1|^2 + \dots + |x_j|^2\right)^{\frac{n}{2}}}$$

For which $p \in [1, \infty]$ is each $f_j \in W^{1,p}(B_1(0))$.

Exercise 9 Show that

$$f(x) := \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

does not belong to $W^{1,p}((-1, 1))$ for any $p > 1$.
