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Partial Differential Equations 1 – Spring 2019
Exercise Sheet 2 — Due Date: February 04

Work in groups, write in LaTeX!

Problem 5 Let $\Omega \neq \emptyset$ be a path-connected set, and assume $A \subset \Omega$ satisfies the following three conditions

- $A \neq \emptyset$
- A is relatively closed with respect to Ω
- A is relatively open with respect to Ω.

Show that $A = \Omega$.

Hint: You can employ the following strategy: Show that for any continuous path $\gamma : [0,1] \to \Omega$ with $\gamma(0) \in A$ we also have $\gamma(1) \in A$. Do this by computing the maximum value $t_0 \in [0,1]$ such that $\gamma([0,t_0]) \subset A$.

Problem 6 Prove the following statement that was left open in the proof of Theorem 2.5.

If $u \in C^2(\Omega)$ satisfies

$$\Delta u \leq 0 \quad \text{in } \Omega,$$

then

$$u(x) \geq \fint_{\partial B(x,r)} u(z)\, d\mathcal{H}^{n-1}(z) \quad \forall \text{ balls } B(x,r) \subset \Omega$$

Problem 7 In Corollaries 2.7 and 2.8 of the lecture we discussed the strong and weak maximum principle for subharmonic functions.

Formulate and prove a strong and weak *minimum* principle for superharmonic functions.

Problem 8 Prove Remark 2.10 from the lecture: Let $\Omega \subset\subset \mathbb{R}^n$ be a connected, bounded, open set and assume that $u \in C^2(\Omega) \cap C^0(\overline{\Omega})$ solves the problem

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ u = g & \text{on } \partial\Omega \end{cases}$$

for some $g \in C^0(\partial\Omega)$ which satisfies $g \leq 0$, but $g \not\equiv 0$.

Show that then $u < 0$ in all of Ω.
