1. Complete these problems from the book:
   (a) P53: Exercise 2.1.3
   (b) P54: Exercise 2.1.15
   (c) P64: Exercise 2.2.7
   (d) P71: Exercise 2.3.7

2. Show that if \((x_n)_{n \in \mathbb{N}}\) is a convergent sequence then every subsequence of \((x_n)_{n \in \mathbb{N}}\) is also convergent. Moreover if
   \[ x := \lim_{n \to \infty} x_n \]
   then for any subsequence \((x_{n_i})_i\),
   \[ x = \lim_{i \to \infty} x_{n_i} \]

3. Let \((x_n)_{n \in \mathbb{N}}\) be a sequence and assume one of the following properties:
   (a) there is some \(x\) such that any subsequence \((x_{n_i})_i\) contains another subsequence \((x_{n_{ij}})_j\) which is convergent to \(x\).
   (b) any subsequence \((x_{n_i})_i\) contains another subsequence \((x_{n_{ij}})_j\) which is convergent (a priori not necessarily to the same \(x\))
   Show in which cases \((x_n)_n\) is convergent. Give a counterexample for the other cases.

4. (bonus) Find the following limit. Show all work.
   \[ \lim_{n \to \infty} \left( \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \ldots + \frac{1}{\sqrt{n^2 + 2n}} \right) \]